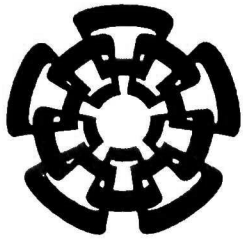




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## DIAGNOSTICO DE FALLAS EN SISTEMAS DE MANUFACTURA DISCRETOS

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**Mildreth Isadora Alcaraz Mejía**

para obtener el grado de:

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en la especialidad de:

**Ingeniería eléctrica**

Directores de Tesis

**Dr. Luis Ernesto López Mellado**

**Dr. Antonio Ramírez Treviño**

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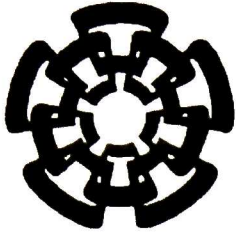
**Tesis de Maestria en Ciencias  
Ingeniería eléctrica**

Por:

**Mildreth Isadora Alcaraz Mejía**  
Ingeniero en Sistemas Computacionales  
Instituto Tecnológico de Colima 1995-2000

Directores de Tesis  
**Dr. Luis Ernesto López Mellado**  
**Dr. Antonio Ramírez Treviño**

CINVESTAV del IPN Unidad Guadalajara, Agosto del 2004.



# CINVESTAV

Centro de Investigación y de Estudios Avanzados del IPN  
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## **Petri Net-Based Fault Diagnosis of Discrete Manufacturing Systems**

A thesis presented by  
**Mildreth Isadora Alcaraz Mejía**

To obtain the degree of:  
**Master in Sciences**

On the subject of:  
**Electrical Engineering**

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**Petri Net-Based Fault Diagnosis of Discrete  
Manufacturing Systems**

Master of Sciences Thesis  
In Electrical Engineering

By:

**Mildreth Isadora Alcaraz Mejía**  
Computer Systems Engineer  
Instituto Tecnológico de Colima 1995-2000

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Thesis Advisors:

**Dr. Luis Ernesto López Mellado**  
**Dr. Antonio Ramírez Treviño**

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# Introduction

The dependency of the performance of a Flexible Manufacturing System (*FMS*) on the equipment efficiency and the process control, join with the constant increasing of the complexity of control systems leads to a higher demand on reliability and safety.

One of the most important tasks to improve reliability, safety and economy is the fault diagnosis and error recovery, which if they are accurate and on time can help to avoid system shutdown, breakdown, and even dangerous operations concerning human losses and material damage.

Fault analysis has been, and still is, a very active area of research both in industry and in academy. Therefore, several modelling methods and a variety of techniques to perform diagnosis have been proposed.

In the Discrete Event Systems (*DES*) context, Finite Automata (*FA*) has been used as modelling formalism and many results on diagnosis have being obtained for *DES* represented as *FA*. In [25] [26], Lin uses a nondeterministic Mealy automaton and a deterministic Moore automaton as models for off-line and on-line diagnostics respectively; the author proposes two algorithms, one to find a minimal observable event set and an algorithm to find a control string that diagnoses the system. However, this last algorithm has an exponential complexity. In [31], Magni, et al. propose a technique describing the components as Finite State Machine (*FSM*) models, but they use composition rules that lead in a state explosion, even when they consider temporal information. In [40] [41], Sampath et al. also use *FSM* as component modelling incorporating the sensor maps for off-line and on-line diagnosis with a finite delay, but the analysis is comparable in complexity with the reachability tree search. In [17], Garcia et al. apply the concepts in [40] [41] to introduce a centralized modular diagnoser using *FSM* as modelling technique.

Nevertheless, when the size of the system increases and its behaviour is complex, the size of *FA* models make the analysis prohibitive, for this reason, the study of diagnosis with an observability approach using Petri Nets (*PN*) as modelling formalism has emerged in the late years. In [18], Giua defines an observer based on the observation of a word of events using *PN* and he also presents algorithms for estimating the state considering two cases, then; from this estimate, he shows how to design a controller to ensure that forbidden markings are not reached but it is not optimal. In [14], Fanni et al. use the algorithms presented in [18] to estimate the actual marking of the system and to design the controller; though this controlled system may result blocked, hence the authors show with an example, but only with an example, how to introduce time-out mechanisms to recover from this blocking.

This thesis addresses the diagnosis problem in Flexible Manufacturing Systems (*FMS*), using Interpreted Petri Nets (*IPN*) as modelling formalism. This allows, in one hand, representing both measurable and non-measurable internal states, and on the other hand, provides

representations that are more compact. Besides the characterization from a language point of view of the diagnosability property, it is also presented a structural characterization, which reduces the complexity in the diagnosability analysis.

In addition, it is presented an extension of the modelling methodology that obtains binary *IPN* models presented by [38] to include faulty markings; it is a bottom-up modular construction technique. The study of diagnosability, the design of the diagnoser and the error recovery approach take advantage of this modelling technique.

Another contribution of this thesis is a fault recovery approach to complement the diagnosis approach modelled with Interpreted Re-Writing Nets (*IRWNets*), an extension of Re-Writing Nets (*RWNets*) [10] [11] based on *IPN*, a formalism introduced here that also helps in task reprogramming.

This thesis is organized as follows. Chapter I presents an overview of *FMS*, the concepts in the context of fault tolerance, and fault tolerance in *DES*. Chapter II presents the *PN* fundamentals, the *IPN* formalism and the extended modelling methodology. Chapter III defines the diagnosability property, its characterization from a language point of view and its structural characterization. Moreover, the design of a diagnoser net and the general scheme for diagnosis are also presented. Finally, chapter IV introduces the *IRWNets* definition and their dynamics in order to illustrate a fault recovery approach. Two cases of study to illustrate this approach and the task reconfiguration as a feature of the *IRWNets* formalism.

# Chapter 1

## Fault Tolerant Manufacturing Systems

---

**Summary.** *This chapter presents an overview of manufacturing systems, in particular, flexible manufacturing systems and their levels, components, processes and goals. It also presents some concepts in the context of fault tolerance as dependability, fault, error, failure concepts, fault types, fault attributes and fault tolerance goals. Finally, this chapter is focused in fault tolerance in discrete event systems, specially the diagnosis methods and related work.*

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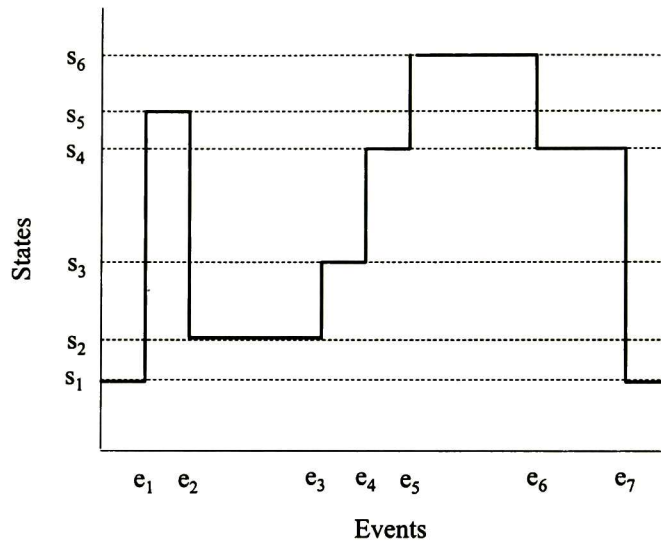


Figure 1.1: A DES Example

## 1.1 Manufacturing Systems

In Discrete Event Systems (*DES*), the state space of a system is described by a discrete set  $\{0, 1, 2, 3, \dots\}$ , and transitions are observed only at discrete points in time. These transitions are associated with "events" Informally, an event can be considered as an occurrence that produces a change from one state to another. The initial and final states may be the same or not.

The Discrete Event Dynamic Systems (*DEDS*), widely recognized by Discrete Event Systems, satisfy two main properties:

- 1) The state space is a discrete set.
- 2) The state transition mechanism is event-driven.

A formal definition of *DES* is presented in the following, and in Figure 1.1 a *DES* is shown:

**Definition 1.** A *Discrete Event System (DES)* is a discrete-state, event-driven system, that is, its state evolution depends entirely on the occurrence of asynchronous discrete events over time [12].

*DES* are common in the real world. Examples of *DES* are: Queuing Systems, Computer Systems, Communication Systems, Manufacturing Systems, Traffic Systems, Database Systems, Software Systems: Telephony, Monitoring and Control of Complex Systems, etc.

Flexible manufacturing systems, a specific type of manufacturing systems, is one form of automation which is an important solution to meet the three main goals managers look for in their enterprises: to increase productivity, to increase quality, and to reduce costs.

### 1.1.1 Manufacturing Systems Classification

In order to visualize **Flexible Manufacturing Systems (FMS)** with respect to all different types of Manufacturing Systems, the Manufacturing System Classification with respect to the shop architecture is presented below [43]:

- *Transfer Lines.* The process is a sequence of operations that take similar amounts of time to complete. Each workstation performs repetitively the same task. The main optimization problem is balancing, which means to make all the operation times in different workstations as similar as possible to reduce inactive times and to balance the workload of stations.
- *Production Lines.* There is some variability required in the workstations. Some intermediate buffers are introduced to filter out the variations. The buffer problem is critical, because large buffers intend to avoid blocking and starvation, but enhance the work in progress with the resulting economical problems.
- *Flow Shop.* There are some products which may be processed in a different way or pursue alternative paths. The sequencing problem tries to minimize inventory and production costs by finding a proper sequence of parts or lots, naturally controlled to satisfy the production demand.
- *Job Shop.* The variety of products in this kind of plant is greater. A production route is defined for each product. However, a greater flexibility results in a lower efficiency. The decision problem addresses the optimization while satisfying the production demands.
- *Flexible Manufacturing.* Capture the main advantages offered by the past production systems. Because this is a very important theme in this thesis, these systems will be widely described in the following.

### 1.1.2 Flexible Manufacturing Systems

In the middle of the 1960's, competition in the market became more intense and the main concern was cost. Later, the priority was quality. During the passing of time, the market became more and more complex, and speed of delivery became an extra concern customer also needed.

Therefore, companies had to look for a new strategy. This new strategy was called customizability, where companies have to adapt to the environment in which they operate, to be more flexible in their operations and to satisfy different market segments.

Thus, *FMS* is related to the effort of gaining competitive advantage.

The idea of an *FMS* was proposed in England in the 60's under the name *System 24*. It was named *System 24* referring to a flexible machining system that could operate without human operators 24 hours a day under computer control. Automation was the main goal of System 24.

Today, Flexible Manufacturing Systems (*FMS*) is a very interesting area and of great importance. Nevertheless, there is not an agreed definition of the term, but the following definition can be considered.



**Definition 2.** *The **Flexible Manufacturing System (FMS)** is a configuration of computer-managed numerical work stations where materials are automatically handled and loaded by machines [45].*

The word that makes the difference between *FMS* and traditional manufacturing systems is "flexible" which means that *FMS* has the flexibility to switch between two or more different parts for their process with little time lost.

## 1.2 Fault Tolerant Concepts

### 1.2.1 Dependability Concept

The dependability of a computer system characterizes its trustworthiness.

**Definition 3.** *A system is said to be **dependable** if the reliance can be placed on the service it delivers [24].*

The dependability attributes are: **availability** (readiness for usage), **reliability** (continuity of services), **safety** (non-occurrence of catastrophic consequences) and **security** (prevention of unauthorized access). These attributes can be considered in isolation but they can be interdependent. For example: if not reliable then not available, if unreliable then probably not safe, and so on.

In order to achieve the goal of dependability, an effort in all the phases of the system's development must be done [19]. At design time, the dependability can be increased through fault avoidance techniques. At implementation time, the dependability can be increased through fault removal techniques. At execution time, fault tolerance and fault evasion techniques are required.

### 1.2.2 Fault, Error and Failure

It is of great importance to have knowledge about the terminology used in the problem context, moreover, to make distinction among them. Therefore, the following basic concepts are presented [28].

**Definition 4.** *A **fault** is the original source of any problem. It may lead to an error directly or indirectly.*

**Definition 5.** *An **error** is the difference between what is specified and what is actually there. In a control system, an error is an observable difference between the actual state and the representation of the pretended state.*

**Definition 6.** *A **failure** occurs when an error affect the service delivered from a system. If the component that has the failure is part of a system, the result is a fault in the system which contains the failing component.*

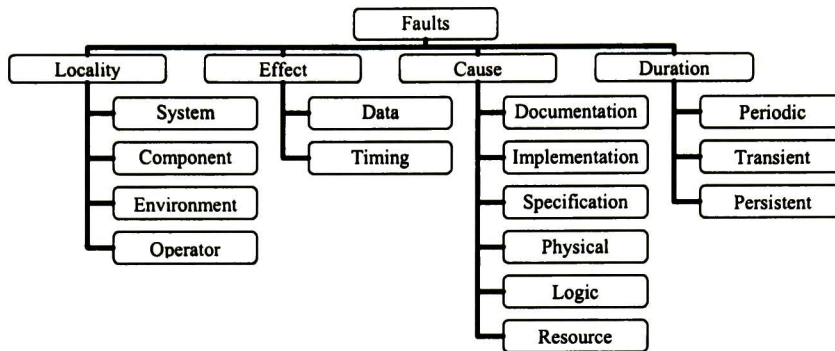


Figure 1.2: Fault Types

### 1.2.3 Fault Types

Since it is not possible to consider all existent faults, it is preferred to classify them based on locality, effect, cause, duration, among others [19]. This classification is shown in Figure 1.2.

### 1.2.4 Fault Attributes

#### Fault Observability

**Definition 7.** A *fault* is said to be **observable** if the system interface shows a *symptom*.

**Definition 8.** A *symptom* is information indicating the existence of a fault. A symptom may be an observed fault or failure, or it may be a change in the system behaviour although it still meets its specification [19].

**Definition 9.** If a fault is found by a fault tolerant mechanism (described later), this fault has been **detected**, otherwise is **latent**, whether it is observable or not.

#### Fault Propagation

**Definition 10.** A *fault* is said to be **active** if its propagation generates more faults or failures, otherwise, it is said to be **dormant**.

When a fault is detected in the system, it is very important to block its propagation, because if not, it may conclude in a chain reaction causing a catastrophe.

### 1.2.5 Fault Tolerant System Concept

Fault tolerance is one way to achieve dependability, which means that it is one way for the system to become available, reliable, safe, and secure. Fault tolerance can be applied in

three levels [19]:

- *Hardware Level*, where fault tolerance has been used to compensate for faults in computing resources.
- *Software Level*, where Software fault tolerance compensates for faults such as changes in program or data structures due to transients or design errors.
- *System Level*, where the computer subsystem may provide functions that compensate for faults in other systems components that are not computer-based.

**Definition 11.** *Fault Tolerant Systems are systems which are able to handle an error or fault without affecting the service delivered [28].*

Fault tolerance involves:

1. *Fault detection.* A fault has occurred.
2. *Fault diagnosis.* What caused the fault.
3. *Fault isolation.* Prevents the propagation of faults.
4. *Fault masking.* Insuring that only correct values are passed to the system boundary.
5. *Fault compensation.* Provide a response to compensate for output of the faulty system.
6. *Fault Repair.* Faults are removed from a system. (Recovery process).

These steps may be covered or synthesized by the following:

1. **Fault Detection.** The process of observing the actual state of the controlled system and comparing it with its specifications in order to find discrepancies as early as possible.
2. **Fault Diagnosis.** The process of finding the original fault which caused the error or which can cause an error.
3. **Fault Recovery.** The process of applying proper corrective actions in order to prevent a possible future error, or reach an error free state.

In each of these subfields there are several principles to achieve these objectives, as well as different methods for representing the information needed. It will be discussed below.

## 1.2.6 Redundancy Management

Fault tolerance is sometimes called Redundancy Management; it is because Redundancy is the provision of functional capabilities that would be unnecessary in a fault-free environment.

The measure of success of fault tolerance is coverage (the probability of a system failure given that a fault occurs). Simplistic estimates measure redundancy by counting the number of redundant success paths in a system, which can be modelled by Markov models, or Petri Nets.

There exist several types of redundancy:

- *Space redundancy*, provides separate physical copies of a source, function, or data item.
- *Time redundancy*, if the expected faults are transient, a function can be rerun with a stored copy of the input data, and if information shifted in time can be analyzed for unwanted changes, it can be corrected to its original value.
- *Fault containment regions*, with few or no common dependencies between them, to prevent propagation.

## 1.2.7 Fault tolerance goals

The main goal of fault tolerance is to prevent faults from propagating to the system boundary, where it becomes observable and, hence, a failure. In general, the further a fault has propagated, the harder it is to deal with. Since fault tolerance is redundancy management, however it becomes a matter of the degree of redundancy desired. Finally, it is important to note that dealing with faults earlier rather than later may be the difference in the service the system delivers.

# 1.3 Fault Tolerance in Discrete Event Systems

## 1.3.1 Diagnosis Methods

The problem of fault diagnosis has received considerable attention in the literature of reliability engineering, control, and computer science, and several methods and techniques have been proposed.

The system is usually modelled using artificial intelligence and mathematical methods. These artificial intelligence methods are those knowledge-based [4], and agent-based [42], where the system behaviour is specified in form of rules and facts, obtained from empirical human observation. Mathematical methods are those process-model-based where the system behaviour is represented as a mathematical model such differential equations [21], state space models [21], FA-model [22] [25] [26], FSM-model [31] [40] [41], PN-model [14] [18] [34], CPN-model [7], and so on.

There exist a variety of studied techniques to perform diagnosis on the modelled system, which can be classified in two main groups: artificial intelligence and mathematical techniques. Artificial intelligence techniques are expert systems for inference [4] [27], neural network classifier [27], fuzzy logic for inference [27], and adaptive or neuro-fuzzy system for

inference [27]. Mathematical techniques are signal models [20] [21] [30], parameter estimation [20] [21], state estimation [18] [20] [21] [22], observers [14] [21] [40] [41], parity equations [21], change detection [21] [34], symptom generation [21], modularity [17], fault trees [21], statistical classifiers [27], geometric classifiers [27], polynomial classifiers [27] and fuzzy logic for inference [27].

### 1.3.2 Related Work

The problem of diagnosis has been investigated by several researchers since different points of view.

#### Diagnosis approaches

- In [25], Lin studies off-line and on-line diagnostics. On-line diagnostic properties are state changing, events not all controllable, observation restricted to outputs, and constraint test sequence. While off-line diagnostic properties are state not changing, events all controllable, observation not restricted to outputs, and not constraint test sequence. He uses a nondeterministic Mealy automaton and a deterministic Moore automaton as models for off-line and on-line diagnostics, respectively, as first component, and a set of controllable events as second component. In addition, he partitions the state space into disjoint subsets, as desired. For off-line diagnostics, Lin assumes that the outputs are events observed and that all events are controllable. Based on the observable events and the desired partition, the state space is partitioned under every possible event. The conjunction of all these partitions must be finer than or as fine as the desired partition in order to determine a positive diagnosability. For on-line diagnostics, not all events are controllable. Lin says that a string of controllable events diagnoses a system with respect to a desired partition, if every pair of reached states having the same outputs belongs to the same desired partition. Therefore, if there exists such string then the system is on-line diagnosable. In [26], this theory is applied to an electronic engine control circuit. *Advantages:* Lin studies on-line and off-line diagnostic, and proposes an algorithm to find a minimal observable event set and an algorithm to find a control string that diagnoses the system. *Disadvantages:* Lin studies on-line and off-line diagnostics separately. For on-line diagnostics, he uses partial state observation and no event observation. For off-line diagnostics, he uses no state observation and partial event observation. The algorithm to find a control string has complexity of  $O(2^{|Q|})$  ( $Q$  is the set of states) in its worst case because is a reachability search.
- In [31], Magni et al. present a methodology for the development of a diagnostic system for complex industrial plants. The proposed technique consists in describing all the elements of the plant and the diagnostic system (plant, devices, sensors, actuators, diagnostic tests, hardware and analytical redundancies) as FSM models represented as event/state and output matrices. Then, they present a formal composition rule under the assumption that faults can occur only one at a time limiting the dimension of the resultant FSM model. This resultant output matrix, called matrix of residuals or fault signature, shows if faults are not distinguishable. If so, authors consider temporal

information and present an algorithm to diagnose the system using the minimum and maximum times of the event or fault to occur and its detection by an alarm.

### Diagnosability and observability approaches

- In [40] [41], Sampath et al. propose a DES approach to the problem of failure diagnosis, where the system is modelled as a FSM that describes normal and failed behaviour, which belong to the event set. If the system has more than one component, first, they build the model of every component and then, they compose these models. With the composite model and sensor maps, they generate the final composite system model including interactions among components and incorporating the sensor maps. Authors introduce the notion of diagnosability and I-diagnosability of DES. Roughly speaking, a system is diagnosable if it is possible to detect with a finite delay occurrences of failure events using the record of observed events. In addition, a system is said to be I-diagnosable if it is possible to diagnose failures, not always, but whenever the failure events are followed by certain observable indicator events that are associated with the failures. They also introduce the diagnoser which is a FSM built from the system model. They first assume that the system starts in a normal state, so they add a label N (Normal). They use the next observed event to make the transition to another state which it includes all possible states every one with its own label (N=Normal, F+#of fault=Fault, and/or I+#of indicator=Indicator) and so on. They use the diagnoser to perform on-line diagnosis. Moreover, they present additional modifications on the diagnoser in order to perform off-line diagnosis.
- In [17], García et al. present a modular decomposition as an approach for failure diagnosis based on DES. The methodology presented is based on DES and on the diagnoser concepts introduced by [40] [41], and starts with the individual modelling as FSM of all components that form the system. In order to introduce modular decomposition, authors present a new architecture, where the system is decomposed in subsystems, each one with its minimum local controller, which send information to a centralized modular diagnoser. This modular composition must consider the functional interrelation and the dimension of the models in order to have as low dependability [19] as possible. Authors also present a solution for the coupling problem of diagnoser that consists of including a conjunction function working only in coupling situations. This function uses non local information coming from the subsystem where a failure has taken place.

### Observability and state estimation approaches

- In [18], Giua defines an observer as the system that computes the estimate of the actual marking of the net based on the observation of a word of events. Two algorithms are presented for estimating the state of the system assuming that only the net structure is known and considering two cases: when the initial marking is unknown and when the initial marking is a macromarking. The estimate computed as a result of these algorithms is always a lower bound on the actual marking of the net. Giua shows how this resultant estimate may be used to design a controller. This controlled is

called state feedback control loop with observer which ensures that the system never enters a set of forbidden states, assuming complete controllability. The controller uses an algorithm to ensure that forbidden markings are not reached. *Advantages:* It is possible to estimate the actual marking event when the initial marking is unknown or it is a macromarking. *Disadvantages:* The algorithm used by the controller to ensure that forbidden markings are not reached is not optimal; it may also prevent transition firings that lead from legal markings to legal markings.

- In [14], Fanni et al. use the algorithms presented in [18] to estimate the actual marking of the system based on event observations. Therefore, they use this estimate to design a state feedback controller as described in [18]. The controlled system may result blocked, as a result of the estimate, hence the authors show with an example how to introduce time-out mechanisms to recovery from this blocking. This time-out mechanism consists in adding a place and a time-out transition. Thus, the timed transition will fire only if no other transition is enabled. Then, the observer, regardless of its previous marking, reconstructs its marking. *Advantages:* The consideration of mutual exclusion constraints as state specifications of the desired behaviour. *Disadvantages:* The authors do not present a general approach to automatically construct the time-out structure, they describe only an example.

# Chapter 2

## Modelling FMS with PN

---

**Summary.** *This chapter presents the PN fundamentals: the PN definition, their properties, and their implication in FMS. The IPN definition is also presented, an extension of PN, used as modelling formalism allowing the representation of measurable and non-measurable states and providing more compact representations than other formalisms. Finally, an extension of a modelling methodology for DES that obtains binary IPN models with faults is also presented.*

---



## 2.1 Petri Net Fundamentals

### 2.1.1 PN Definition

**Definition 12.** A *Petri Net structure*  $G$  is a bipartite digraph represented by the 4-tuple  $G = (P, T, I, O)$  where [29] [39].

- $P = \{p_1, p_2, \dots, p_n\}$  is a finite set of vertices called **places**,
- $T = \{t_1, t_2, \dots, t_m\}$  is a finite set of vertices called **transitions**,
- $I : P \times T \longrightarrow \mathbb{Z}^+$  is a function representing the **weighted arcs** going from places to transitions, and
- $O : P \times T \longrightarrow \mathbb{Z}^+$  is a function representing the **weighted arcs** going from transitions to places.  $\mathbb{Z}^+$  is the set of nonnegative integers.

Usually  $\cdot t_j$  denotes the set of all places  $p_i$  such that  $I(p_i, t_j) \neq 0$  and  $t_j \cdot$  the set of all places  $p_i$  such that  $O(p_i, t_j) \neq 0$ . Analogously,  $\cdot p_i$  denotes the set of all transitions  $t_j$  such that  $O(p_i, t_j) \neq 0$  and  $p_i \cdot$  the set of all transitions  $t_j$  such that  $I(p_i, t_j) \neq 0$ . Graphically, places are represented by circles, transitions by rectangles, and arcs are depicted as arrows.

The **incidence matrix** of  $G$  is  $C = [c_{ij}]$ , where  $c_{ij} = O(p_i, t_j) - I(p_i, t_j)$ . The **marking** function  $M : P \longrightarrow \mathbb{Z}^+$  is a mapping from each place to the nonnegative integers representing the number of tokens (depicted as dots) residing inside each place. The marking of a PN is usually expressed as an  $n$ -entry vector.

**Definition 13.** A *p-invariant*  $Y$  of a PN is a rational-valued solution of equation  $Y^T C = 0$ . The **support of a p-invariant**  $Y_i$  is the set  $\|Y_i\| = \{p \mid Y_i(p) \neq 0\}$ .

**Definition 14.** A *Petri Net system* (PN) is the pair  $N = (G, M_0)$ , where  $G$  is a PN structure and  $M_0$  is an initial token distribution.

In a PN system, a transition  $t_j$  is **enabled** at marking  $M_k$  if  $\forall p_i \in P, M_k(p_i) \geq I(p_i, t_j)$ . An enabled transition  $t_j$  can be **fired** reaching a new marking  $M_{k+1}$  which can be computed as  $M_{k+1} = M_k + C v_k$ , where  $v_k(i) = 0, i \neq j, v_k(j) = 1$ , this equation is called the **PN state equation**. The **reachability set** of a PN is the set of all possible reachable marking from  $M_0$  firing only enabled transitions; this set is denoted by  $\mathbf{R}(G, M_0)$ .

**Example 15.** Consider the PN of figure 2.1. For this net the PN System  $N = (G, M_0)$ , where  $G = (P, T, I, O)$  and  $M_0$  are given below:

- $P = \{p_1, p_2, p_3, p_4, p_5\}$ ;
- $T = \{t_1, t_2, t_3, t_4, t_5\}$ ;
- $I(p_1, t_1) = 1, I(p_5, t_1) = 1, I(p_2, t_2) = 1, I(p_3, t_3) = 1, I(p_5, t_3) = 1, I(p_4, t_4) = 1, I(p_3, t_1) = 1, \text{ and } I(p_i, t_j) = 0 \text{ for the rest of pairs } (p_i, t_j), \text{ with } i, j = 1, \dots, 5$ ;
- $O(p_2, t_1) = 1, O(p_3, t_2) = 1, O(p_5, t_2) = 1, O(p_4, t_3) = 1, O(p_1, t_4) = 1, O(p_5, t_4) = 1, O(p_1, t_5) = 1, \text{ and } O(p_i, t_j) = 0 \text{ for the rest of pairs } (p_i, t_j), \text{ with } i, j = 1, \dots, 5$ ;

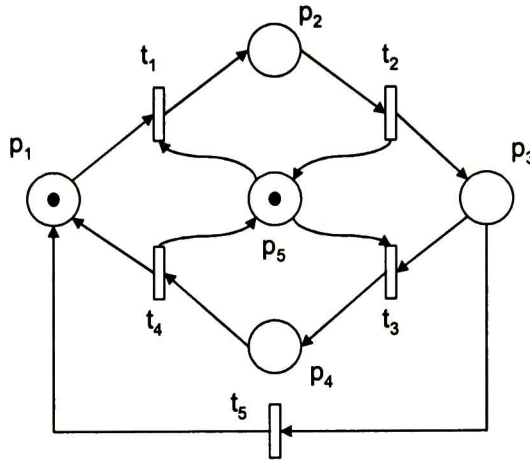


Figure 2.1: A Petri Net

- $M_0(p_1) = 1, M_0(p_5) = 1$  and  $M_0(p_2) = M_0(p_3) = M_0(p_4) = 0$ .

The input function matrix is  $I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$

The output function matrix is  $O = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

The incidence matrix is  $C = \begin{bmatrix} -1 & 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 \\ -1 & 1 & -1 & 1 & 0 \end{bmatrix}$

The initial marking is  $M_0 = [1 \ 0 \ 0 \ 0 \ 1]^T$

There is only one enabled transition, which is transition  $t_1$ . After the firing of the transition  $t_1$ , the resulting PN marking is  $M_1 = [0 \ 1 \ 0 \ 0 \ 0]^T$  where the next enabled transition is  $t_2$ . If transition  $t_2$  is fired, the resulting PN marking is  $M_2 = [0 \ 0 \ 1 \ 0 \ 1]^T$

### 2.1.2 PN Properties

Petri nets models may exhibit several properties: *Behavioural properties* that depend on the initial marking of a PN, and *Structural properties* that do not depend on the initial marking of a PN, but they do depend on the PN topology.

Let us review the most important properties.

**Definition 16.** Given a  $N = (G, M_0)$ , a marking  $M$  is **reachable** from marking  $M_0$  if there exists a sequence of transitions firings that transforms  $M_0$  to  $M$ . The reachability set  $\mathbf{R}(G, M_0)$  is formed by all the reachable markings  $M_i$  from  $M_0$ .

Reachability is considered a behavioural property, since it depends on the initial marking.

**Definition 17.** Given a  $N = (G, M_0)$ , and its reachability set  $\mathbf{R}(G, M_0)$ , a place  $p \in P$  is **B – bounded** if  $M(p) \leq B, \forall M \in \mathbf{R}(G, M_0)$ , where  $B$  is a positive integer. PN is **B – bounded** if each place in  $P$  is **B – bounded**. **Safeness** is **1 – bounded**.  $G$  is **structurally bounded** if  $G$  is bounded given any finite initial marking  $M_0$ .

**Definition 18.** A **transition**  $t$  is **live** if at any marking  $M \in \mathbf{R}(G, M_0)$ , there is a sequence of transitions whose firing reach a marking that enables  $t$  or, that there is a transition firing sequence that includes  $t$ . A PN is **live** if every transition in it is live. A PN is **structurally live** if there is a finite initial marking that makes the net live.

**Definition 19.** A **transition**  $t$  is **dead** if there is an  $M \in \mathbf{R}(G, M_0)$ , such that there is no sequence of transition firings to enable  $t$  starting from  $M$ . A PN contains a **deadlock** if there is an  $M \in \mathbf{R}(G, M_0)$  at which no transition is enabled. Such a marking is called a **dead marking**.

**Definition 20.** A PN **System**  $N = (G, M_0)$  is said to be **reversible** if  $\forall M \in \mathbf{R}(G, M_0)$ ,  $M_0 \in \mathbf{R}(G, M)$ . That is, the fact that from every marking reached from the initial marking, there is a transition firing sequence that ends up with the initial marking. A PN is said to be **structurally reversible** if there is a finite initial marking that makes the net reversible.

The properties just mentioned (Boundedness/safeness, liveness, reversibility) are independent each other. A PN may have or not any of them.

**Definition 21.** A PN with an initial marking  $M_0$  is strictly **conservative** iff  $\forall M \in \mathbf{R}(G, M_0)$ ,

$$\sum_{i=1}^{|p|} M(p_i) = \sum_{i=1}^{|p|} M_0(p_i)$$

This means that  $|t| = |t \cdot| \forall t \in T$ .

In other words, a PN is conservative if the number of tokens in the initial marking does not change through the firing of transitions.

**Definition 22.** A PN is **repetitive** iff there exists a marking  $M$  and a firing sequence  $s$  such that:

- After the firing of the sequence  $s$  from  $M$ , it ends up in  $M$ . This is  $M \xrightarrow{s} M$ .
- Every transition occurs in  $s$ .

**Definition 23.** A PN is said to be **consistent** if there exists a marking  $M$ , and a firing sequence  $s$  from  $M$  that ends up with  $M$ , such that every transition occurs at least once in  $s$ .

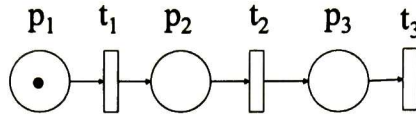


Figure 2.2: Sequential Relation

### 2.1.3 PN's interpretation in FMS

According with [45], there are two interpretations of  $PN$  in the context of  $FMS$ . In the *first interpretation*, **places** model resource status and operations, **transitions** model the start and/or end of operations, **directed arcs** model material, resource, information, and/or control flow direction, and **tokens** model material, resources, or information. In the *second interpretation*, **places** model resource status and conditions, **transitions** model operations, processes, activities, and events, **directed arcs** model material, resource, information, and/or control flow direction, and **tokens** model material, resources, or information.

There exists a modelling convention with  $PN$  for  $FMS$ . For every concept in manufacturing, a  $PN$  modelling is presented in the following [45]:

- *Moving of production lot size.* Weight of directed arcs models the production size that must be moved.
- *Number of resources (AGVs, machines, workstation, and robots).* The number of tokens in places models quantity of the corresponding resource.
- *Capacity of a workstation.* The number of tokens in places models the availability of the workstation.
- *Work-in-process.* The number of tokens in places models the buffers and operations of all machines.
- *Production volume.* The number of tokens in places models the counter for or the number of firings of transitions modelling the end of a product.
- *The time of an operation (setup, processing, and loading).* Time delays associated with the place or transition modelling the operation.
- *Conveyance or transportation time.* Time delays associated with the directed arc, place or transition models the conveyance or transportation.
- *System state.*  $PN$  marking (plus the timing information for timed  $PN$ )
- *Sequence.* One operation follows the other. It is shown in 2.2.
- *Concurrency.* Two operations are initiated by one event forming one parallel structure starting with a transition. It is shown in 2.3
- *Conflict.* If any of two operations can indistinctly follows an operation. It is shown in 2.4

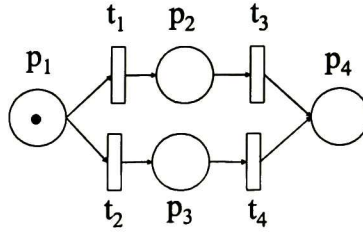


Figure 2.3: Concurrent Relation

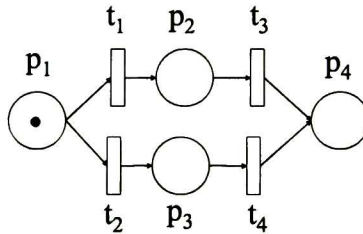


Figure 2.4: Conflicting Relation

- *Mutually exclusive.* Processes can not be performed at the same time due to a shared resource. There are parallel and sequential mutual exclusion shown in 2.5 and 2.6, respectively.

The *PN* properties have an interpretation in the context of the modelled manufacturing systems and it is described below [45]:

- *Reachability.* A certain state can be reached from the initial conditions.
- *Boundedness.* No capacity (of buffer, storage area, and workstation) overflows.
- *Safeness.* Availability of a single resource; or no request to start an ongoing process.
- *Conservativeness.* Conservation of non-consumable resources, as machines and AGVs.
- *Liveness.* Freedom from deadlock and guarantee the possibility of a modelled event, operation, process or activity to be ongoing.

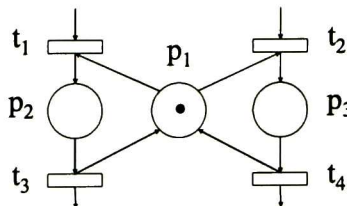


Figure 2.5: Parallel Mutual Exclusion Relation

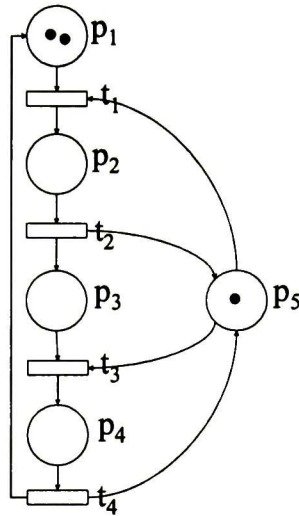


Figure 2.6: Sequential Mutual Exclusion Relation

- *Reversibility.* Re-initialization and cyclic behaviour.
- *Repetitiveness.* Existence of repetitive operations/activities/events for some marking.
- *Consistency.* Existence of cyclic behaviour for some marking.

See [33] for a deeper revision of properties, analysis and applications of PN.

### 2.1.4 IPN Formal Definition

Now, Interpreted Petri Nets (*IPN*) [1] [2] [32], the extension to *PN* used in this work as modelling formalism, is given. This extension allows to associate input and output alphabets to *PN* models.

**Definition 24.** An *Interpreted Petri Net (IPN)* is the 5-tuple  $Q = (N, \Sigma, \Phi, \lambda, \varphi)$  where

- $N = (G, M_0)$  is a **PN system**,
- $\Sigma = \{\alpha_1, \alpha_2, \dots, \alpha_r\}$  is the **input alphabet** of the net, where  $\alpha_i$  is an **input symbol**,
- $\Phi = \{\zeta_1, \zeta_2, \dots, \zeta_n\}$  is the **output alphabet** associated to places, where  $\zeta_i$  is an **output symbol**,
- $\lambda : T \rightarrow \Sigma \cup \{\varepsilon^i\}$  is a **labelling function** of transitions with the following restriction:
  - $\forall t_j, t_k \in T, j \neq k$  if  $I(p_i, t_j) = I(p_i, t_k) \neq 0$  and both  $\lambda(t_j), \lambda(t_k) \neq \varepsilon^i$ , then  $\lambda(t_j) \neq \lambda(t_k)$ . In this case  $\varepsilon^i$  represents an **internal system event**,
- $\varphi : \mathbf{R}(G, M_0) \rightarrow \{\Phi\}^q$  is an **output function**, where  $\mathbf{R}(G, M_0)$  is the **reachability set** defined as in the *PN* and  $q$  is the number of available outputs associated to places.

**Remark.** From now on,  $(Q, M_0)$  will be used instead of  $Q = (N, \Sigma, \Phi, \lambda, \varphi)$  to emphasize the fact that there is an initial marking in an IPN.

**Remark.** The function  $\varphi$  is linear and can be represented as a  $q \times n$  matrix.  $\varphi = [\varphi_{ij}]$ , where the  $i$ -th row vector  $\varphi_i$  is the transpose of an elemental vector  $e_j$ , and  $n$  is the number of places.

**Definition 25.** A transition  $t \in T$  is said to be **controllable**, if  $\lambda(t_j) \neq \varepsilon^i$ , and **uncontrollable**, otherwise.

**Definition 26.** A place  $p_i \in P$  is said to be **measurable**, if  $\exists j \in \{1, 2, \dots, r\}$  such that  $\varphi_j = e_i^T \neq 0$ , where  $\varphi_j$  is the  $j$ -th row of  $\varphi$ ; otherwise,  $p_i$  is called **non-measurable**. The set  $P_m = \{p \mid \exists j \in \{1, 2, \dots, r\} \text{ such that } \varphi_j = e_i^T \neq 0\}$  is the **set of measurable places** and  $P_{nm} = P \setminus P_m$  is the **set of non-measurable places**.

**Remark.** A measurable place is depicted as an unfilled circle, while a non-measurable place is depicted as a filled circle.

Let  $\sigma = t_i t_j t_k \dots$  be a firing transition sequence. The **Parikh vector**  $\vec{\sigma} : T \rightarrow \mathbb{Z}^+$  of  $\sigma$  maps every transition  $t$  of  $T$  (the transitions of the net) to the number of occurrences of  $t$  in  $\sigma$ . In an IPN, a transition  $t_j \in T$  is **enabled** at a marking  $M$  if  $\forall p_i \in P, M(p_i) \geq I(p_i, t_j)$ . However, if  $t_j$  is an uncontrollable transition then  $t_j$  can be **fired**; otherwise, the input symbol  $\lambda(t_j) = \alpha_i \neq \varepsilon$  must be present in order to **fire**  $t_j$ . Using the Parikh vector, the **state equation** of an IPN can be written as:

$$\begin{aligned} M_{k+1} &= M_k + C \vec{\sigma} \\ y_k &= \varphi M_k \end{aligned} \tag{2.1}$$

This work assumes that just one transition is fired at a time.

**Definition 27.** Let  $(Q, M_0)$  be an IPN. The **firing language** of  $(Q, M_0)$  is  $\mathcal{L}(Q, M_0) = \{\sigma \mid \sigma = t_i t_j \dots t_k \wedge M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \dots \xrightarrow{t_k} M_k\}$ .

**Definition 28.** The **input language** of  $(Q, M_0)$  is  $\mathcal{L}_{in}(Q, M_0) = \{\lambda(t_i)\lambda(t_j)\dots\lambda(t_k) \mid t_i t_j \dots t_k \in \mathcal{L}(Q, M_0)\}$ . The **output language** of  $(Q, M_0)$  is  $\mathcal{L}_{out}(Q, M_0) = \{\varphi(M_0)\varphi(M_1)\dots\varphi(M_w)\dots \mid M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \dots M_w \xrightarrow{t_k} M_k \wedge t_i t_j \dots t_k \in \mathcal{L}(Q, M_0)\}$

**Definition 29.** Let  $(Q, M_0)$  be an IPN and  $w \in \mathcal{L}_{in}(Q, M_0)$  be an input word of the net. The **set of firing sequences generated by  $w$**  is  $\Gamma_w = \{\sigma = t_i t_j \dots t_k \in \mathcal{L}(Q, M_0) \mid \lambda(t_i)\lambda(t_j)\dots\lambda(t_k) = w\}$ . The **set of output words generated by  $w$**  is

$$\Psi_w = \{\psi = \varphi(M_0)\varphi(M_1)\dots\varphi(M_k) \mid M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \dots \xrightarrow{t_k} M_k \wedge t_i t_j \dots t_k \in \Gamma_w\}.$$

**Example 30.** Consider the IPN shown in figure 2.7. For this net, the 5-tuple  $Q = (N, \Sigma, \Phi, \lambda, \varphi)$  is given below:

- $N$  is the PN System described in example 15.
- $\Sigma = \{Start, Stop\}$

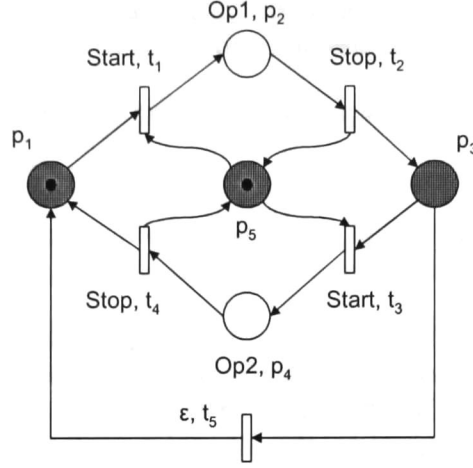


Figure 2.7: An Interpreted Petri Net

- $\Phi = \{Op1, Op2\}$
- $\lambda(t_1) = Start, \lambda(t_2) = Stop, \lambda(t_3) = Start, \lambda(t_4) = Stop, \lambda(t_5) = \varepsilon.$
- $\varphi = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Transitions  $t_1, t_2, t_3, t_4,$  are controllable since  $\lambda(t_i) \neq \varepsilon^i, 1 \leq i \leq 4.$  Hence, transition  $t_5$  is uncontrollable since  $\lambda(t_5) = \varepsilon.$

Places  $p_2$  and  $p_4$  are both measurable places since the transpose of the elemental vector  $e_2$  and  $e_4$  of  $\varphi$  are not null. While places  $p_1, p_3, p_5$  are non-measurable since  $e_1^T$  and  $e_5^T$  of  $\varphi$  are null.

There is only one enabled transition at marking  $M_0,$  which is transition  $t_1.$  If the input symbol *Start* is present in the system, then  $t_1$  is fired resulting in a new marking  $M_1$  and a new output signal  $y_1.$  This dynamic is described by the state equation 2.1 as described in the following:  $M_1 = [1 \ 0 \ 0 \ 0 \ 1]^T + C [1 \ 0 \ 0 \ 0 \ 0]^T = [0 \ 1 \ 0 \ 0 \ 0]^T$  and  $y_1 = \varphi [0 \ 1 \ 0 \ 0 \ 0]^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$

The firing language of  $(Q, M_0)$  is described by  $\mathcal{L}(Q, M_0) = (t_1 t_2 t_3 t_4 \cup t_1 t_2 t_5)^*$  Hence, the input language of  $(Q, M_0)$  is described by  $\mathcal{L}_{in}(Q, M_0) = (StartStop)^+$  and the output language is described by  $\mathcal{L}_{out}(Q, M_0) = (Op1Op2 \cup Op1)^*.$

Consider  $w = StartStop.$  Then, the set of firing sequences generated by  $w$  is  $\Gamma_w = \{t_1 t_2, t_1 t_2 t_5\}$  and the set of output words generated by  $w$  is  $\Psi_w = \{Op1\}.$

The following definitions relate the input sequence  $\alpha_0 \alpha_1 \cdots \alpha_n$  with its respective produced output sequence  $y_0 y_1 \cdots y_n$  in an IPN.

**Definition 31.** A sequence of input-output symbols of  $(Q, M_0)$  is a sequence  $\omega = (\alpha_0, y_0) (\alpha_1, y_1) \cdots (\alpha_n, y_n),$  where  $\alpha_j \in \Sigma \cup \{\varepsilon\}$  and  $\alpha_{i+1}$  is the current input of the IPN



when the output changes from  $y_i$  to  $y_{i+1}$ . It is assumed that  $\alpha_0 = \varepsilon$ ,  $y_0 = \varphi(M_0)$  and  $(\alpha_{i+1}, y_{i+1})$  belongs to the sequence when:

- $(\alpha_i, y_i)$  belongs to the sequence,
- $y_{i+1} \neq y_i$ , and
- there exists no  $y_j \neq y_i$ ,  $y_j \neq y_{i+1}$  occurring after the occurrence of  $y_i$  and before the occurrence of  $y_{i+1}$ .

**Definition 32.** If  $\omega = (\alpha_0, y_0) (\alpha_1, y_1) \cdots (\alpha_n, y_n)$  is a sequence of input-output symbols, then the **transition sequence**  $\sigma \in \mathcal{L}(Q, M_0)$  whose firing actually generates  $\omega$  is denoted by  $\sigma_\omega$ . The set of all possible firing transition sequences that could generate the word  $\omega$  is  $\Omega(\omega) = \{\sigma \mid \sigma \in \mathcal{L}(Q, M_0) \wedge \text{the firing of } \sigma \text{ produces } \omega\}$ .

**Definition 33.** Let  $(Q, M_0)$  be an IPN. The set of all sequences of input-output symbols of  $(Q, M_0)$  will be denoted by  $\Lambda(Q, M_0)$ . The set of all input-output sequences of length greater or equal than  $n$  will be denoted by  $\Lambda^n(Q, M_0)$ , i.e.,  $\Lambda^n(Q, M_0) = \{\omega \in \Lambda(Q, M_0) \mid |\omega| \geq n\}$ .

**Definition 34.** The set of all input-output sequences that leads to an ending marking in the IPN (markings enabling no transition or only self-loop transitions) is denoted by  $\Lambda_B(Q, M_0)$ , i.e.,  $\Lambda_B(Q, M_0) = \{\omega \in \Lambda(Q, M_0) \mid \exists \sigma \in \Omega(\omega) \text{ such that } M_0 \xrightarrow{\sigma} M_j \text{ and if } M_j \xrightarrow{t_i} \text{ then } C(\bullet, t_i) = 0\}$ .

**Definition 35.** Let  $\omega = (\alpha_0, y_0) (\alpha_1, y_1) \cdots (\alpha_n, y_n) \in \Lambda(Q, M_0)$  be a sequence of input-output symbols. The marking sequence set corresponding to  $\omega$  is defined as

$$I_\omega = \{M_0 M_1 \cdots M_k \mid M_i \in \mathbf{R}(Q, M_0), \quad (2.2)$$

$$M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \cdots \xrightarrow{t_m} M_k \wedge \sigma_\omega = t_i t_j \cdots t_m \in \Omega(\omega)\}$$

### 2.1.5 Event-Detectability property

The event-detectability property of an IPN is definitely essential, as it will be seen in chapter 3, in the context of the diagnosability property the focus of this thesis. Therefore, it is important to define it.

**Definition 36.** An IPN given by  $(Q, M_0)$  is **event-detectable** if any transition firing can be uniquely determined by the output signals that it produces [2].

The following lemma states when an IPN is event-detectable [2].

**Lemma 37.** A live IPN given by  $(Q, M_0)$  is event-detectable if and only if

1.  $\forall t_i, t_j$  it holds that  $\varphi C(\bullet, t_i) \neq \varphi C(\bullet, t_j)$  and
2.  $\forall t_k \in T$  it holds that  $\varphi C(\bullet, t_k) \neq 0$ .

For further information about the proof of this lemma see [2].

**Example 38.** Consider the IPN  $(Q, M_0)$  shown in figure 2.7. Based on lemma 37, there are two conditions to be analyzed in order to define event-detectability:

1.  $\forall t_i, t_j$  it holds that  $\varphi C(\bullet, t_i) \neq \varphi C(\bullet, t_j)$  and,
2.  $\forall t_k \in T$  it holds that  $\varphi C(\bullet, t_k) \neq 0$ .

This means that all columns of  $\varphi \bullet C$  must be not null and different from each other.

Let  $\varphi$  and  $C$  the output functions matrix and the incidence matrix described in examples 30 and 15, respectively. Therefore,  $\varphi \bullet C = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

In this case, it can be concluded by the result of  $\varphi \bullet C$  that  $(Q, M_0)$  is not event-detectable because there exists one null column. This is because transition  $t_5$  cannot be detected when  $(Q, M_0)$  changes from  $p_3$  to  $p_1$  by observing the outputs. Therefore, if the output signal  $OS$  is added either to place  $p_1$  or place  $p_3$ , then transition  $t_5$  can be detected by observing the outputs.

### 2.1.6 Marking-Detectability property

Another property that is useful in the design of diagnoser nets is the marking detectability [37].

**Definition 39.** Let  $(Q, M_0)$  be an IPN.  $(Q, M_0)$  is marking detectable if  $\forall \omega \in \mathcal{L}_{in}(Q, M_0) \exists z$  such that  $\omega z \in \mathcal{L}_{in}(Q, M_0)$ ,  $|z| < k < \infty$ , where  $M_0 \xrightarrow{\omega z} M_i$  and  $M_i$  can be computed.

In order to characterize an IPN exhibiting this property, the concepts of conservative marking laws and synchronic distance are now introduced [37].

**Definition 40.** Let  $(Q, M_0)$  be an IPN and  $M(p_j)$  be any reachable marking of place  $p_j$  in  $(Q, M_0)$ . The set of equations  $CML = \{\sum_{j=1}^n \alpha_j^i \cdot M(p_j) = k_i \mid i \in [1, \dots, s], \alpha_j^i \in Z^+\}$  where  $\forall \alpha_k^i \neq 0$ , it holds that  $k_i/\alpha_k^i$  is an integer value, form a set of conservative marking laws when all nonmeasurable places of the net are contained in at least one equation (i.e., if  $p_j$  is nonmeasurable, then  $\exists_i$  such that  $\alpha_j^i \neq 0$ ).

The upper marking bound  $K_j$  for a nonmeasurable place  $p_j$  is defined as  $K_j = \min\{k_i/\alpha_j^i \mid \alpha_j^i \neq 0, i \in [1, \dots, s]\}$ . Note that  $K_j$  is always defined.

**Definition 41.** The synchronic distance of transition  $t_i$ , with respect to transition  $t_j$  in an IPN  $(Q, M_0)$ , is the maximum value of the difference between the number of firings of  $t_i$  and  $t_j$ , considering all possible firing sequences. This is expressed as:

$$SD(Q, M_0; t_i, t_j) = \max_{\sigma \in \mathcal{L}(Q, M_0)} \{\vec{\sigma}(t_i) - \vec{\sigma}(t_j)\}.$$

In [37], the concept of synchronic distance is extended to a set of transitions in the following sense. If  $S_1, S_2$  are sets of transitions, then  $SD(Q, M_0; S_1, S_2)$  means the maximum value of the difference between the number of firings of transitions in  $S_1$  and transitions  $S_2$ , considering all possible firing sequences.

The next theorem, defined in [37], characterizes the IPN exhibiting the marking detectable property.

**Theorem 42.** *Let  $(Q, M_0)$  be a cyclic, live and bounded IPN where a CML is defined. If  $(Q, M_0)$  is event detectable and  $\forall p_j$  nonmeasurable place it holds that  $SD(Q, M_0; \cdot p_j, p_j \cdot) \geq K_j$ , then  $(Q, M_0)$  is marking detectable.*

For further information about the proof of this theorem see [37].

## 2.1.7 Observability

This property definition deals with the possibility of finding out the system initial state in a finite time using only the knowledge of the system structure, input and output.

In this work, the observability property [38] is used to characterize the diagnosability property as described later.

**Definition 43.** *An IPN given by  $(Q, M_0)$  is **observable** if there exists an integer  $k < \infty$  such that  $\forall \omega \in \Lambda^k(Q, M_0)$  it holds that the information provided by  $\omega$  and  $(Q, M_0)$  suffices to uniquely determine the initial marking  $M_0$  and the marking  $M_i$  reached by the firing of the underlying transition firing sequence  $\sigma_\omega$ .*

## 2.2 The Modelling Methodology

The model of a *FMS* is represented using binary *IPN*. This modelling methodology [3] [38] is based on the identification of the components of the *FMS*, then it includes the description of the relationships among them using two operators, synchronic and permissive compositions, and finally, adding an interpretation with the association of input symbols to transitions and output symbols to places. The extension here presented is specifically to model the probably faults that the system can have or the ones that are important for us to know about their occurrence.

This modelling methodology includes three steps. The extension to model faults is made on step 1 as described in the following:

1. **Set of values.** For each state variable  $sv_j^i \in \text{STATE\_VARIABLES}_i$ , the set  $\text{VALUE}_{sv_j^i}$  includes all  $val_p^{ij}$  of fault that  $sv_j^i$  can have.
2. **Codification.** Because of the existence of faulty values, the set of places can be partitioned into the subsets  $P_{sv_j^i}^F$  and  $P_{sv_j^i}^N$ , representing the faulty and normal values of the state variable  $sv_j^i$ , respectively.
3. **Event Modelling.** Each place  $p_n^{ij} \in P_{sv_j^i}^F$  must have only one input transition and self-loops transitions representing all the events related with this state variable. We denote by  $P^F$  the set of all faulty places, i.e.  $P^F = \bigcup_{sv_j^i} P_{sv_j^i}^F$ .

**Remark.** *In this work, for every faulty place  $p_i \in P^F$   $\varphi(p_i) = \varphi(p_k)$ , where  $p_k \subseteq \cdot \cdot p_i$ . This case considers the worst one, i.e. when the system enters a fault state and the sensors do not show it. However, this is not the only case it can be considered.*

**Remark.** *Remember that there is only one input transition to every faulty place.*

**Remark.** For the complete modelling methodology see [3] [38].

In order to exemplify the modelling methodology introduced by [38] with the extension to model faults here established, it will be presented the following case.

### 2.2.1 Application of the extended modelling methodology

Case Study 1: "Manufacturing couplers and brackets".

#### System Description

This *FMS* processes two different parts, couplers and brackets. The two original process plans are described in the following.

The original process plan for a Coupler is:

1. Chuck the part on 0.900 diameter.
2. Face one side.
3. Turn 0.750 X 0.500 diameter segment.

The process plan for a Bracket is:

1. Mill outside dimensions ensuring a 2.300 X 0.95 X 0.950 part.

In order to compliance with this process plans, the following components are needed:

- An engine lathe or turning machine, as generic name, for facing and turning processes.
- A milling machine.
- Two conveyors.
- One robot.
- A part selector.

The system layout is depicted in figure 2.8, and the process of a coupler is described as follows:

1. Robot takes the part from the input pallet to the engine lathe.
2. Engine lathe performs the corresponding processes of facing and, then, turning on the part.
3. Robot takes the part from the engine lathe to the output pallet.
4. The part is finished.

The process of a bracket is described as follows:

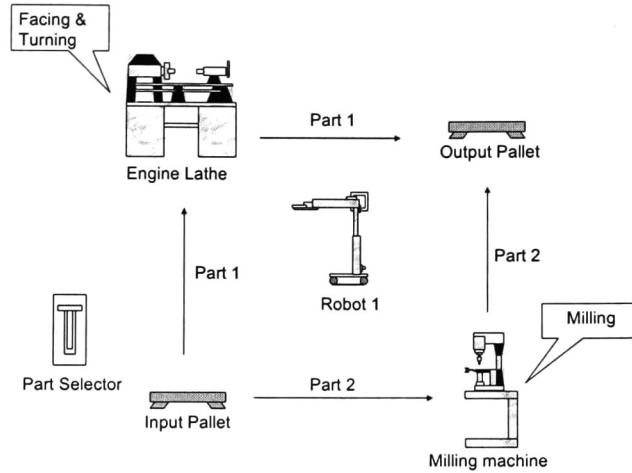


Figure 2.8: System1: Layout

1. Robot takes the part from the input pallet to the milling machine.
2. Milling machine performs the corresponding process of milling on the part.
3. Robot takes the part from the milling machine to the output pallet.
4. The part is finished.

## System Model

The *FMS* model is represented by a binary *IPN* which it will be built up following the modelling methodology introduced by [38] with the extensions to model faults.

**Step 1: Building PN** The model methodology is based on building *PN* modules representing the behaviour of a system component

Applying algorithm *Building-PN*

1. *System components.* The identified system components are the engine lathe, the milling machine, the robot, the buffers and the part selector. Therefore, it is defined the following set.  $\text{SYSTEM\_COMPONENTS} = \{\text{EngineLathe}, \text{MillingMachine}, \text{Robot}, \text{InputPallet}, \text{OutputPallet}, \text{Selector}\}$ .
2. *State variables.* The  $\text{STATE\_VARIABLES}$   $EL, MM, R, IP, OP, S$  are associated to the *EngineLathe, MillingMachine, Robot, InputPallet, OutputPallet, Selector* system components, respectively.
3. *Set of values.* The sets  $\text{VALUE}_{EL} = \{EL\_available, EL\_facing, EL\_turning, EL\_fault, In\_facing, EL\_faultIn\_turning\}$ ,  $\text{VALUE}_{MM} = \{MM\_available, MM\_milling, MM\_$

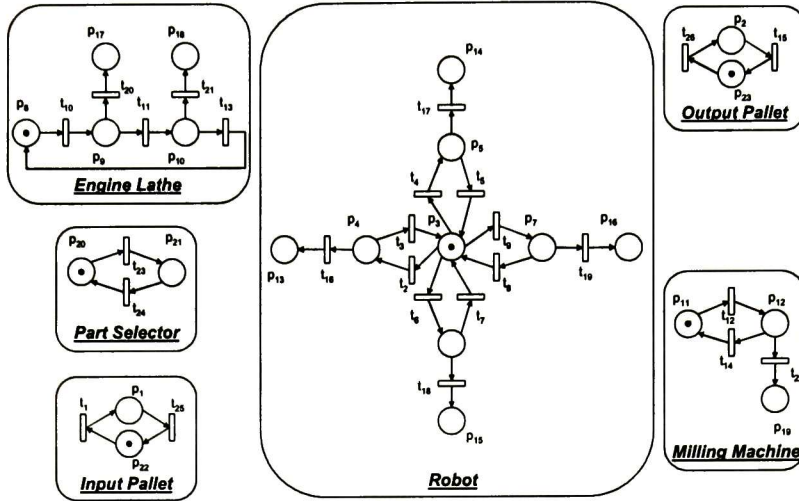


Figure 2.9: System1: PN Modules

$\text{faultIn\_milling}$ ,  $\text{VALUE}_R = \{R\_available, R\_operation1a, R\_operation2a, R\_faultIn\_operation1a, R\_faultIn\_operation2a, R\_operation1b, R\_operation2b, R\_faultIn\_operation1b, R\_faultIn\_operation2b\}$ ,  $\text{VALUE}_{IP} = \{IP\_available, IP\_notavailable\}$ ,  $\text{VALUE}_{OP} = \{OP\_available, OP\_notavailable\}$ ,  $\text{VALUE}_S = \{S\_p1, S\_p2\}$  are the values that the state variables  $EL$ ,  $MM$ ,  $R$ ,  $IP$ ,  $OP$ , and  $S$  can take, respectively. Observe that every state variable includes its own fault values.

4. *Codification.* Places  $p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}, p_{21}, p_{22}, p_{23}$  correspond to the values  $IP\_available, OP\_available, R\_available, R\_operation1a, R\_operation1b, R\_operation2a, R\_operation2b, EL\_available, EL\_facing, EL\_turning, MM\_available, MM\_milling, R\_faultIn\_operation1a, R\_faultIn\_operation1b, R\_faultIn\_operation2a, R\_faultIn\_operation2b, EL\_faultIn\_facing, EL\_faultIn\_turning, MM\_faultIn\_milling, IP\_notavailable, OP\_notavailable$ , respectively.
5. *Event modelling.* Transitions  $t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13}, t_{14}, t_{15}, t_{16}, t_{17}, t_{18}, t_{19}, t_{20}, t_{21}, t_{22}, t_{23}, t_{24}, t_{25}, t_{26}$  are created to represent the change of values. In addition, for a shorter representation of the self-loops of every  $p_n^{ij} \in P_{sv_j}^F$ , they will be omitted.
6. *Initial Marking.* In order to obtain the initial marking of the modules, it is assumed that the initial state of the system is  $EL=EL\_available, MM=MM\_available, R=R\_available, S=S\_p1, IP\_notavailable, OP\_notavailable$ . Therefore, the initial marking is  $M_0(p_3) = 1, M_0(p_8) = 1, M_0(p_{11}) = 1, M_0(p_{20}) = 1, M_0(p_{22}) = 1, M_0(p_{23}) = 1$  and  $M_0(p_i) = 0, i = 1, 2, 4, \dots, 7, 9, 10, 12, \dots, 19, 21$ .
7. *Output.* The output of this algorithm is a set of PN modules shown in figure 2.9.

Up to now, isolated  $PN$  modules  $G^{ij}$  for each state variable  $sv_j^i$  have been built. Notice that all these models are state machines.

**Step 2: PN Modules Composition** The state variable  $PN$  module composition is made through two basic operators: synchronic and the permissive composition. Synchronic composition is used to establish a relation among two or more modules containing transitions representing the same event i.e. this composition merges two or more transitions when they have the same physical meaning. Permissive composition is used to establish a relation among two or more modules when the marking of a place enables the firing of a transition without modify the marking of this places. See [38] for the formal introduction of this operators.

Applying algorithm *PN Module-composition*.

1. *Labelling modules.* The transitions representing the start of the facing operation at the engine lathe  $EL$  get the same label  $tlab(t_3) = tlab(t_{10}) = x_3$ . Also, the transitions representing the start of the milling operation at the milling machine  $MM$  get the same label  $tlab(t_7) = tlab(t_{12}) = x_7$ . The transitions representing the stop of the turning operation at the engine lathe  $EL$  get the same label  $tlab(t_4) = tlab(t_{13}) = x_4$ . Analogously,  $tlab(t_8) = tlab(t_{14}) = x_8$  that represents the stop of the milling operation at the milling machine  $MM$ . The transition  $t_{26}$  representing the unload of the part to the output pallet  $OP$  has two different physical meaning, therefore it is replicated into two different transitions,  $t_{26a}$  for the unload of part 1 and  $t_{26b}$  for the unload of part 2. The transitions representing the unload of the part 1 to the output pallet  $OP$  get the same label  $tlab(t_5) = tlab(t_{26a}) = x_5$ . The transitions representing the unload of the part 2 to the output pallet  $OP$  get the same label  $tlab(t_9) = tlab(t_{26b}) = x_9$ . The transition  $t_{25}$  representing the load of a part from the input pallet  $IP$  has two different meaning, the load to the engine lathe  $EL$  and the load to the milling machine  $MM$ , therefore it is replicated into two different transitions,  $t_{25a}$  for the load to the engine lathe and  $t_{25b}$  for the load to the milling machine. Since  $x_2$  is associated with the loading of a part 1 from the input pallet  $IP$  to the engine lathe  $EL$  and  $p_{20}$  represents that the system is processing parts 1, then  $plab(p_{20}) = \{x_2\}$ . Analogously,  $plab(p_{21}) = \{x_6\}$ . In figure 2.10, the modules after the application of the  $tlab$  and  $plab$  functions are shown.
2. *Module composition.* In the synchronic composition of all modules, transitions with the same label fusion in one transition. In the permissive composition, for every label  $l_i$  in a place, an arc is added from this place to every transition labelled as  $l_i$  and viceversa.
3. *Output.* The output of this algorithm is a labelled  $PN$ , shown in 2.11, which represents the behaviour of the  $FMS$ .

**Step 3: Building IPN** Finally, the next algorithm performs the task of building the  $IPN$ .

Consider the output of the algorithm *PN Module-composition* shown in figure 2.11 to apply the algorithm *IPN building*.

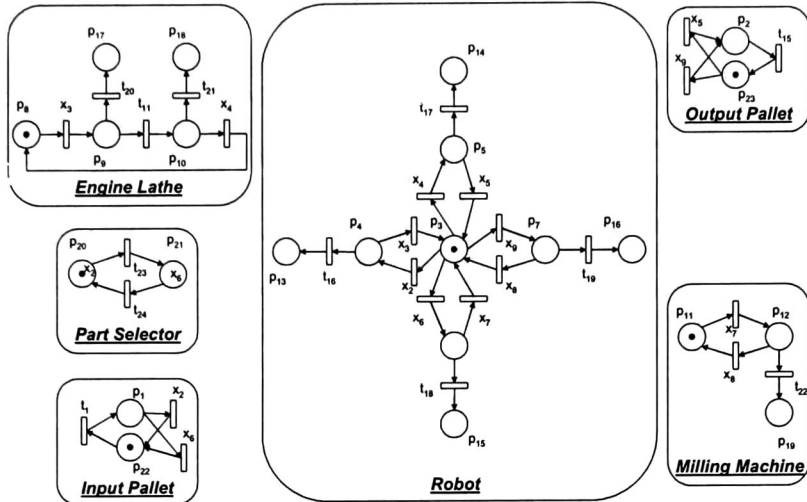


Figure 2.10: System1: *tlab* and *plab* Functions

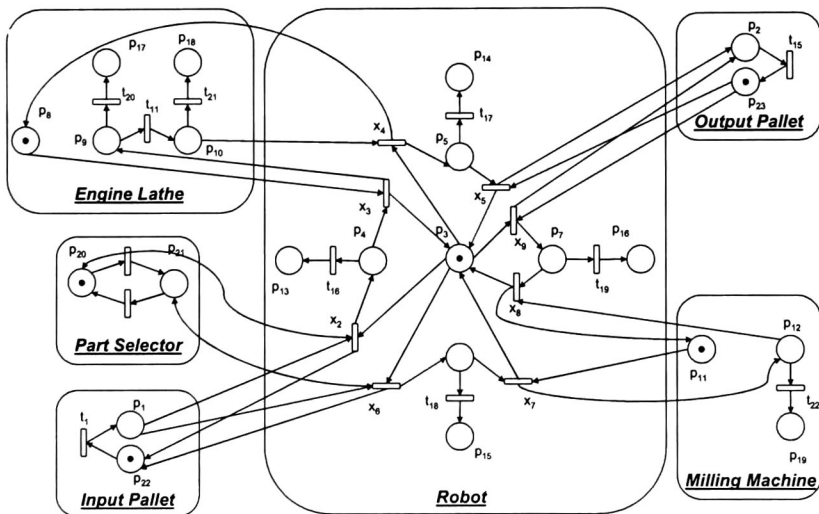


Figure 2.11: System1: Labelled *PN*



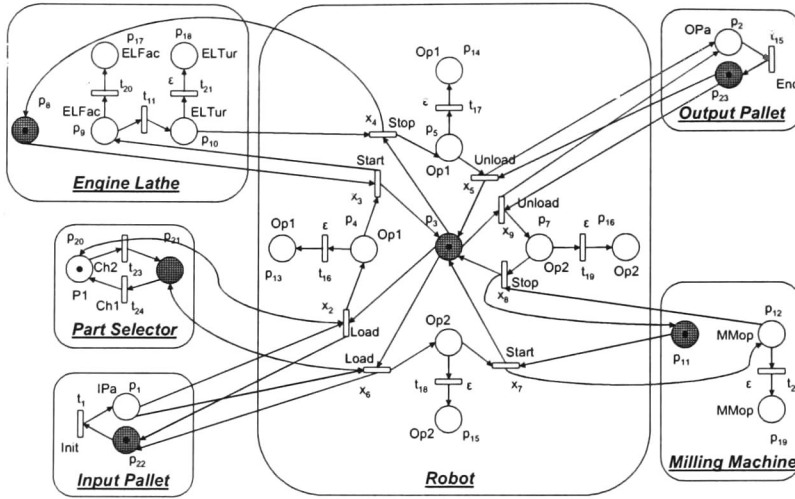


Figure 2.12: System1: IPN

### Applying algorithm *IPN Building*

1. *Input and output IPN functions.* The input function  $\lambda$  associate to each transition in the model, the input symbol, if any, used to change the state variable value. The set of input symbols is  $\Sigma = \{Init, Load, Start, Stop, Load, Unload, FacTur, End, Ch2, Ch1\}$ . Then,  $\lambda(t_1) = Init$ ,  $\lambda(x_2) = Load$ ,  $\lambda(x_3) = Start$ ,  $\lambda(x_4) = Stop$ ,  $\lambda(x_5) = Unload$ ,  $\lambda(x_6) = Load$ ,  $\lambda(x_7) = Start$ ,  $\lambda(x_8) = Stop$ ,  $\lambda(x_9) = Unload$ ,  $\lambda(t_{11}) = FacTur$ ,  $\lambda(t_{15}) = End$ ,  $\lambda(t_{23}) = Ch2$ ,  $\lambda(t_{24}) = Ch1$  and  $\lambda(t_{15}) = \lambda(t_{16}) = \lambda(t_{17}) = \lambda(t_{18}) = \lambda(t_{19}) = \lambda(t_{20}) = \lambda(t_{21}) = \lambda(t_{22}) = \varepsilon$ . The set of output symbols is  $\Phi = \{IPa, OPa, Op1, Op2, ELFac, ELTur, MMop, P1\}$ . Therefore, the output function is represented by the matrix  $\varphi$ .

$$\varphi = \begin{bmatrix} 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

2. *Output.* The output of this algorithm is the IPN shown in figure 2.12, which represents the behaviour of the FMS, including the input commands and output signals.

The complete **modelling methodology** consists then in the following steps:

1. Apply *Building-PN* algorithm.
2. Apply *PN Module-composition* algorithm.
3. Apply *IPN-building* algorithm.

## Chapter 3

# Fault Detection and Diagnosis of FMS

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**Summary.** *This chapter defines the diagnosability property in IPN, its characterization from a language point of view and its structural characterization. Moreover, it also presents general scheme for diagnosis on line. Finally, it exemplifies the test of the diagnosability property in IPN with both presented approaches: characterization based on its language and characterization based on its structure.*

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### 3.1 Diagnosability

The following definitions extend the concept of *IPN* previously presented 24 and these are applied to the *IPN* obtained by the modelling methodology presented in 2.2 in order to perform the analysis of the diagnosability property.

**Definition 44.** Let  $(Q, M_0)$  be an *IPN* and  $\mathbf{R}(Q, M_0)$  be its reachability graph.  $\mathfrak{F}=\{M \mid M(p_k) > 0, M \in \mathbf{R}(Q, M_0) \text{ and } p_k \in P_{sv_i}^F \text{ for every state variable } sv_i\}$  is named the **faulty marking set** and  $\mathfrak{N}=\mathbf{R}(Q, M_0) - \mathfrak{F}$  is named the **normal marking set**.

**Definition 45.** Let  $(Q, M_0)$  be an *IPN*,  $M_f \in \mathfrak{F}$  be a faulty marking and  $\mathcal{L}_{in}(Q, M_f)$  be the input language of  $(Q, M_0)$  from  $M_f$ . A word  $\omega = \lambda(t_a) \lambda(t_b) \cdots \lambda(t_x) \in \mathcal{L}_{in}(Q, M_f)$  is **controllable** if  $\forall t_s \in T$  such that  $\lambda(t_s) = \varepsilon$  it holds that  $\bar{\omega} \lambda(t_s) \cap \mathcal{L}_{in}(Q, M_f) \subseteq \mathcal{L}_{in}(Q, M_f)$ , where  $\bar{\omega}$  is the prefix of  $\omega$ .

#### 3.1.1 Diagnosable IPN

The following definition partitions the reachability set into faulty and normal markings and uses the faulty states to define the diagnosability property for *IPN*.

**Definition 46.** An *IPN* given by  $(Q, M_0)$  is said to be **weak input-output diagnosable** at  $k$  steps if  $\forall \omega \in \mathcal{L}_{in}(Q, M_f) \exists z$ , such that  $\omega z \in \mathcal{L}_{in}(Q, M_f)$ ,  $|z| < k < \infty$  and the information provided by  $\omega z$ , the output word generated by  $\omega z$  and the structure of the system  $(Q, M_0)$  are enough to distinguish any marking  $M_f \in \mathfrak{F}$  from any other state.

**Definition 47.** An *IPN* given by  $(Q, M_0)$  is said to be **input-output diagnosable** in a finite  $k < \infty$  steps if any marking  $M_f \in \mathfrak{F}$  is distinguishable from any other state using a controllable input word  $\zeta \in \mathcal{L}_{in}(Q, M_f)$  such that  $|\zeta| \leq k$ , and the generated output word  $\zeta_\omega \in \mathcal{L}_{out}(Q, M_f)$ .

#### 3.1.2 Characterization of the IPN based on its language

Now, the *IPN* exhibiting the input-output diagnosability property are characterized from a language point of view.

**Definition 48.** Two different places  $p_i, p_j$  are **input-output related**, denoted as  $(p_i, p_j) \in \text{IOR}$  if the following conditions are held:

1. *Reachability:*  $M_0 \xrightarrow{\sigma_{in}^a} M_i, M_0 \xrightarrow{\sigma_{in}^b} M_j$  where  $M_i(p_i) \geq 1, M_j(p_j) \geq 1$ , i.e. there exist two sequences of transitions that leads to two different markings that mark places  $p_i$  and  $p_j$ .
2. *Prefix equivalence:*  $\sigma_{in}^a \neq \sigma_{in}^b$  and  $\sigma_{in}^a, \sigma_{in}^b \in \Omega(\omega_{in})$ , for an  $\omega \in \Lambda(Q, M_0) \cup \Lambda_B(Q, M_0)$ , the two different sequences of transitions generate the same input-output word.
3. *Suffix equivalence:*  $\forall k > 0$  there exist  $\sigma_{out}^a \neq \sigma_{out}^b$  such that  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b \in \Omega(\omega_T)$ , for an  $\omega_T \in \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0)$ , i.e. there exist one input-output word generated from two different sequences of transitions that concatenated with the sequences described in step 2 generate an input-output word part of the *IPN*.

**Proposition 49.** *An IPN model  $(Q, M_0)$  is weak input-output diagnosable iff for  $\forall p_i \in P^F \nexists p_j \in (P^N \cup P^F)$  such that  $(p_i, p_j) \in IOR$ .*

*Proof.* (Sufficiency). Assume that  $(p_i, p_j) \in IOR$  and  $p_i \in P^F$  then there exist sequences  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b \in \Omega(\omega_T)$ , for  $\omega_T \in \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0)$  such that  $M_0 \xrightarrow{\sigma_{in}^a} M_i \xrightarrow{\sigma_{out}^a}, M_0 \xrightarrow{\sigma_{in}^b} M_j \xrightarrow{\sigma_{out}^b}$  where  $M_i(p_i) \geq 1, M_j(p_j) \geq 1$ . Then  $M_i \in \mathfrak{F}$  and cannot be distinguished from  $M_j$  when  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b$  are fired. Thus  $(Q, M_0)$  is not input-output diagnosable.

(Necessity). For any  $p_i \in P^F$  there exists no  $p_j$  such that  $(p_i, p_j) \in IOR$ . Then for no marking  $M_i$ , such that  $M_i(p_i) \geq 1$  (i.e.  $M_i \in \mathfrak{F}$ ), there exists another marking  $M_j$  such that both markings  $M_i, M_j$  are reached with the same input-output word and enable firing transition sequences with the same input-output words. Thus  $M_i$  is distinguishable from any other marking and  $(Q, M_0)$  is input-output diagnosable.  $\square$

### Testing Diagnosability

In order to show how to test the diagnosability of a system off-line based on its language, consider the case study 1 described in section 2.2.1, which IPN is shown in figure 2.12. It is also presented a second case, which is a variant of case study 1.

**Case Study 1** In order to test the input-output diagnosability property on the IPN shown in figure 2.12 from a language point of view, the proposition 49 will be used.

Let define  $P^N = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{20}, p_{21}, p_{22}, p_{23}\}$  and  $P^F = \{p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}\}$ . For all  $p_i \in P^F$  there exists no  $p_j$  such that  $(p_i, p_j) \in IOR$ . Then, the three conditions defined in 48 must be analyzed:

For  $p_{13}$  : if  $M(p_{13}) = 1$  and  $\omega_{in} = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (\varepsilon, Op1P1) = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1)$  then there exists a sequence  $\sigma_{in}^a = t_1 x_2 t_{16}, \sigma_{in}^a \in \Omega(\omega_{in})$ . Marking  $M(p_4) = 1$  can be reached with  $\omega_{in} = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1)$ , then there exists a sequence  $\sigma_{in}^b = t_1 x_2, \sigma_{in}^b \in \Omega(\omega_{in})$ . Therefore, the pair  $(p_{13}, p_4)$  fulfils condition 1 and condition 2, but it does not fulfil condition 3. It is because there not exist two sequences  $\sigma_{out}^a, \sigma_{out}^b$ , such that  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b$  generate the same word. In fact, the next input symbol is *Start*, but the output signals are different for these two sequences.

For  $p_{17}$  : if  $M(p_{17}) = 1$  and  $\omega_{in} = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1) (\varepsilon, ELFacP1) = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1)$  then there exists a sequence  $\sigma_{in}^a = t_1 x_2 x_3 t_{20}, \sigma_{in}^a \in \Omega(\omega_{in})$ . Marking  $M(p_9) = 1$  can be reached with  $\omega_{in} = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1)$ , then there exists a sequence  $\sigma_{in}^b = t_1 x_2 x_3, \sigma_{in}^b \in \Omega(\omega_{in})$ . Therefore, the pair  $(p_{17}, p_9)$  fulfils condition 1 and condition 2, but it does not fulfil condition 3. It is because there not exist two sequences  $\sigma_{out}^a, \sigma_{out}^b$ , such that  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b$  generate the same word. The next input symbol is *FacTur*, but the output signals are different for these two sequences.

For  $p_{18}$  : if  $M(p_{18}) = 1$  and  $\omega_{in} = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1) (FacTur, ELTurP1) (\varepsilon, ELTurP1) = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1) (FacTur, ELTurP1)$  then there exists a sequence  $\sigma_{in}^a = t_1 x_2 x_3 t_{11} t_{21}, \sigma_{in}^a \in \Omega(\omega_{in})$ . Marking  $M(p_{10}) = 1$  can be reached with  $\omega_{in} = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1) (FacTur, ELTurP1)$ , then there exists a sequence  $\sigma_{in}^b = t_1 x_2 x_3 t_{11}, \sigma_{in}^b \in \Omega(\omega_{in})$ .

Therefore, the pair  $(p_{18}, p_{10})$  fulfils condition 1 and condition 2, but it does not fulfil condition 3. It is because there not exist two sequences  $\sigma_{out}^a, \sigma_{out}^b$ , such that  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b$  generate the same word. The next input symbol is *Stop*, but the output signals are different for these two sequences.

For  $p_{14}$  : if  $M(p_{14}) = 1$  and  $\omega_{in} = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1) (FacTur, ELTurP1) (Stop, Op1P1) (\varepsilon, Op1P1) = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1) (FacTur, ELTurP1) (Stop, Op1P1)$  then there exists a sequence  $\sigma_{in}^a = t_1 x_2 x_3 t_{11} x_4 t_{17}$ ,  $\sigma_{in}^a \in \Omega(\omega_{in})$ . Marking  $M(p_5) = 1$  can be reached with  $\omega_{in} = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1) (FacTur, ELTurP1) (Stop, Op1P1)$ , then there exists a sequence  $\sigma_{in}^b = t_1 x_2 x_3 t_{11} x_4$ ,  $\sigma_{in}^b \in \Omega(\omega_{in})$ . Therefore, the pair  $(p_{14}, p_5)$  fulfils condition 1 and condition 2, but it does not fulfil condition 3. It is because there not exist two sequences  $\sigma_{out}^a, \sigma_{out}^b$ , such that  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b$  generate the same word. The next input symbol is *Unload*, but the output signals are different for these two sequences.

For  $p_{15}$  : if  $M(p_{15}) = 1$  and  $\omega_{in} = (\varepsilon, -) (Init, IPa) (Load, Op2) (\varepsilon, Op2) = (\varepsilon, -) (Init, IPa) (Load, Op2)$  then there exists a sequence  $\sigma_{in}^a = t_1 x_6 t_{18}$ ,  $\sigma_{in}^a \in \Omega(\omega_{in})$ . Marking  $M(p_6) = 1$  can be reached with  $\omega_{in} = (\varepsilon, -) (Init, IPa) (Load, Op2)$ , then there exists a sequence  $\sigma_{in}^b = t_1 x_6$ ,  $\sigma_{in}^b \in \Omega(\omega_{in})$ . Therefore, the pair  $(p_{15}, p_6)$  fulfils condition 1 and condition 2, but it does not fulfil condition 3. It is because there not exist two sequences  $\sigma_{out}^a, \sigma_{out}^b$ , such that  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b$  generate the same word. The next input symbol is *Start*, but the output signals are different for these two sequences.

For  $p_{19}$  : if  $M(p_{19}) = 1$  and  $\omega_{in} = (\varepsilon, -) (Init, IPa) (Load, Op2) (Start, MMop) (\varepsilon, MMop) = (\varepsilon, -) (Init, IPa) (Load, Op2) (Start, MMop)$  then there exists a sequence  $\sigma_{in}^a = t_1 x_6 x_7 t_{22}$ ,  $\sigma_{in}^a \in \Omega(\omega_{in})$ . Marking  $M(p_{12}) = 1$  can be reached with  $\omega_{in} = (\varepsilon, -) (Init, IPa) (Load, Op2) (Start, MMop)$ , then there exists a sequence  $\sigma_{in}^b = t_1 x_6 x_7$ ,  $\sigma_{in}^b \in \Omega(\omega_{in})$ . Therefore, the pair  $(p_{19}, p_{12})$  fulfils condition 1 and condition 2, but it does not fulfil condition 3. It is because there not exist two sequences  $\sigma_{out}^a, \sigma_{out}^b$ , such that  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b$  generate the same word. The next input symbol is *Stop*, but the output signals are different for these two sequences.

For  $p_{16}$  : if  $M(p_{16}) = 1$  and  $\omega_{in} = (\varepsilon, -) (Init, IPa) (Load, Op2) (Start, MMop) (Stop, Op2) (\varepsilon, Op2) = (\varepsilon, -) (Init, IPa) (Load, Op2) (Start, MMop) (Stop, Op2)$  then there exists a sequence  $\sigma_{in}^a = t_1 x_6 x_7 x_8 t_{19}$ ,  $\sigma_{in}^a \in \Omega(\omega_{in})$ . Marking  $M(p_7) = 1$  can be reached with  $\omega_{in} = (\varepsilon, -) (Init, IPa) (Load, Op2) (Start, MMop) (Stop, Op2)$ , then there exists a sequence  $\sigma_{in}^b = t_1 x_6 x_7 x_8$ ,  $\sigma_{in}^b \in \Omega(\omega_{in})$ . Therefore, the pair  $(p_{16}, p_7)$  fulfils condition 1 and condition 2, but it does not fulfil condition 3. It is because there not exist two sequences  $\sigma_{out}^a, \sigma_{out}^b$ , such that  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b$  generate the same word. The next input symbol is *Unload*, but the output signals are different for these two sequences.

Since there not exists any pair  $(p_i, p_j)$ ,  $p_j \in P^F$ , input-output related  $IOR = \emptyset$ , and for every pair  $(p_i, p_j)$  considered takes just one more fire of a transition to notice that an error has occurred in the system, then it can be concluded that the system is weak input-output diagnosable in  $k=1$ .

**Case Study 2** Consider the system described in case study 1, in section 2.2.1, and the obtained *IPN* from the modeling methodology shown in figure 2.12, but with a change in the output function. Instead of *ELFac* be the output symbol of places  $p_9, p_{17}$  and *ELTur* be the output symbol of places  $p_{10}, p_{18}$ , now, the four places have the same output symbol

*ELop*.

The sets  $P^N$  and  $P^F$  are the same as the sets in case 1.

The analysis for all  $p_i \in P^F$  in order to verify that there not exists  $p_j$  such that  $(p_i, p_j) \in IOR$  is the same for every place except for place  $p_{17}$ . Then, the three conditions defined in 48 must be analyzed for place  $p_{17}$ :

For  $p_{17}$  : if  $M(p_{17}) = 1$  and  $\omega_{in} = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1) (\varepsilon, ELFacP1) = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1)$  then there exists a sequence  $\sigma_{in}^a = t_1 x_2 x_3 t_{20}$ ,  $\sigma_{in}^a \in \Omega(\omega_{in})$ . Marking  $M(p_9) = 1$  can be reached with  $\omega_{in} = (\varepsilon, P1) (Init, IPaP1) (Load, Op1P1) (Start, ELFacP1)$ , then there exists a sequence  $\sigma_{in}^b = t_1 x_2 x_3$ ,  $\sigma_{in}^b \in \Omega(\omega_{in})$ . Therefore, the pair  $(p_{17}, p_9)$  fulfils condition 1 and condition 2, and it does fulfil condition 3 when the next input symbol *FacTur* is present in the system due the output signal is the same signal *ELop* for both sequences. But, when it comes the next input symbol *Stop*, the pair  $(p_{17}, p_9)$  does not accomplish with condition 3. It is because there not exist two sequences  $\sigma_{out}^a, \sigma_{out}^b$ , such that  $\sigma_{in}^a \sigma_{out}^a, \sigma_{in}^b \sigma_{out}^b$  generate the same output word.

Since there not exists any pair  $(p_i, p_j), p_j \in P^F$ , input-output related  $IOR = \emptyset$ , and since there exist one pair  $(p_{17}, p_9)$  that takes two more fires of a transition to notice that an error has occurred in the system, then it can be concluded that the system is weak input-output diagnosable in  $k=2$ .

### 3.1.3 Structural Characterization of the IPN

Proposition 49 uses the reachability set and language information to characterize the *IPN* exhibiting the input-output diagnosability property. This fact leads to *NP* complete algorithms. In the case when the non faulty part of the *IPN* exhibits the event detectability property [1] [2] [38] [36], the characterization can be determined from the structure of the *IPN*. This is proved in the following lines.

**Remark.**  $\cdot P^F = \cdot P^F \cup P^F$

**Definition 50.** Let  $(Q, M_0)$  be an *IPN*. The *underlying normal behaviour*  $(Q^N, M_0^N)$  of  $(Q, M_0)$  is the *IPN* derived from  $(Q, M_0)$  when  $P^F$  and  $\cdot P^F$  are removed from the set of places and transitions respectively and the marking is restricted to the new set of places.

**Proposition 51.** Let  $(Q, M_0)$  be an *IPN* and  $(Q^N, M_0^N)$  be its underlying normal behaviour *IPN*. If  $(Q^N, M_0^N)$  is event-detectable and for every  $p_i \notin P^F$  such that  $p_i \subseteq \cdot \cdot P^F$  there is a transition  $t_i \in (p_i) \cdot$  with  $\lambda(t_i) \neq \varepsilon$ , then  $(Q, M_0)$  is weak input-output diagnosable.

*Proof.* Since  $(Q^N, M_0^N)$  is event detectable there are not two different transition sequences in  $(Q^N, M_0^N)$  generating the same input-output word  $\omega$ . Therefore, for every input-output word  $\omega$  the set  $\Omega(\omega)$  is a singleton.

Assume without loss of generality that  $M_0 \xrightarrow{\sigma} M_i \xrightarrow{t_e} M_j$  and  $M_i \notin \mathfrak{F}, M_j \in \mathfrak{F}$ . Since  $t_e \in \bullet P^F$  then  $\lambda(t_e) = \varepsilon$  and  $\varphi_C(\bullet, t_e) = 0$ , hence  $\sigma, \sigma t_e \in \Omega(\omega)$  for a given  $\omega \in \Lambda(Q, M_0) \cup \Lambda_B(Q, M_0)$ . Thus there exist places  $p_k \notin P^F$ , and  $p_e \in P^F$  such that  $M_i(p_k) \geq 1$  and  $M_j(p_e) \geq 1$  fulfilling conditions ‘‘Reachability’’ and ‘‘Prefix equivalence’’ of definition 48.

However condition “Suffix equivalence” will never be satisfied. Indeed since  $M_i$  is a live marking in  $(Q^N, M_0^N)$  then there exists an enabled transition  $t_m \neq t_e$  enabled such that  $\varphi C(\bullet, t_m) \neq 0$  and  $\lambda(t_m) \neq \varepsilon$ . By the modelling methodology, there exists  $t'_m$  enabled at  $M_j$  such that  $\lambda(t_m) = \lambda(t'_m)$  and  $\varphi C(\bullet, t'_m) = 0$ . Then the output words are different  $\varphi(M_0 \dots M_i M_m) \neq \varphi(M_0 \dots M_i M_j M_j)$  and  $\sigma t_m$  and  $\sigma t_e t'_m$  cannot belong to the same set  $\Omega(\omega)$ . Thus  $p_e \in P^F$  cannot be related with any other place in  $IOR$  and by previous proposition  $(Q, M_0)$  is input-output diagnosable.  $\square$

**Theorem 52.** *Let  $(Q, M_0)$  be an IPN. If  $(Q, M_0)$  is observable, then  $(Q, M_0)$  is input-output diagnosable.*

*Proof.* Since  $(Q^N, M_0^N)$  is observable then there exists an integer  $k < \infty$  such that  $\forall \omega \in \Lambda^k(Q, M_0)$  it holds that the information provided by  $\omega$  and  $(Q, M_0)$  suffices to uniquely determine the marking  $M_i$  reached by the firing of the underlying transition firing sequence  $\sigma_\omega$  (the initial marking is known). Suppose that  $M_i$  is a faulty marking, then it can be distinguished from any other using the input-output word  $\omega$ . Therefore,  $(Q, M_0)$  is input-output diagnosable.  $\square$

### Testing diagnosability

In this approach, the diagnosability property can be determined from its structure, i.e. when the underlying normal behaviour of the IPN shown in figure 2.12 exhibits the event detectability property, as it will be seen in the following.

**Case Study 1** Based on proposition 51, there are two general conditions to be analyzed in order to define the diagnosability of the IPN :

1.  $(Q^N, M_0^N)$  is event detectable. In order for an IPN, in this case for the marking evaluator  $(Q^N, M_0^N)$  of  $(Q, M_0)$  shown in figure 3.1, to be event-detectable based on lemma 37, all columns of  $\varphi \bullet C$  must be not null and different from each other, but if two columns are the same, then they must have different input symbols. From the result of  $\varphi \bullet C$  shown in figure 3.2, it can be concluded that  $(Q^N, M_0^N)$  is event-detectable, and it is not necessary to analyze the input symbols.
2. For every  $p_i \in P^F$  such that  $p_i \subseteq P^F$  there is a transition  $t_i \in (p_i)$  with  $\lambda(t_i) \neq \varepsilon$ . For  $p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}$  there exist transitions  $x_3, x_5, x_7, x_9, t_{11}, x_4, x_8$  respectively, fulfilling this condition.

Therefore, the system is weak input-output diagnosable.

**Case Study 2** Based on proposition 51, there are two general conditions to be analyzed in order to define the diagnosability of the obtained IPN for the case 2.

1.  $(Q^N, M_0^N)$  is event detectable. In order for an IPN, in this case for the marking evaluator  $(Q^N, M_0^N)$  of  $(Q, M_0)$  shown in figure 3.3, to be event-detectable based on lemma 37, all columns of  $\varphi \bullet C$  must be not null and different from each other, but if two columns are the same, then they must have different input symbols. From the

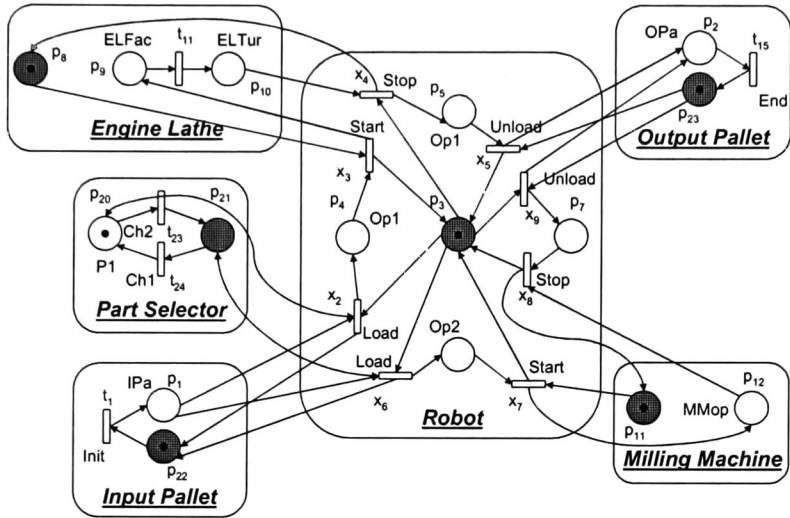


Figure 3.1: System1: The underlying normal behaviour  $(Q^N, M_0^N)$  of the IPN  $(Q, M_0)$

$$\varphi \cdot C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 3.2: System1:  $\varphi \cdot C$





the underlying normal behaviour  $IPN$  which is given the same inputs that are given to the system. By establishing that enabled transitions  $t_i$  in the diagnoser, such that  $\lambda(t_i) = \alpha_i \neq \varepsilon$  fire whenever the input signal  $\alpha_i$  is present, the differences in the output signals generated by the system and the diagnoser allows to determine the fault occurrences.

**Definition 53.** Let  $(Q, M_0)$  be input-output diagnosable  $IPN$  in  $k < \infty$  steps. The  $IPN$   $(Q', M'_0)$  is a **diagnoser net** for  $(Q, M_0)$  if for all transition sequence  $\sigma$  such that  $M_0 \xrightarrow{\sigma} M_f$  where  $M_f$  is a fault marking, the same input word associated to  $\sigma$ , when given to  $(Q', M'_0)$  generates a transition sequence  $\sigma'$  such that  $M'_0 \xrightarrow{\sigma'} M'_f$  and for all input word  $\zeta \in \mathcal{L}_{in}(Q, M_f)$  such that  $|\zeta| > k$ , it holds that  $\varphi(M_k) \neq \varphi(M'_k)$  where  $M_k$  and  $M'_k$  are the markings reached when  $\zeta$  is applied from  $M_f$  and  $M'_f$ , respectively. i.e. When a faulty marking is reached then the output is different.

**Definition 54.** The underlying normal behaviour  $N_B = (Q^N, M_0^N)$   $IPN$  is called **marking evaluator** of the  $N = (Q, M_0)$  if  $N_B$  fulfils the following two conditions:

1. The initial marking  $M_0^N$  of  $N_B$  is  $M_0^N(p_i) = M_0(p_i), \forall p_i$ .
2. If  $\lambda(t_j) = \alpha$ ,  $t_j$  is enabled in  $N_B$  and  $\alpha \in \Sigma$  is given, then  $t_j$  is fired.

**Theorem 55.** Let  $N = (Q, M_0)$  be an  $IPN$  and  $N_B = (Q^N, M_0^N)$  be a marking evaluator  $IPN$  of  $(Q, M_0)$ . If  $(Q^N, M_0^N)$  is event-detectable and for every  $p_i \notin P^F$  such that  $(p_i) \cdot \in P^F$  there is a transition  $t_j \in (p_i) \cdot$  with  $\lambda(t_j) \neq \varepsilon$ , then marking evaluator net is a diagnoser net for  $(Q, M_0)$ .

*Proof.* By definition of  $N_B$  it holds that  $M_0^N = M_0|_{P^N}$  and, as long as no faulty markings are reached, it holds that  $M_k^N = M_k$ . Let a fault transition  $t_j \in (p_i) \cdot$ , fire in  $N$  reaching a new marking  $M_{k+1} \in \mathfrak{F}$ , since is a failure state it holds that  $\varphi(M_k) = \varphi(M_{k+1})$ . Moreover, since no input symbol  $\alpha \in \Sigma$  was given in  $N$ , then  $N_B$  does not change its marking, thus there exists a difference between the marking of  $N$  and  $N_B$ . This error can be computed using the input word  $\lambda(t_i) \neq \varepsilon$  because, since  $M_{k+1}$  is an error state, then the marking in  $N$  does not change. In  $N_B$ , however,  $t_i$  is fired and a new marking  $M_{k+1}^N$  is reached. Being  $N_B$  event-detectable, it holds that  $\varphi(M_k^N) \neq \varphi(M_{k+1}^N)$  and therefore  $\varphi(M_{k+1}) \neq \varphi(M_{k+1}^N)$ . Thus, the error state can be detected.  $\square$

The global diagnoser scheme is shown in figure 3.5, where the Evaluator/Selector performs the difference between the outputs from the diagnoser net and from the system model. This difference is represented in a vector called error vector  $E$  with a length of the number of outputs in the  $IPN$ . The error vector determines the set of decision rules that are part of the Evaluator/Selector component. A decision rule is formed by two parts: the condition and the operations. The following algorithm shows how to construct the rules.

**Algorithm 56.** Construction of rules.

1. For every  $E_i[j] \neq 0$  do
  - (a) condition: where  $\varphi(j, \cdot) = E_i[j]$ .
  - (b) operation: where FAULTTYPE is in the component associated to the output  $\varphi(j, \cdot)$ .

**Case Study 1** Consider the case study described in section 2.2.1. The set of rules derived from the algorithm 56 to decide the type of failure are the following:

1. If  $Op1 = 1$  and  $ELFac = -1$  then  $FaultType = Robot\_error\_Op1$ .
2. If  $Op2 = 1$  and  $MMop = -1$  then  $FaultType = Robot\_error\_Op2$ .
3. If  $ELFac = 1$  and  $ELTur = -1$  then  $FaultType = Engine\_lathe\_error\_Fac$ .
4. If  $ELTur = 1$  and  $Op1 = -1$  then  $FaultType = Engine\_lathe\_error\_Tur$ .
5. If  $MMop = 1$  and  $Op2 = -1$  then  $FaultType = Milling\_machine\_error$ .
6. If  $Op1 = 1$  and  $Opa = -1$  then  $FaultType = Robot\_error\_Op1\_2$ .
7. If  $Op2 = 1$  and  $Opa = -1$  then  $FaultType = Robot\_error\_Op2\_2$ .

**Case Study 2** For this case the set of if-then rules to decide the type of failure are the following:

1. If  $Op1 = 1$  and  $ELFac = -1$  then  $FaultType = Robot\_error\_Op1$ .
2. If  $Op2 = 1$  and  $MMop = -1$  then  $FaultType = Robot\_error\_Op2$ .
3. If  $ELop = 1$  and  $Op1 = -1$  then  $FaultType = Engine\_lathe\_error$ .
4. If  $MMop = 1$  and  $Op2 = -1$  then  $FaultType = Milling\_machine\_error$ .
5. If  $Op1 = 1$  and  $Opa = -1$  then  $FaultType = Robot\_error\_Op1\_2$ .
6. If  $Op2 = 1$  and  $Opa = -1$  then  $FaultType = Robot\_error\_Op2\_2$ .

In order to build the complete diagnoser scheme for an specific system consider the following algorithm.

**Algorithm 57.** *Building Diagnoser\_Scheme*

1. Make a copy of the underlying normal behaviour of the system, i.e. diagnoser net.
2. Connect the input commands to the diagnoser net and to the system.
3. Connect the outputs of the diagnoser net and of the system to the evaluator/selector.
4. Construct the rules.

**Case Study 1** The final diagnoser scheme derived from the algorithm 57 is shown in figure 3.6.

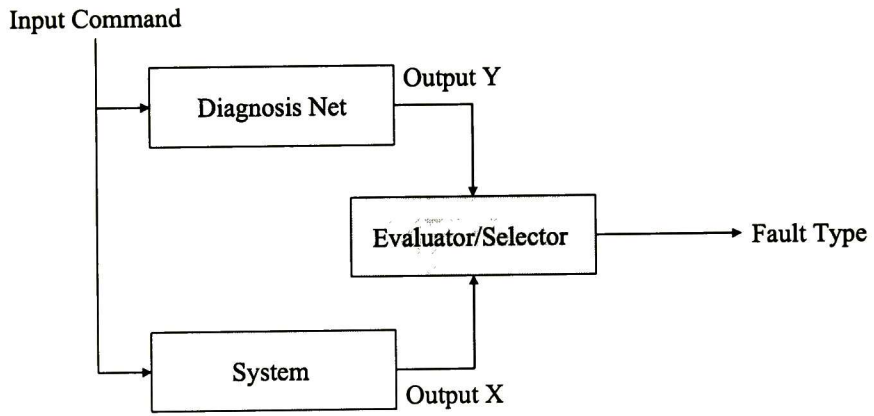


Figure 3.5: Global diagnoser scheme

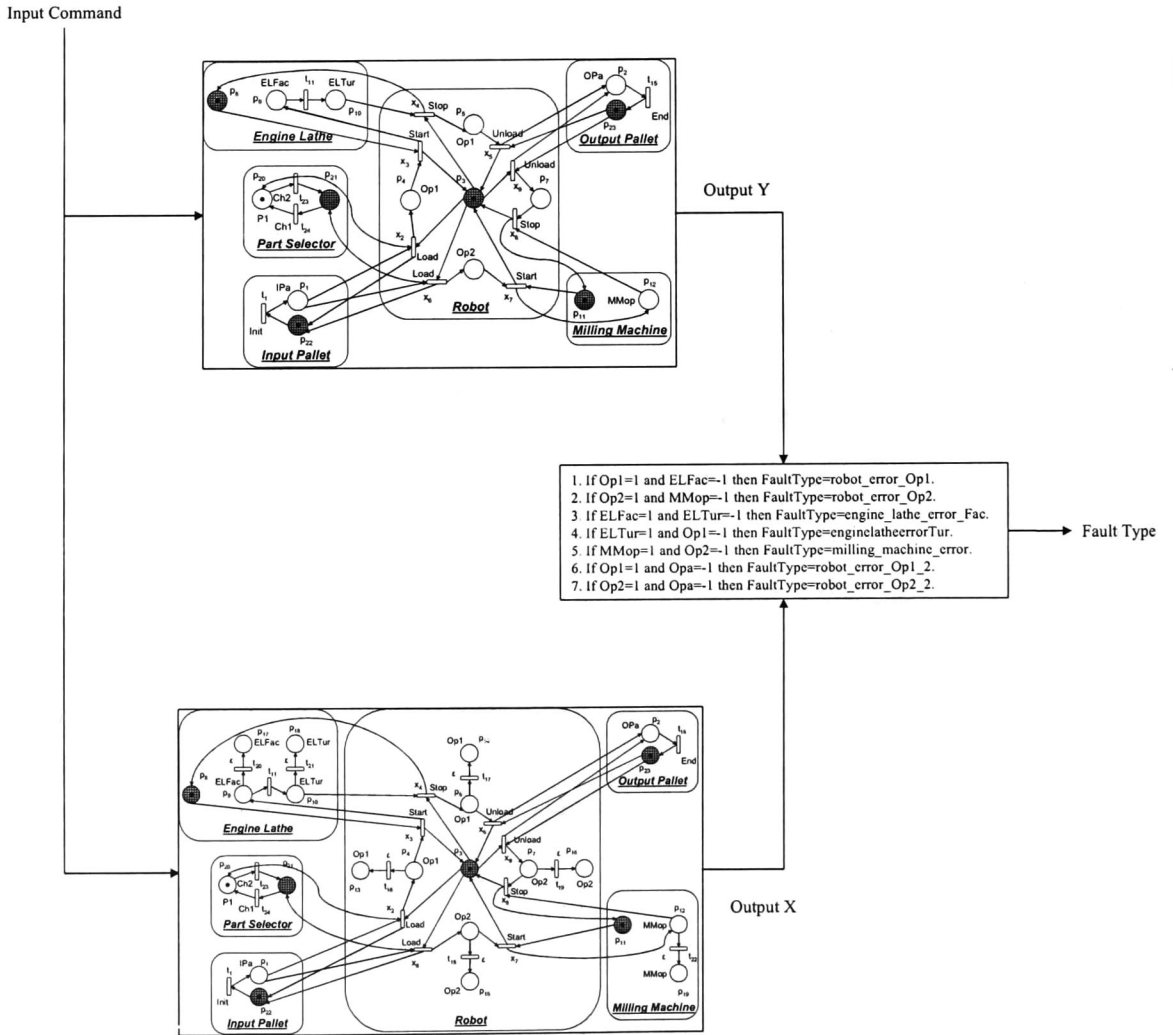


Figure 3.6: Diagnoser Scheme of Case 1

# Chapter 4

## Fault Recovery in FMS

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**Summary.** *This chapter presents the definition of Interpreted ReWriting Nets (IRWNets) and their dynamics. The IRWNets formalism is an extension of the RWNets formalism that this chapter also presents. Finally, a study of the application of IRWNets to task reconfiguration for addressing a fault recovery approach is presented.*

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## 4.1 Error Recovery Review

There exist several techniques addressing different types of failures presented by diverse authors.

Fanni et al. in [14] use the algorithms presented in [18] to estimate the actual marking which is used to design a state feedback controller. The controlled system may result blocked, as a result of the estimate, hence the authors show with an example how to introduce time-out mechanisms to recovery from this blocking. This time-out mechanism consists in adding a place and a time-out transition. Thus, the timed transition will fire only if no other transition is enabled. Then, the observer, regardless of its previous marking, reconstructs its marking.

Heimerdinger et al. in [19] present the two common mechanisms for fault recovery which are acceptance test techniques and comparison techniques. Acceptance tests usually uses recovery blocks, which provide backward fault recovery by rolling back to the state before the fault occurred repairing the state and the result. Comparison-based technique uses multiple pairs of threads or processors performing the same task. When an error occurs, the recovery uses the values from the “good” pair usually by voting.

Khatab et al. in [22] uses an automaton augmented by data variables to control the system and by time information to model timed discrete event systems. They present two types of failure recovery procedures, one that takes the system back to its normal functioning mode, and one that takes the system back to a degraded functioning mode. The first procedure consists on driving the system from a failure state back to a previous normal state. The second procedure assumes that the system reaches a failure state and there is no way to return the system back to its normal functioning without loss of performance. Therefore, for every failure, a degraded behaviour is specified as a set of states to which the system is taken back from a fault state. The system evolves in this specified set until the return to the normal behaviour is possible that naturally occurs after the failure disappears.

Loborg in [28] presents a basic categorization for fault recovery. One is the backward error recovery that consists on find a previous error free state of the system and return there, by undoing what has been done since then. The other is the forward error recovery that consists on find an error free state that the system is supposed to eventually reach, and perform actions to reach that state, by predefined alternative actions or replanning of actions.

Zhou et al. in [44] propose the concept of a controller modelled by PN for automatic error recovery, and present four basic augmentation methods for different types of error recovery guaranteeing the properties of the controller when the PN augmentation methods are applied. The first construct is the Input Condition, which if this condition is satisfied will favour the selection of a specific transition in the case when several transitions leave a place, otherwise rejects that transition. The second construct is the Alternative Path, which uses the input condition to define an alternative path through the PN. The third construct is the Backward Error Recovery, which uses the input condition and the alternative path to trap an error and execute corrective actions returning to a previous state in the PN. Finally, the fourth construct is the Forward Error Recovery, which is analogous to backward error recovery but the corrective action will solve the problem and return the control to the same place as it was when the error was originated or detected.

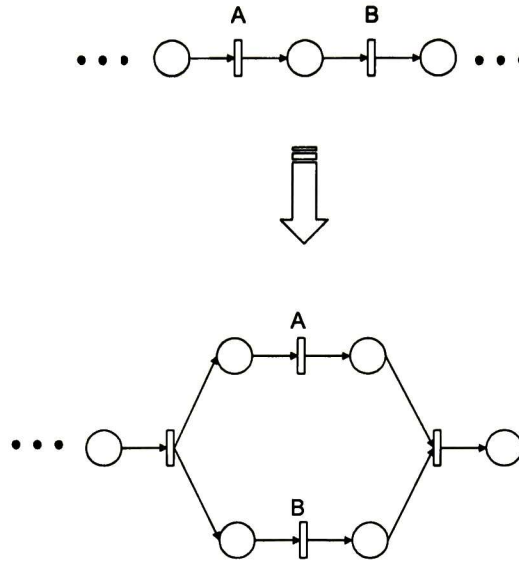


Figure 4.1: Production Reprogramming

## 4.2 An Approach for Fault Recovery

The fault recovery approach here proposed uses the Interpreted Re-Writing Nets (*IRWNets*) formalism which is an extension of the Re-Writing Nets (*RWNets*) formalism [10] [11]. The *RWNets* formalism is an extension of Mobile nets [9], Dynamic nets [5], and Reconfigurable nets [6], which are classes of high level Petri nets. The *IRWNets* formalism will be used for the reconfiguration of the *IPN* model by adding, deleting, and replacing subnets. This reconfiguration can be applied to changes imposed by modifications on the production requirements (production reprogramming) or changes inherent to recovery actions from failure situations (fault recovery).

Figure 4.1 shows the case in which a piece of a task execution model must be modified: the operations *A* and *B* that were sequentially performed must be now executed in parallel. Figure 4.2 shows the case in which a failure during the execution of the activity *A* leads to an unknown state from which a recovery operation *R* must be performed to bring the controller back to an operational state into the original workflow; then, the new path is added to the model. This process can be considered as an adaptation mechanism.

The *RWNets* allow realizing dynamic changes within a *PN* structure [10], as shown in figure 4.3. The firing of transition *r* consumes the left side *PN*, which involves four activities, *Notify*, *Take\_Left*, *Take\_Right* and *Eat*, within a causal relationship. The firing of transition *r* produces the right side *PN*, which establishes the change of the *Take\_Left* and *Take\_Right* sequential activities into two parallel ones.



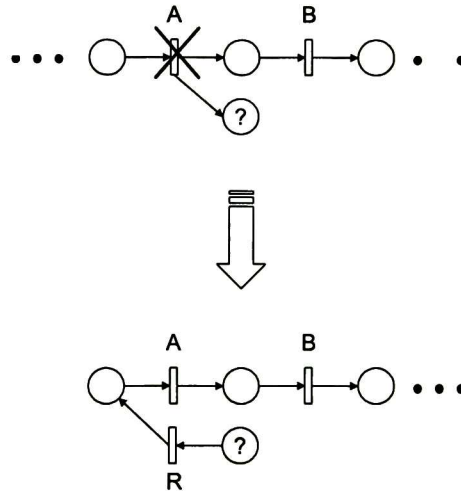


Figure 4.2: Failure Recovery

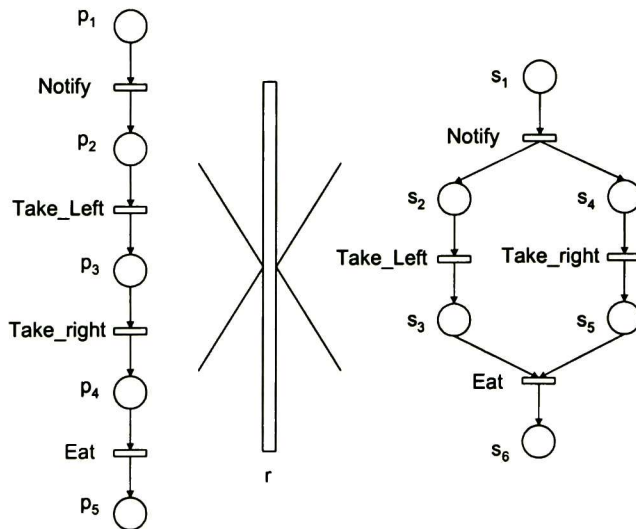


Figure 4.3: Dynamic Changes within a  $PN$  model

## 4.3 Interpreted RWNets

### 4.3.1 IRWNets Formal Definition

In this section, the *Interpreted Marked Re-Writing Nets (IRWNets)* are introduced providing similar modelling capabilities of *IPN*. The definition of a *IRWNet* uses the definition of a *RWNet*, therefore it will be defined below.

**Definition 58.** *The set of Re-Writing Nets (RWNets) is defined recursively as follows [10]:*

1. *A marked PN is a RWNet*
2. *The pair  $(N, M_0)$ , where  $N = (P, T, I, O)$  such that:*
  - *$P$  is a finite set of places or RWNets,*
  - *$T$  is a finite set of transitions,*
  - *$I, O : P \times T \rightarrow Z^+$  are the Pre and Post functions, respectively,*
  - *$M_0 : P \upharpoonright_{Places} \rightarrow Z^+$  is the initial marking, restricted to places.*

Now, the definition of a *IRWNet* can be now introduced.

**Definition 59.** *The set of Interpreted Re-Writing Nets (IRWNets) is defined as follows:*

1. *A marked IPN is a IRWNet*
2. *The pair  $(Q, M_0)$ ,  $Q$  is 5-tuple  $Q = (N, \Sigma, \Phi, \lambda, \varphi)$  where*
  - *$N = (P, T, I, O)$  is a RWNet,*
  - *$\Sigma = \{\alpha_1, \alpha_2, \dots, \alpha_r\}$  is the **input alphabet** of the net, where  $\alpha_i$  is an **input symbol**,*
  - *$\Phi = \{\zeta_1, \zeta_2, \dots, \zeta_n\}$  is the **output alphabet** associated to places, where  $\zeta_i$  is an **output symbol**,*
  - *$\lambda : T \rightarrow \Sigma \cup \{\varepsilon^i\}$  is a **labelling function** of transitions with the following restriction:*
    - *$\forall t_j, t_k \in T, j \neq k$ , if  $p_i \in I(t_j)$  and  $p_i \in I(t_k)$  and both  $\lambda(t_j), \lambda(t_k) \neq \varepsilon^i$ , then  $\lambda(t_j) \neq \lambda(t_k)$ . In this case  $\varepsilon^i$  represents an internal system event,*
  - *$\varphi : R(Q, M_0) \rightarrow \{\Phi\}^q \upharpoonright_{Places}$  is an **output function** restricted to places, where  $R(Q, M_0)$  is the **reachability set** defined as in the PN and  $q$  is the number of available outputs associated to places.*
  - *$M_0 : P \upharpoonright_{Places} \rightarrow Z^+$  is the initial marking, restricted to places.*

### 4.3.2 Dynamics of IRWNets

**Definition 60.** An IRWNet  $(Q^s, M_0^s)$  is a **subnet** of another IRWNet  $(Q, M_0)$ , denoted  $(Q^s, M_0^s) \subseteq (Q, M_0)$ , iff:

1.  $P^s \subseteq P$ ,
2.  $T^s \subseteq T$ ,
3.  $I^s \subseteq I \upharpoonright_{T^s}$ ,
4.  $O^s \subseteq O \upharpoonright_{T^s}$ ,
5.  $\lambda(T^s) \subseteq \lambda(T)$ ,
6.  $\varphi(P^s) \upharpoonright_{Places} \subseteq \varphi(P) \upharpoonright_{Places}$
7. And, recursively,  $\forall t_j \in T^s \quad \{\cup p_i \mid (p_i, t_j) \in I^s\} \subseteq \{\cup p_k \mid (p_k, t_j) \in I\}$  and  $\{\cup p_l \mid (p_l, t_j) \in O^s\} \subseteq \{\cup p_m \mid (p_m, t_j) \in O\}$ .

**Definition 61.** Two IRWNets,  $(Q, M_0)$  and  $(Q', M'_0)$  are **isomorphic**, denoted  $(Q, M_0) \cong (Q', M'_0)$ , iff there exists a pair of isomorphism  $\langle f, g \rangle$ , where  $f : T \rightarrow T'$ ,  $g : P \rightarrow P$ , such that  $I' \circ f = g \circ I$   $O' \circ f = g \circ O$ . and, recursively,  $g(W) \cong W$ ,  $W$  and  $I' \circ f(t) \cong W \upharpoonright (W, t_j) \in I$ ,  $\forall t_j \in T$ .

**Definition 62.** Let  $(Q, M_0) \in IRWNets$ . A **binding**  $b$  for  $W \in RWNNets$  is a function  $b : RWNNets \rightarrow RWNNets$ , such that  $W \cong b(W)$ ,  $\lambda(t) \cong b(\lambda(t))$ , and  $\varphi(W) \cong b(\varphi(W))$ ,  $\forall t \in T$ ,  $\forall p \in P \upharpoonright_{Places}$

**Definition 63. Substitution.** Let  $b$  be a binding. Let  $\langle f, g \rangle$  be the pair isomorphism defined by  $b$ . For a  $(Q, M_0) \in IRWNets$ ,  $(Q, M_0)[b]$  denotes  $(Q, M_0) \langle f, g \rangle$ , i.e., the application of the pair isomorphism  $\langle f, g \rangle$  to  $(Q, M_0)$ .

**Definition 64. Transition enabling.** Let  $(Q, M_0) \in IRWNets$ . Let  $t \in T$ ;  $t$  is enabled in a marking  $M_i$  under the binding  $b$ , denoted  $(N, M_i)[t >_b$ , iff there exists a binding  $b$  for the pre of  $t$ , namely  $W$ , such that  $b(W) \subseteq N$ . However, if  $t$  is an uncontrollable transition then  $t$  can be **fired**; otherwise, the input symbol  $\lambda(t) = \alpha_i \neq \varepsilon$  must be present in order to **fire**  $t$ .

**Definition 65. Firing Rule.** Let  $\_ [ >_b, \_ \in IRWNet \times IRWNet$ , be the smallest substitutive relation generated by:

$(N, M_0)[t > b \Rightarrow (N', M'_0)$ , where  $(N', M'_0) \in \_ [ >_b, (W^1, t) \in I \mid W^1 = (Q^1, M_0^{w^1})$ , and  $(W^2, t) \in O \mid W^2 = (Q^2, M_0^{w^2})$ ,

Therefore,

$M'_0 = M_0 \setminus (M_0^{w^1} \setminus M_0^{w^2}) \oplus (M_0^{w^2} \setminus M_0^{w^1})$  and  $N' = N \setminus (W^1[b] \setminus W^2[b]) \oplus (W^2[b] \setminus W^1[b])$ .

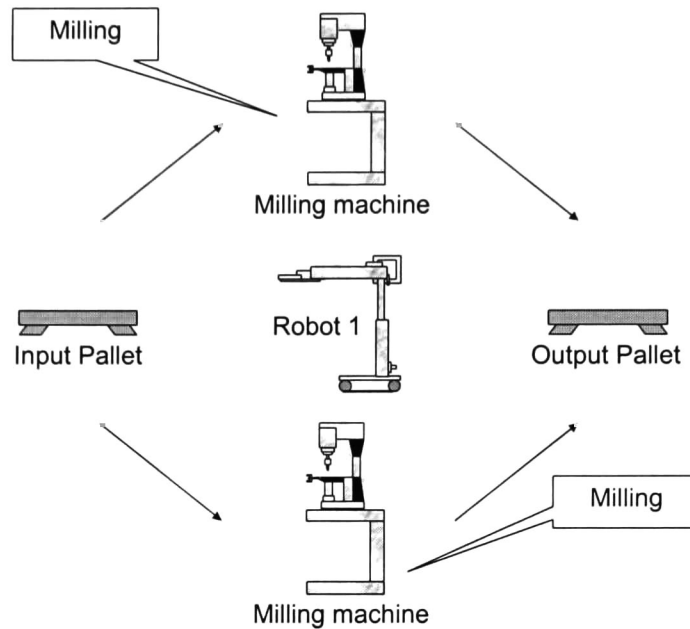


Figure 4.4: System3: Layout

## 4.4 Case Study 1: Fault Recovery in FMS

For the application of *IRWNets* to fault recovery, *IRWNets* can model the following and other mechanisms:

a) *IPN* rewriting system. In form of rules, pieces of *IPN* can be replaced by other *IPN*. The pre and the post conditions of a transition are both *IPN* structures.

b) *IPN* recovery system. The Pre is a reachable marking of an *IPN* describing a task; the Post is an *IPN* structure and/or a marking that must be added to the original net for recovery purposes.

### System Description

The system consists of one robot, two milling machines, and two pallets, one input pallet and one output pallet. As shown in figure 4.4, the process plan for a part is the following:

1. The robot takes a raw part from the input pallet, and loads it at the milling machine.
2. A milling machine performs its tasks on the part. There are two milling machines, therefore there exist two ways to accomplish the process plan.
3. The robot unloads the part from the milling machine and moves it on to the output conveyor.

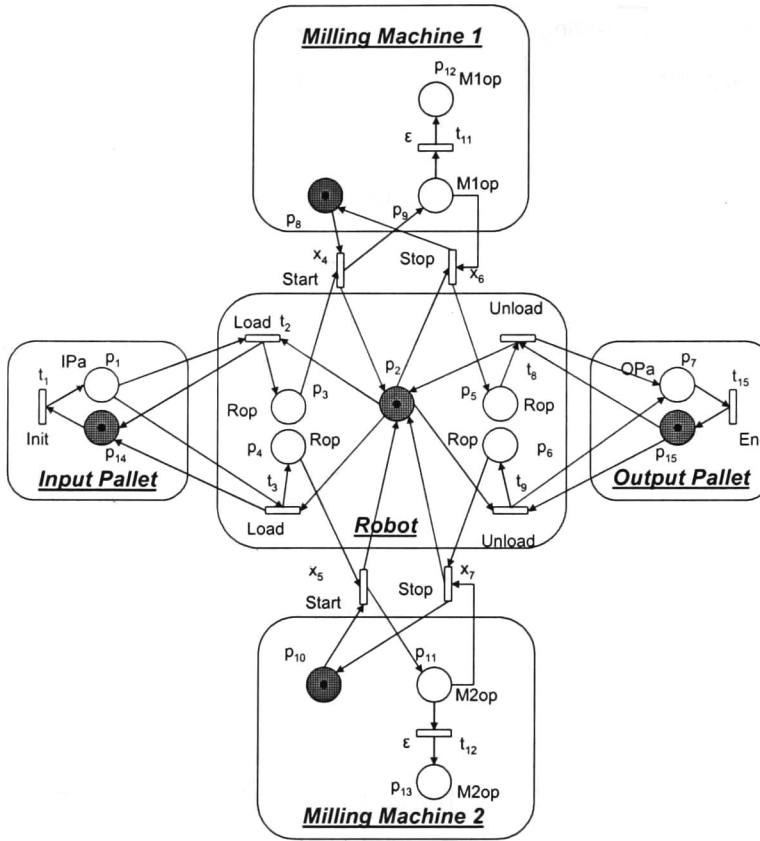


Figure 4.5: System3: *IRWNet*

### System Model

The *IPN* shown in figure 4.5 is the system model corresponding to the system description presented before using the extended modelling methodology presented in this work. This *IPN* has been tested for diagnosability property and it is input-output diagnosable in  $k=1$ .

**Remark.** Call that an *IPN* is an *IRWNet*. See definition 59.

### IRWNet and RW Rules

In the proposed system, several faulty states could exist. For example, in place  $p_9$  where the milling machine  $M1$  is supposed to perform certain task on the part, for any reason, the  $M1$  task cannot be completed causing a fault on the system, represented by  $p_{12}$ . In order to implement error recovery in the system due to this case, a few changes are made to the system model for converting it into an *IRWNet* named  $W = (Q, M_0)$ .

**Definition of  $W$**  The *IRWNet*  $W = (Q, M_0)$ , where  $Q = (N, \Sigma, \Phi, \lambda, \varphi)$ , is defined as below:

- $N = (P, T, I, O)$ , where:

$$- P = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, W^1\}.$$

$$T = \{t_1, t_2, t_3, x_4, x_5, x_6, x_7, t_8, t_9, t_{10}, t_{11}, t_{12}, r_1\}.$$

$$I = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$O = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

- $\Sigma = \{Init, Load, Start, Stop, Unload, End, Milling\_machine\_error\}$ .

- $\Phi = \{IPa, Rop, M1op, M2op, Opa\}$ .

- $\lambda(t_1) = Init, \lambda(t_2) = \lambda(t_3) = Load, \lambda(x_4) = \lambda(x_5) = Start, \lambda(x_6) = \lambda(x_7) = Stop, \lambda(t_8) = \lambda(t_9) = Unload, \lambda(t_{10}) = End, \lambda(t_{11}) = \lambda(t_{12}) = \varepsilon, \lambda(r_1) = Milling\_machine\_error$

$$\bullet \varphi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\bullet M_0 = [0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0]^T$$

Because the definition of an *IRWNet* is recursive, see definition 59, it is necessary to define  $W^1$ .

Let  $W^1 = (Q^1, M_0^1)$ , where  $Q^1 = (N^1, \Sigma^1, \Phi^1, \lambda^1, \varphi^1)$ , is defined as below:

•  $N^1 = (P^1, T^1, I^1, O^1)$ , where:

$$- P^1 = \{p_1, p_3, p_5, p_8, p_9, p_{12}\}.$$

$$T^1 = \{t_2, x_4, x_6, t_8, t_{11}\}.$$

$$I^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$O^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

•  $\Sigma = \{Load, Start, Stop, Unload\}$ .

•  $\Phi = \{IPa, Rop, Mop\}$ .

•  $\lambda(t_2) = Load, \lambda(x_4) = Start, \lambda(x_6) = Stop, \lambda(t_8) = Unload, \lambda(t_{11}) = \varepsilon$ .

$$\bullet \varphi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\bullet M_0 = [0 \ 0 \ 0 \ 0 \ 0 \ 1]^T$$

Transition  $r_1$  (shown in figure 4.6) is enabled when a token is present in place  $p_{12}$  and when the input command *Milling\_machine\_error* is emitted from the diagnoser system as a fault type, then  $r_1$  is fired and  $Q^1$  is deleted from  $Q$  resulting in a new *IRWNet*, as shown in figure 4.7. This means that if the system is working properly, and suddenly a fault occurs in the system detected by the diagnoser, the evaluator/selector component issues a command to fire the corresponding transition in order to recover the system to a fault free state.

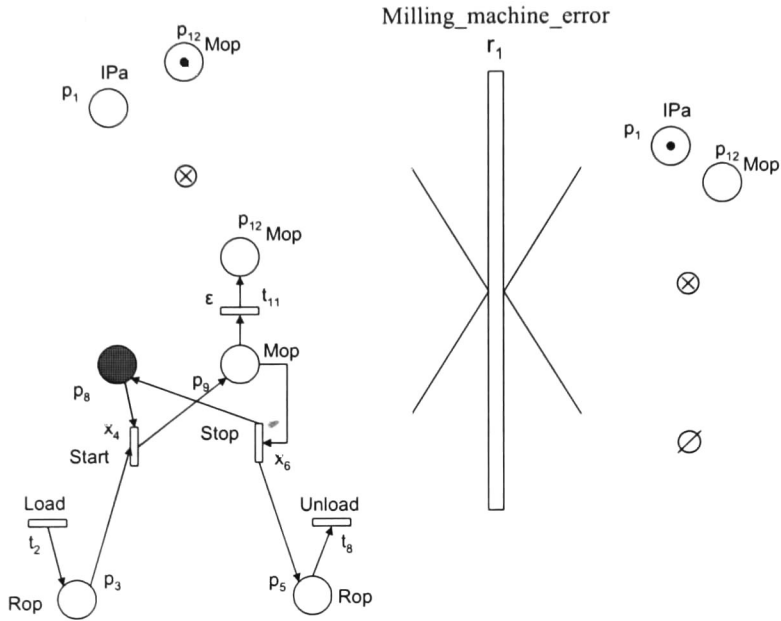


Figure 4.6: System3: Re-Writing Transition  $r_1$

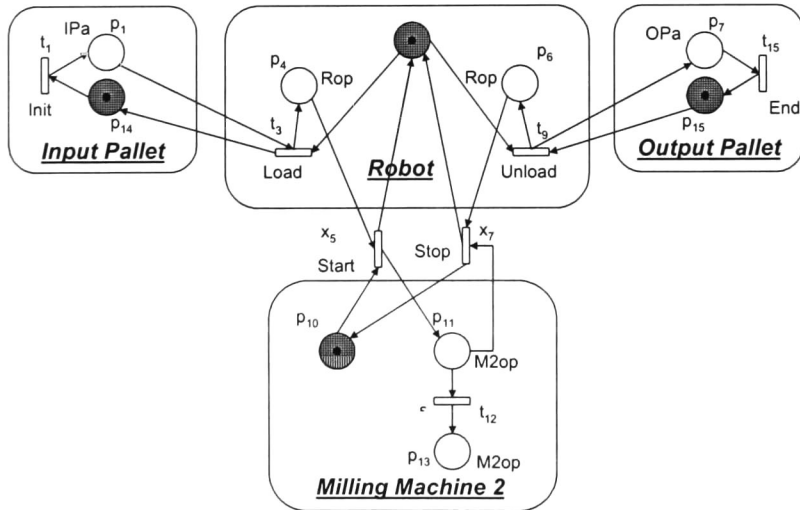


Figure 4.7: System3:  $Q$  after the firing of  $\sigma$



## 4.5 Case Study 2: Production Reprogramming in FMS

The *IRWNets* formalism is also used to production reprogramming in *FMS*.

### System Description

The process plan for a Bracket is the following:

1. Mill outside dimensions ensuring a 2.300 X 0.95 X 0.950 part.
2. Clamp on Two smooth sides.
3. Face top and one end.
4. Face opposite end and cut to length.

In order to compliance with this process plan, the following components are needed:

- A millin.g machine.
- An engine lathe for facing process.
- Three conveyors.
- Two robots.

The system layout is depicted in figure 4.8, and the process of a part is described as follows:

1. Robot 1 takes the part from the input pallet to the milling machine.
2. Milling machine performs the corresponding process of milling on the part.
3. Robot 1 takes the part from the milling machine to the buffer pallet.
4. Robot 2 takes the part from the buffer pallet to the engine lathe.
5. Engine lathe performs the corresponding processes of facing both sides on the part.
6. Robot 2 takes the part from the engine lathe to the output pallet.
7. The part is finished.

### System Model

The *IPN* shown in figure 4.9 is the system model corresponding to the system description presented before. This *IPN* has been tested for diagnosability property and it is input-output diagnosable in  $k=1$ .

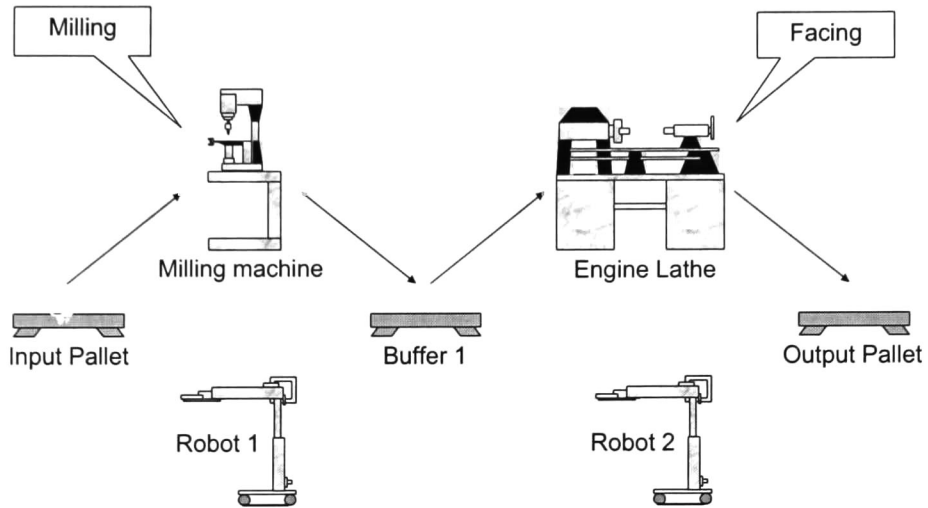


Figure 4.8: System4: Layout

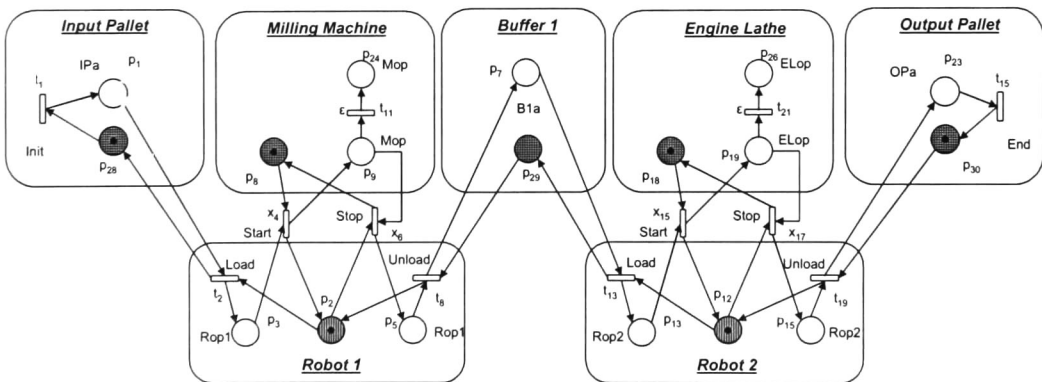


Figure 4.9: System4: IRWNet

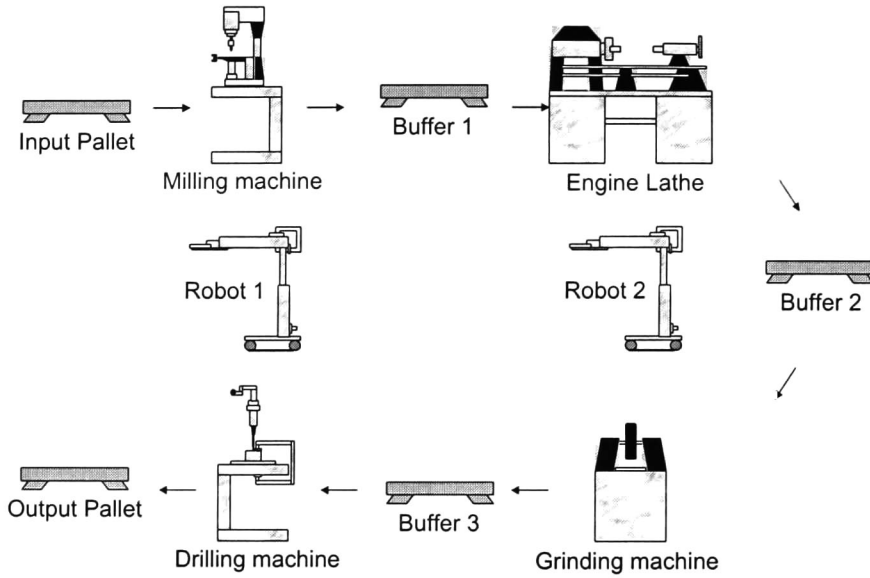


Figure 4.10: System4: Layout

### IRWNet and RW Rules

Suppose the system requirements have changed. The actual system layout is shown in figure 4.10. Therefore, it is needed to reconfigure the system, and the *IRWNets* can support it.

**Definition of  $W$**  The *IRWNet*  $W = (Q, M_0)$ , where  $Q = (N, \Sigma, \Phi, \lambda, \varphi)$ , is defined as below:

- $N = (P, T, I, O)$ , where:

$$- P = \{p_1, p_2, p_3, p_5, p_7, p_8, p_9, p_{12}, p_{13}, p_{15}, p_{18}, p_{19}, p_{23}, p_{24}, p_{26}, W^1, W^2\}.$$

$$T = \{t_1, t_2, x_4, x_6, t_8, t_{10}, t_{11}, t_{13}, x_{15}, x_{17}, t_{19}, t_{21}, r_1\}.$$

$$I = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$O = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

•  $\Sigma = \{Init, Load, Start, Stop, Unload, End, Reprogramming\}$ .

•  $\Phi = \{IPa, Rop1, Rop2, Mop, ELop, B1a, Opa\}$ .

•  $\lambda(t_1) = Init, \lambda(t_2) = \lambda(t_{13}) = Load, \lambda(x_4) = \lambda(x_{15}) = Start, \lambda(x_6) = \lambda(x_{17}) = Stop, \lambda(t_8) = \lambda(t_{19}) = Unload, \lambda(t_{10}) = End, \lambda(t_{11}) = \lambda(t_{21}) = \varepsilon, \lambda(r_1) = Reprogramming$ .

$$\bullet \varphi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\bullet M_0 = [0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0]^T$$

Because the definition of a *RWNet* is recursive, see definition 59, it is necessary to define  $W^1$  and  $W^2$ .

The *IRWNet*  $W^1 = (Q^1, M_0^1)$ , where  $Q^1 = (N^1, \Sigma^1, \Phi^1, \lambda^1, \varphi^1)$ , is defined as below:

$\bullet N^1 = (P^1, T^1, I^1, O^1)$ , where:

$$- P^1 = \{p_2, p_3, p_5, p_{12}, p_{13}, p_{15}, p_{23}\}.$$

$$T^1 = \{t_2, t_8, t_{10}, t_{13}, t_{19}\}.$$

$$I^1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$O^1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\bullet \Sigma^1 = \{Load, Unload, End\}$ .

$\bullet \Phi^1 = \{Rop1, Rop2, Opa\}$ .

$\bullet \lambda^1(t_2) = \lambda^1(t_{13}) = Load, \lambda^1(t_8) = \lambda^1(t_{19}) = Unload, \lambda^1(t_{10}) = End$ .

$$\bullet \varphi^1 = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bullet M_0^1 = [1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0]^T$$

The *IRWNet*  $W^2 = (Q^2, M_0^2)$ , where  $Q^2 = (N^2, \Sigma^2, \Phi^2, \lambda^2, \varphi^2)$ , is defined as below:





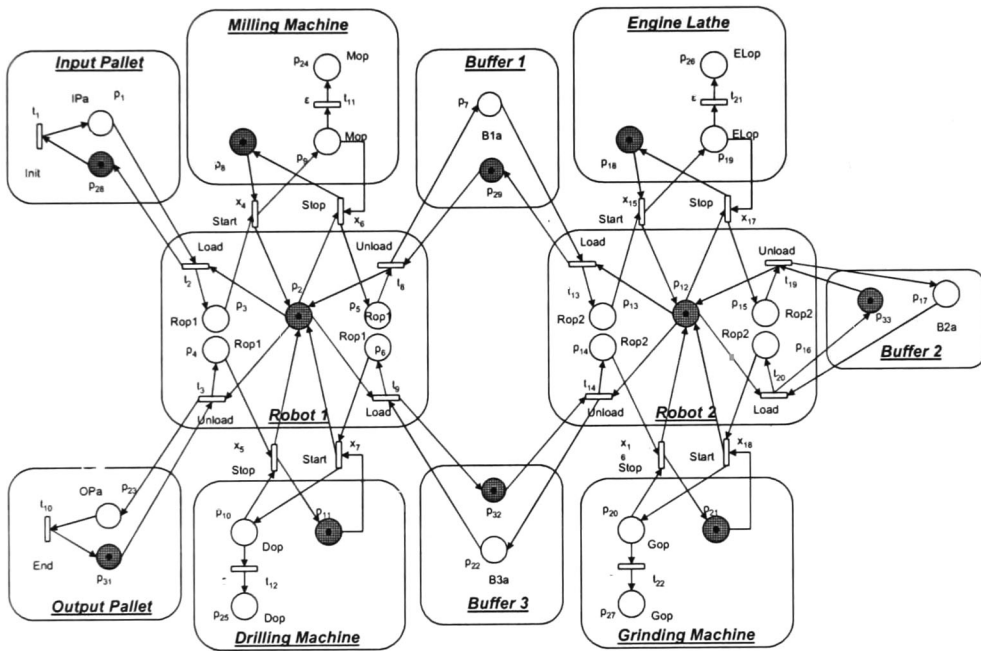


Figure 4.12: System4:  $Q$  after de firing of  $r_1$





# Conclusions

This thesis presented an extension of a modelling methodology for DES that obtains binary IPN models that includes the faulty markings of the system. The advantage of the use of this modelling methodology is to provide the necessary models to analyse the property of diagnosability, which this thesis also introduces. Specifically, the advantage of the use of IPN in the diagnosis problem is the structural manipulation that this formalism provides.

The concept of diagnosability of IPN models is a novel contribution in the context of PN since this concept has been applied only for AF in the DES. In order to test the diagnosability of the IPN two characterizations are introduced. The first characterization uses the reachability set and the language information to characterize the IPN exhibiting the input-output diagnosability property. This fact leads to NP complete algorithms. Therefore, a second characterization was introduced. In this characterization, in the case when the non-faulty part of the IPN exhibits the event detectability property, the diagnosability of the IPN can be determined from its structure leading to a P algorithm.

It is also proposed the general scheme to perform on-line diagnosis. This scheme is formed by the system, the diagnoser net, and an evaluator/selector, which performs the difference between the system and the diagnoser outputs and decides the type of fault or localization of the fault through a set of if-then rules. This scheme is important because it is structured to dynamically add a fault recovery block. As an example, a first approach for fault recovery is proposed, which its input is the output of the diagnoser scheme. This fault recovery scheme is based on IRWNets formalism, another contribution of this thesis, which its application is to task reconfiguration, in this case, for addressing fault recovery. Even when this recovery scheme is a first approach, it promises to be a good formalism for this proposal.

The study of the diagnosability as a property is an area very slightly exploited in the context of PN since this work presents the first approach in IPN. Therefore, it could be more deeply studied.

The fault recovery issue is another important matter that merits a deeper study, because the approach here presented promises to be a good technique to focus in fault recovery due the use of IRWNets that seems to be well adapted to support on-line task reconfiguration.



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**DIAGNOSTICO DE FALLAS EN SISTEMAS DE MANUFACTURA DISCRETOS**

del (la) C.

Mildreth Isadora ALCARAZ MEJÍA

el día 27 de Agosto de 2004.

Dr. Luis Ernesto López Mellado  
Investigador CINVESTAV 3A  
CINVESTAV Unidad Guadalajara

Dr. Félix Francisco Ramos Corchado  
Investigador CINVESTAV 2B  
CINVESTAV Unidad Guadalajara

Dr. Luis Isidro Aguirre Salas  
Profesor  
Universidad de Guadalajara, Centro  
Universitario de la Costa Sur,  
Departamento de Ingenierías





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