CENTRO DE INVESTIGACIÓN Y DE ESTUDIOS AVANZADOS
DEL INSTITUTO POLITÉCNICO NACIONAL

UNIDAD ZACATENCO
DEPARTAMENTO DE FÍSICA

"Fenomenología del Modelo Cosmológico SO(1,1)"

Tesis que presenta

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para obtener el Grado de

Doctor en Ciencias

en la Especialidad de

Física

Director de tesis: Dr. Abdel Pérez Lorenzana

Ciudad de México Julio, 2020

CENTER FOR RESEARCH AND ADVANCED STUDIES OF THE NATIONAL POLYTECHNIC INSTITUTE

PHYSICS DEPARTMENT

"Phenomenology of the Cosmological Model SO(1,1)”

by

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In order to obtain the

Doctor of Science

degree, speciality in

Physics

Advisor: Ph. D. Abdel Pérez Lorenzana

Mexico City July, 2020

Acknowledgments

I especially thank my advisor, Dr. Abdel Pérez Lorenzana, for his invaluable guidance, support, and patience.

I thank the reviewers of this dissertation, Dra. Nora Eva Bretón Báez, Dr. Pablo Roig Garcés, Dr. Josue De Santiago Sanabria, and Dr. José Alberto Vázquez González, for agreeing to be my reviewers and for their important comments and corrections.

For your financial support through grant number 375318/243387, I am grateful to the Consejo Nacional de Ciencia y Tecnología (CONACYT).

I also thank the personnel at the Physics Department of CINVESTAV for their academic and administrative support.

Last but not least, I express my deepest gratitude to all my friends who have accompanied me during these years.

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Resumen

En el presente trabajo sugerimos que las masas de los neutrinos pesados que entran en el mecanismo seesaw son generadas por el valor de expectación de vacío del campo de quintessence, el cual es responsable de la expansión acelerada del universo observada actualmente. Esta idea puede implementarse naturalmente en el modelo de unificación cosmológica basado en la simetría global $SO(1,1)$, el cual presentamos en este trabajo. En este modelo, la inflación temprana del universo y la aceleración reciente están gobernadas por los grados de libertad pertenecientes a un doblete escalar. La simetría protectora $SO(1,1)$ provee naturalmente los acoplamientos entre el campo de quintessence y los sigletes fermiónicos del modelo estándar mínimamente extendido. Dichos acoplamientos dan origen a las masas de los fermiones, estos a su vez proveen el mecanismo para el recalentamiento del universo primigenio a través de sus acoplamientos tipo Yukawa con el Higgs y los leptones del modelo estándar. También demostramos en el presente trabajo, que el número de partículas asociadas al campo de quintessence producidas por aniquilación de pares de neutrinos está altamente suprimido, de tal manera que nuestro modelo es consistente con las constricciones requeridas por Big Bang Nucleosíntesis.

Abstract

In this work, we suggest that heavy neutrino masses that feed the seesaw mechanism, are induced by the very large vacuum expectation value of the quintessence field which drives the currently observed accelerated expansion of the Universe. This idea is naturally implemented in the Cosmological Unification model based on the global $SO(1,1)$ symmetry, where early inflation and late accelerated expansion of the Universe are driven by the degrees of freedom of a doublet scalar field. In this model, the $SO(1,1)$ custodial symmetry naturally provides the quintessence to Standard Model singlet fermion couplings that sources neutrino masses, which, in turn, mediate reheating through Yukawa couplings to the SM Higgs and leptons. We also show that particle excitations of quintessence produced via heavy neutrino annihilation are highly suppressed, such that the model is consistent with the constraints of Big Bang Nucleosynthesis.

Chapter 1

Introduction

At the end of the 20th century, the cosmological surveys led by Perlmuter, Schmidt, and Ries[1] found that the expansion rate of our Universe is accelerating. For this discovery, they were awarded the Nobel Prize in Physics 2011.

Finding an explanation for the accelerated expansion is one of the biggest problems in cosmology today. The simplest solution, in the framework of the general relativity, could be the cosmological constant (Λ) introduced by Einstein in 1917, however, as we will explain later, it carries its own inconveniences.

An alternative is replacing the cosmological constant with an exotic kind of matter that nowadays behaves like Λ, with the advantage that it could be phenomenologically richer. In any case, whatever the source of the current acceleration, either, the cosmological constant or exotic matter, has been named as Dark Energy (DE).

Exotic matter as DE is commonly implemented through scalar fields, from which the most known, and likely the most studied, is quintessence (Q), first introduced by Caldwell in 1998. We will delve into this in the next chapter.

Cosmic acceleration driven by a scalar field is also the grounding idea behind inflationary cosmology, proposed by Guth and Linde in the early '80s. As we

[1] All references will be included in the next chapter where this and other topics, only mentioned here, will be expanded.

will see, it is thought that before the radiation dominated age, the Universe underwent another accelerating expansion era.

In this work, we present a model intended for unifying, in the field theory sense, both of the cosmic acceleration sectors. To build our model, we follow symmetry principles as guidance, to this end, we choose the $SO(1,1)$ symmetry group as the custodial symmetry of the model.

The fundamental doublet scalar field representation transforming under this symmetry, gives rise, in a very natural way, to a couple of effective fields with a very marked hierarchy of masses, one of them quite large and the other extremely small, this last yielding to a very flat potential, this allows identify such fields with the inflaton and quintessence correspondingly.

By linearly combining the bilinear invariants of symmetry, the quadratic potentials for both sectors can be written, more general potentials can be obtained by including higher-order invariants, although, here we restrict to the former.

Since our model also admits a fundamental doublet fermionic representation, as well a singlet, it naturally provides fermion to scalar couplings. We identify those fermions with right-handed neutrinos, thus, the inflaton can disintegrate into right-handed neutrinos, which in turn, couple to quintessence field.

Due to this coupling, our model allows generating masses for the right-handed neutrinos by using the false vacuum expectation value in which the quintessence field keeps trapped during all the Hubble time, turning out that, such a mass is in the range needed to feed the seesaw mechanism.

Because of the seesaw mechanism, right-handed neutrinos also couple to both, the Higgs field and left-handed neutrinos, therefore, our model includes the channels for the disintegration of inflaton into standard model (SM) particles by passing through the heavy neutrinos.

After building the model, in this work, we study its phenomenology by checking the consequences of the couplings in two fronts.

First, we study the decay and annihilation of right-handed neutrinos into SM particles and we probe that none heavy relic that could be observed today is left, also we show that these processes are efficient enough as necessary for thermalization of the primordial plasma of SM particles.

Then, because pairs of heavy neutrinos also can annihilate into quintessence quanta contributing to the total ultra-relativistic energy density, which could increase such that it might conflict with the predictions of Big Bang Nucleosynthesis (BBN), we study the production of this energy density. Nevertheless, as we will show hereafter, the process is already so inefficient, that no additional constraints are needed on our model, in such a way that it appears as consistent with BBN.

Finally, since the seesaw mechanism yields effective couplings between the quintessence field and light active neutrinos, the temperature of these latter induces a thermal correction to the scalar field mass. Consequently, we calculate the thermal corrections, then we verify these do not spoil the flatness of the potential in such a way that the quintessence field keeps behaving as DE during all the Universe history.

All these themes are carried out throughout this work as follows.

Chapter 2 is devoted to the theoretical framework, there we make a short exposition on cosmology and explain the problems that have motivated inflation as well as dynamic DE. Also, we mention briefly the issues related to neutrinos that motivated the seesaw mechanism.

Chapter 3 is devoted to set the Lagrangian of our model. We start that chapter by explaining the bilinear invariants under the two-dimensional representation of the $SO(1,1)$ group, then we write the whole Lagrangian including fermions and then diagonalize it to identify the physical fields, then by using the false vacuum of Q we endowed with mass the right-handed neutrinos.

The part of this work devoted to phenomenology is developed in Chapter 4. There, we study the reheating and the production of SM particles and quintessence quanta. Next, we study the effect of the thermal corrections to the quintessence mass. Finally, we study the dynamics of the background Universe. All the results will serve to establish the validity of our model.

Chapter 5 contains a summary and the final remarks.

Finally, several appendices are included to support the previous chapters. Appendix A contains, in some detail, basic cosmological topics to complement that was said in Chapter 2. Also to support this chapter as well the whole work, in Appendix B we explain the notation that we used, this appendix also includes a short introduction to the Standard Model of Particle Physics and a section devoted to the seesaw mechanism. To support Chapter 3, we include Appendix C which contains algebraic details on the model building. To support and make more readable Chapter 4, we include Appendix D which contains most of the technical calculations related to the phenomenology. Finally, since our main tool for calculating the particle production is the Boltzmann equation, we have included Appendix E which is devoted to the Boltzmann equation as well as the thermally averaged cross-section (in Appendix B we included a section devoted to cross-section theory).

Chapter 2

Theoretical Framework

The General Relativity (GR) theory, formulated in 1915 by Einstein, settled the basis on which the modern physical cosmology is built.

In its original form the Einstein field equations are given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{M_{\rm pl}^2}T_{\mu\nu}^{(ord)} \ , \tag{2.1}$$

where $1/M_{\rm pl}^2 = 8\pi G$, with $M_{\rm pl}$ the reduced Planck mass (with $c = 1$), and G the gravitational Newton's constant. The left-hand side of the previous equation is the Einstein's tensor, which is defined as

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \ , \tag{2.2}$$

where $R_{\mu\nu}$ is the Ricci tensor, R its trace (Ricci scalar) and $g_{\mu\nu}$ the metric tensor[1] or simply, *the metric*. The $G_{\mu\nu}$ tensor can be obtainded by variation of the Einstein-Hilbert action

$$S[g_{\mu\nu}] = \frac{M_{\rm pl}^2}{2}\int dx^4\sqrt{-g}R \ , \tag{2.3}$$

where g is the determinant of the metric.

Likewise this, the tensor appearing on the right hand side of (2.1) which is

[1]In GR $g_{\mu\nu}$ corresponds to the gravitational potential and the gravitational field corresponds to the Riemann tensor, from which the Ricci tensor is its trace $R_{\mu\nu} = Riem^{\alpha}{}_{\mu\alpha\nu}$.

known as the *energy momentum* tensor, can be obtained by variation of the action

$$S[g_{\mu\nu}] = \int dx^4 \sqrt{-g}\mathcal{L}^{(ord)} \ . \tag{2.4}$$

As denoted by the superscript $^{(ord)}$, the $T_{\mu\nu}^{(ord)}$ tensor accounts for the *ordinary* matter contained by the system, understanding by ordinary either, the non-relativistic matter (or simply matter) or the ultra-relativistic matter (or simply radiation).

Together with GR, another foundational concept of the modern physical cosmology is the *cosmological principle*, which states that the Universe is spatially both, homogeneous and isotropic at big scales[2]. The cosmological principle is grounded on astronomical observations, as well as conceptual arguments, as for example that the properties of the Universe must be the same for all observers wherever they are (Copernican principle).

As Friedmann noted in 1923, the application of the cosmological principle to GR implies the universe can evolve in time, contrary to the Einstein hypothesis of a static Universe (1917). The same result was derived by Lemaître in 1927 and posteriorly confirmed by the observations of Hubble in 1929, who found that distant galaxies recede from Earth with a velocity proportional to their distance, which means that the Universe expands.

The metric that better describes a homogeneous, isotropic and expanding Universe is the Friedmann-Lemaître-Robertson-Walker[3] (FLRW) metric,

$$ds^2 = dt^2 - a^2(t) \left\{ \frac{dr^2}{1 - \kappa r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right\} \ , \tag{2.5}$$

with $a = a(t)$ the universal scale factor and κ the *spatial curvature*, such that it can take the values $\kappa = -1, 0, 1$ for negative, zero and positive spatial curvature respectively.

By substitution of (2.5) into equation (2.1), and by assuming the cosmological principle on the right-hand side, i.e., by considering that $T_{\mu\nu}^{(ord)}$ is the one for

[2]Big scales refer to scales beyond the Mpc, where $1pc = 3.26$ light-years.
[3]Robertson and Walker showed in 1935 that this metric is unique.

a perfect fluid, it is obtained

$$\mathsf{H}^2 = \frac{1}{3M_{\rm pl}^2}\rho_{ord} - \frac{\kappa}{a^2} \ , \tag{2.6}$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm pl}^2}(\rho_{ord} + 3p_{ord}) \ . \tag{2.7}$$

In the equation (2.6), H is the Hubble parameter, which is defined as

$$\mathsf{H} := \frac{\dot{a}}{a} \ , \tag{2.8}$$

and the p_{ord} appearing in (2.7) is the pressure of the perfect fluid, which is connected to the energy density by the *equation of state*, which in turn is barotropic, i.e. the pressure only depends on ρ_{ord}, not on the temperature, further, the functional dependence is linear, such that

$$p_{ord} = \omega\rho_{ord} \ , \tag{2.9}$$

with ω a constant that can be calculated from statistical mechanical methods, turning out that, for matter $\omega_m = 0$ and for radiation, $\omega_r = 1/3$.

At the time when Einstein formulated the GR, he was interested in static Universe solutions [in the context of Eq. (2.5) it means $\dot{a} = 0$]. He was aware of his theory lead to an expanding (or contracting) universe even before knowing the FLRW solutions. Einstein solved this "problem" by introducing its famous cosmological constant in 1917 [1, 2]. Here we illustrate this issue by means of (2.5) (although Einstein addressed his analysis using a Minkowsky like metric).

Taking into account equation (2.8) and provided $\dot{a} = 0$, equation (2.6) is fulfilled for universes with positive spatial curvature, however, provided that $p_{ord} > 0$, equation (2.7) is incompatible with a static Universe.

This seems to make no sense as long as either, ordinary matter or radiation is considered, therefore, in order to stabilize the Universe, the equation (2.1) has to be altered, that was exactly what Einstein did in 1917 by introducing the *cosmological constant* Λ, to get

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{1}{M_{\rm pl}^2}T_{\mu\nu}^{(ord)} \ . \tag{2.10}$$

The cosmological constant Λ can be introduced by hand because the Einstein tensor (2.2) satisfies the contracted differential Bianchi identity $(\nabla_\alpha G)^{\alpha\beta} = 0$, where ∇ is a metric-compatible connection, i.e. $\nabla_\alpha g^{\alpha\beta} = 0$, but, more formally, the left-hand side of (2.10) can also be obtained from the Einstein-Hilbert action with cosmological constant

$$S[g_{\mu\nu}] = \frac{M_{\rm pl}^2}{2}\int dx^4\sqrt{-g}(R-2\Lambda)\ , \tag{2.11}$$

which is the most general action built out of the metric and its first and second derivatives.

By substitution of the metric (2.5) into (2.10) the Friedmann equations obtained are

$$\mathsf{H}^2 = \frac{1}{3M_{\rm pl}^2}\rho_{ord} - \frac{\kappa}{a^2} + \frac{\Lambda}{3}\ , \tag{2.12}$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm pl}^2}(\rho_{ord} + 3p_{ord}) + \frac{\Lambda}{3}\ , \tag{2.13}$$

which is a system compatible with $\dot{a} = 0$ for universes with positive spatial curvature wherein

$$\Lambda = \frac{p_{ord}}{M_{\rm pl}^2} + \frac{\kappa}{a^2} \quad \text{and} \quad \frac{p_{ord} + \rho_{ord}}{2M_{\rm pl}^2} = \frac{\kappa}{a^2}\ .$$

As said above, Hubble found in 1929 that the Universe expands and the static solution was considered wrong, this led Einstein to claim that Λ had been his biggest mistake, then he removed it from his equations.

Without Λ on stage, the Friedmann equations (2.6) and (2.7) were enough to provide a quite satisfactory description of the Universe at big scale. Such a description served as the foundational basis for the development of the standard cosmology model of which Big Bang Nucleosynthesis (BBN) is a fundamental part [3, 4, 5]. This last is the most solid theory for explaining today's observed abundances of light elements, because it explains how they were formed during the first minutes of the Universe. Aside from the correct prediction on the relic abundances of light elements, BBN is compatible with the residual thermal energy that permeates the Universe in form of the Cosmic Microwave Background (CMB) which was first detected by Penzias and Wilson in 1965.

Despite these successes, standard cosmology was unable to respond to several problems from which the main ones were the problems of flatness, the horizon, and the galaxy formation.

Next, we give a brief explanation of these problems. A bit more extended exposition is given in Appendix A.4.

Cosmological observations indicate that the Universe energy density almost equals the critical density [6] ($\rho_{TOT} \approx \rho_{\text{crit}} = 3.69 \times 10^{-47}$ GeV^4), i.e., the necessary one to have a spatially plain Universe . However, a spatially plain Universe is an unstable solution of the Friedmann equations (2.6) and (2.7), since the energy densities of radiation, matter, and effective curvature, scale respectively as

$$\rho_r \sim a^{-4}, \qquad \rho_m \sim a^{-3}, \qquad \rho_\kappa \sim a^{-2} \ .$$

This implies that the hierarchies between curvature and matter or curvature and radiation increase with time as,

$$\frac{\rho_\kappa}{\rho_r} \sim a^2, \qquad \frac{\rho_\kappa}{\rho_m} \sim a \ ,$$

nevertheless, today the spatial curvature is highly suppressed respect to other densities.

In order to explain this, in the framework of standard cosmology, the initial condition on the hierarchy of radiation to curvature has to be fine-tuned such that at the beginning of the Universe it fulfills

$$\rho_\kappa(a_i) = \rho_r(a_i) \times 10^{-64} \ .$$

This fine tunning is known as *the flatness problem.*

Similarly, the horizon problem arises when the Friedmann equations (2.6) and (2.7) are used to calculate the evolution of the horizon in a Universe dominated by either, matter or radiation.

It turns out that, there exists a serious inconsistency between the sizes of the causally connected regions at the time when the CMB was emitted ($t = t_{dec}$), and the today's ($t = t_0$) observed isotropy on the sky, since it appears nowadays totally thermalized.

The comoving horizon (the physical distance divided by the scale factor), that a photon can travel between t_i and t_f, is given by [see equation (A.17)]

$$X_H(t_i, t_f) = \int_{t_i}^{t_f} \frac{dt}{a(t)} \ ,$$

according to this, the diameter of the causally connected region that could have thermalized at $t = t_{dec}$ can be denoted as

$$\mathcal{D} = X_H(t_i, t_{dec}) \ ,$$

this region corresponds to the arc sector on the sky far away from us by a distance

$$d = X_H(t_{dec}, t_0)/2 \ .$$

If such a region subtends an angle θ (see the more detailed calculation of this in Appendix A.4), then

$$\theta = \frac{2X_H(t_i, t_{dec})}{X_H(t_{dec}, t_0)} \lesssim 10^\circ \ .$$

This means that the regions that could have been in causal contact, and so that could have thermalized at the time of the CMB emission, could not subtend angles greater than ten degrees in the today's sky, however, nowadays we observe it completely thermalized, even for opposite regions which appear with temperature differences of only a part in a million. This inconsistency is known as *the horizon problem.*

As for the galaxy formation problem, it states that the cosmic structure cannot exist in an initially homogeneous and isotropic Universe (as it is assumed by the standard cosmological model). In order to form galaxies and cumulus, it is necessary introducing inhomogeneities which can not be explained from currents and/or turbulences coming only from the expansion. This means that it is necessary providing other sources for the perturbations that could serve as initial conditions for the cosmic structure formation.

The previous problems motivated the formulation of the inflationary cosmology [7, 8, 9, 10, 11], in which it is assumed that prior to the beginning of the radiation dominated age, there was one in which, in a very short time, the Universe increased its size exponentially.

As shown in more detail in Appendix A.5, inflation solves both the flatness and the horizon problems, if the universe would have increased its size during inflation, at least as many times as it has increased it since the end of inflation until now, namely

$$\frac{a_0}{a_f} \leq \frac{a_f}{a_i} \ ,$$

where a_0 is the current value of the scale factor and a_i and a_f are its values at the beginning and at the end of inflation respectively.

In terms of the e-folding number (it defines the quantity for which the Universe increases its size by a factor of e. See more on Appendix A.6), which is written as

$$N := log(a) \ ,$$

and by assuming the Planck energy scale for the beginning of inflation, the Universe had to have unfolded itself the quantity of

$$\Delta N \approx 74 \ ,$$

or equivalently,

$$\frac{a_f}{a_i} \approx 10^{32} \ .$$

In its more used formulation, inflation is sourced by the energy density of a scalar field called the inflaton, which dominates the Universe totally during the inflationary epoch.

As shown in more detail in Appendix A.7, in the homogeneous limit, the energy density and the pressure of the field are given by

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \qquad \text{and} \qquad p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \ ,$$

where $V(\varphi)$ is the potential that depends on the inflationary model.

To perform inflation, the field must evolve under the slow-roll regime, which is achieved when is fulfilled that

$$\frac{1}{2}\dot{\varphi}^2 \ll V(\varphi), \qquad \text{and} \qquad |\ddot{\varphi}| \ll \left|\frac{\partial V}{\partial \varphi}\right| .$$

These are known as the first and second slow-roll conditions from which, the former means that the inflaton potential energy must overcome its kinetic energy and the latter ensures that the first condition is compatible with the equation of motion (2.15).

From here we can see that the equation of state of the inflaton becomes

$$\omega_\varphi = \frac{p_\varphi}{\rho_\varphi} \approx -1 ,$$

so that, during inflation the strong energy condition is violated (for details on it see Appendix A.3).

It is important to note that, the slow-roll evolution is in correspondence with the scalar field vacuum state domination [12, 13]. This can be illustrated by using the equations (A.53) and (A.57), which, by neglecting the kinetic energy terms respect to the potential, lead to

$$T_{\mu\nu}(\varphi) = g_{\mu\nu} V(\varphi) \approx g_{\mu\nu}\rho_\phi ,$$

which is a configuration equivalent to that of the minimum energy density, such that, if φ_0 is the value of φ which minimizes $V(\varphi)$, we can write the vacuum-energy momentum tensor as

$$T^{\mathrm{vac}}(\varphi_0) = g_{\mu\nu}\rho_{\mathrm{vac}} \qquad \text{wherein} \qquad \rho_{\mathrm{vac}} = V(\varphi_0) .$$

Consequently, slow-roll evolution (which implies $\omega_\varphi \approx -1$), is equivalent to evolution with scalar-field vacuum energy domination.

As an additional remark, notice that the slow-roll conditions can also be expressed in terms of the Hubble parameter as (see Appendix A.9)

$$|\ddot{\mathsf{H}}| \ll |6\mathsf{H}\dot{\mathsf{H}}| \ll 18\mathsf{H}^3 .$$

Under the slow-roll regime, during inflation, the Friedmann equations (2.6) and (2.7) become

$$\mathsf{H}^2 \approx \frac{V(\varphi)}{3M_{\rm pl}^2}, \qquad \text{and} \qquad \dot{\mathsf{H}} = -\frac{\dot{\varphi}^2}{2M_{\rm pl}^2} \approx 0 \; . \tag{2.14}$$

The inflaton background dynamic is completed by involving the Klein-Gordon equation

$$\ddot{\varphi} + 3\mathsf{H}\dot{\varphi} + V_{,\varphi}(\varphi) \; . \tag{2.15}$$

The slow-roll conditions allow defining the slow-roll parameters in terms of the potential as (see Appendix A.8)

$$\epsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\varphi}}{V}\right)^2, \qquad \epsilon \ll 1 \; , \tag{2.16}$$

and

$$\eta = M_{\rm pl}^2\left(\frac{V_{,\varphi\varphi}}{V}\right), \qquad |\eta| \ll 1 \; , \tag{2.17}$$

such that, the inflationary process ends when the value of these parameters approaches to unity [14].

As for the galaxy formation problem, inflation brings a solution when the inflaton's quantum perturbative effects are taking into account. It turns out that, the structure of the cosmic distribution of matter as observed today, is in accordance with the evolution of the primordial density perturbations generated by the quantum fluctuations of the inflaton [15, 16, 17]. It is thought that the cosmic structure was originated due to such perturbations, which are gaussian distributed, adiabatic and almost scale-invariant, as it is shown by the scalar spectral index that characterizes them [18, 19].

This possible solution to the galaxy formation problem is a feature of inflation that endows it with predictive power.

Inflationary theory together with standard cosmology became a fairly robust model for the understanding of the first moments and subsequent evolution of our Universe, however, it does not provide an explanation for the late 20th century discoveries carried out through observations of high redshift supernovae by Riess and Perlmutter [20, 21].

In accordance with that work and posterior ones that confirmed it [19, 22, 23, 24], the Universe is currently undergoing an accelerated expansion phase, which in the framework of GR, can only be explained by the cosmological constant or by the existence of an extra and exotic source of energy aside from the ordinary ones. Whatever its nature, the source of this acceleration has been dubbed Dark Energy (DE).

As said above, the simplest explanation for DE is done by reintroducing the cosmological constant Λ, but with a different value than its original, such that it can overcome the pull of Newtonian gravity.

Let us illustrate how this works by considering a Universe consisting of only matter ($p_{ord} = 0$) and by using equation (2.13), which after restoring $1/M_{\rm pl}^2 = 8\pi G$, becomes

$$\ddot{a} = -\left(\frac{4}{3}\pi a^3 \rho_{ord}\right)\frac{G}{a^2} + \frac{\Lambda}{3}a \ ,$$

which in turn, by defining $\mathcal{R} := a$, clearly can be rewritten as

$$\mathcal{F} = -\frac{mMG}{\mathcal{R}^2} + \frac{\Lambda}{3}\mathcal{R}m \ .$$

From here, we can see that the Λ-term dominates over the Newtonian for large $\mathcal{R}$ accelerating the universe.

Note that the same acceleration behavior can be obtainded from (2.12), which, for a Λ-dominated universe, becomes

$$\mathsf{H}^2 = \frac{\Lambda}{3} \ ,$$

with this and taking into account equation (2.8), it is obtained

$$a(t) = a(t_i)\exp\{(t - t_i)\Lambda/3\} \ ,$$

i.e., the universal scale factor grows exponentially with time, where t_i is the time at which Λ starts to dominate.

Thus, the necessity for explaining the accelerated expansion motivated the reintroduction of Λ into the Friedmann equations to get back to equations

(2.12) and (2.13). The standard cosmology plus inflation and the cosmological constant is known as the Λ-CDM model.

Notice that the equations (2.12) and (2.13) can be rewritten as

$$\mathsf{H}^2 = \frac{1}{3M_{\rm pl}^2}\left(\rho_{ord} - \rho_\kappa + \rho_\Lambda\right) \ , \tag{2.18}$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm pl}^2}(\rho_{ord} + 3p_{ord}) + \frac{\rho_\Lambda}{3M_{\rm pl}^2} \ , \tag{2.19}$$

where it has been defined the curvature density

$$\rho_\kappa := 3M_{\rm pl}^2\frac{\kappa}{a^2} \ , \tag{2.20}$$

together with the cosmological constant density

$$\rho_\Lambda := \Lambda M_{\rm pl}^2 \ , \tag{2.21}$$

so that Λ can be treated as an effective form of matter which turns out to fulfill an *exotic* equation of state,

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -1 \ .$$

It can be easily checked from the continuity equation,

$$\dot{\rho}_\Lambda + 3\mathsf{H}(\rho_\Lambda + p_\Lambda) = 0 \ , \tag{2.22}$$

that, provided $\omega_\Lambda = -1$ leads to $\rho_\Lambda = \text{const.}$

As said above, an energy density following such a state equation can be identified with the vacuum energy density. This has led to identify Λ with the vacuum energy. However, this identification has raised a new cosmological problem, which is known as the *cosmological constant problem* [25, 26, 27, 28].

The problem arises when one tries to reconcile the value of the observed vacuum energy density [6]

$$\rho_\Lambda^{obs} \sim (10^{-12}\ \text{GeV})^4 \ , \tag{2.23}$$

with the value calculated in the framework quantum field theory (QFT).

In principle, this energy could be infinite, but assuming a validity range for QFT, such that the value of the momentum does not exceed a certain cutoff value k_{cut}, the zero-point energy can be roughly estimated as near to the Plank scale as

$$\rho_{cut} \sim \hbar k_{cut}^4 \sim (10^{18} \text{ GeV})^4 \ . \tag{2.24}$$

Furthermore, to this value should be added the shifts in the potential energy coming from the phase transitions in the early Universe, as for example, the electroweak phase transition, which contributes with

$$\rho^{EW} \sim (200 \text{ GeV})^4 \ ,$$

or, the QCD phase transition, that contributes with

$$\rho^{QCD} \sim (0.3 \text{ GeV})^4 \ .$$

It is also thought that this transitios could have been preceded by others belonging to higer energy scales, for instance, the grand unification transition (GUT),

$$\rho^{GUT} \sim (10^{15} \text{ GeV})^4 \ .$$

In any case, in the framework of QFT, such transitions cannot contribute with energies greater than those of the Plank scale.

In conclusion, from (2.23) and (2.24), the expected value of the vacuum energy from QFT differs from the observed one by a factor of 10^{120}. This discrepancy is known with many names, for instance, the smallness problem, the vacuum catastrophe, or the most common, the cosmological constant problem.

There is a second problem related to Λ, which is known as the coincidence problem. It turns out that, from the today observed density parameters [6],

$$\Omega_\Lambda = 0.685 \qquad \text{and} \qquad \Omega_M = 0.265 \ , \tag{2.25}$$

and by knowing the critical density of the universe

$$\rho_{crit} = 3.689 \times 10^{-47} (\text{GeV})^4 \ ,$$

which in turn, by using Eq. (D.5) can be written as

$$\rho_{crit} = 1.047 \times 10^{-120}\ M_{\rm pl}^4\ ,$$

it is obtained that

$$\rho_\Lambda \approx 0.717 \times 10^{-120}\ M_{\rm pl}^4 \qquad \text{and} \qquad \rho_M \approx 0.329 \times 10^{-120}\ M_{\rm pl}^4\ ,$$

from which, we can see a coincidence of the scales of the current energy densities.

In order to illustrate why this scale coincidence is considered a problem, let us use the equation (A.35), which stipulates the radiation energy density at the Planck age, thus by using (D.5) it can be written as

$$\rho_r(a_i) = 2.844 \times 10^2\ M_{\rm pl}^4\ .$$

For a constant Λ-energy density, the initial condition on the hierarchy of the radiation energy density to Λ-energy density has to be of

$$\frac{\rho_r(a_i)}{\rho_\Lambda(a_i)} \approx 10^{118}\ .$$

This means that (similar to the problem of flatness), it is necessary to impose a fine-tuning on the initial condition. That is why the problem of coincidence is also known as the problem of the *cosmological constant fine-tuning*.

The search for solutions to the problems above mentioned has motivated the introduction of dynamic DE theories, which thereby prescind from the cosmological constant, but instead invoke the existence of exotic forms of matter characterized by a varying state equation, in such a way that nowadays $\omega_{DE} \approx -1$, and thus, it can mimic Λ.

The feasibility of dynamic DE has been widely studied, see for instance [29, 30]. It is also expected to be verified in the near future by means of high precision surveys like eBOSS [31], Euclid [32] or DESI [33].

Similar to the inflationary scenery, matter with $\omega_{DE} \approx -1$ can be implemented by means of scalar fields.

The role of scalar fields in many contexts of cosmology has been widely explored, see for instance [34, 35, 36] for early works. Several have been proposed as DE, for instance, Kessence [37, 38], Chaplygin Gas [39, 40], Phantom [41, 42], Hessence [43, 44], but among them likely the most known and studied is Quintessence (Q) [45, 46, 47, 48, 49, 50, 51].

Quintessence is a canonical scalar field, minimally coupled to the gravity whose action is given by

$$S[\mathrm{Q}] = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\nu \mathrm{Q}\partial_\mu \mathrm{Q} - V(Q)\right) d^4x \ . \tag{2.26}$$

By varying this action, it is obtained

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\ \partial^\mu \mathrm{Q}\right) + \frac{dV(\mathrm{Q})}{d\mathrm{Q}} = 0 \ ,$$

which, in a flat FLRW universe (wherein $\sqrt{-g} = a^3$), becomes

$$\ddot{\mathrm{Q}} + 3\mathsf{H}\dot{\mathrm{Q}} = -\frac{dV(\mathrm{Q})}{d\mathrm{Q}} \ . \tag{2.27}$$

The dynamics of Q is completed by the Friedmann equation (2.18) which, by restriction to spatially flat universes, and by writing ρ_{DE} instead of ρ_Λ, becomes

$$\mathsf{H}^2 = \frac{1}{3M_{\mathrm{pl}}^2}\left(\rho_{ord} + \rho_{DE}\right) \ . \tag{2.28}$$

The DE density and the pressure are given respectively by

$$\rho_{DE} = \frac{1}{2}\dot{\mathrm{Q}}^2 + V(\mathrm{Q}) \qquad \text{and} \qquad p_{DE} = \frac{1}{2}\dot{\mathrm{Q}}^2 - V(\mathrm{Q}). \tag{2.29}$$

Similar to the case of inflation, the Universe accelerates when the system evolves under the slow-roll regime

$$\frac{1}{2}\dot{\mathrm{Q}}^2 \ll V(\mathrm{Q}) \quad \Rightarrow \quad \omega_{DE} = \frac{p_{DE}}{\rho_{DE}} \approx -1. \tag{2.30}$$

The previous defines the first slow-roll condition for Q, however, in order to account for the presence of the ordinary matter, it is necessary to define a more generic form of it. This is done through the definition of the first slow-roll parameter for Q, which, as shown in Appendix A.11 is given by [see equation (A.99)]

$$\epsilon = \frac{1}{6\mathsf{H}^2}\frac{V_{,\mathrm{Q}}^2}{V}, \qquad \epsilon \ll 1 \ . \tag{2.31}$$

Notice that, during the pure DE domination age, the equation (2.28) becomes

$$\mathsf{H}^2 = \frac{V(\mathrm{Q})}{3M_{\mathrm{pl}}^2} \ , \tag{2.32}$$

hence, during such an epoch, the parameter (2.31) coincides with that of inflation given in (2.16).

As in inflation, because of consistence between the first slow-roll conditon and the equation of motion, it is also necessary to define the second slow-roll parameter for Q. It is done in detail in the Appendix A.11, then one obtains [see equation (A.107)]

$$\eta \equiv \frac{V_{,\mathrm{QQ}}}{3\mathsf{H}^2} \ . \tag{2.33}$$

Once again, we can see that during the DE domination one can substitute (2.32) into the previous equation to obtain an equivalent definition to that for inflation [see equation (2.17)], however, as explained next, unlike the inflationary case, due to the presence of the ordinary matter, the second slow-roll parameter for Q is allowed taking other values aside than those much smaller than unity.

To better explain this, let us define the parameter

$$\beta \equiv \frac{\ddot{\mathrm{Q}}}{3\mathsf{H}\dot{\mathrm{Q}}} \ , \tag{2.34}$$

which is useful for comparing the first and second terms of the equation of motion (2.27).

As deeply explained in Appendix A.11, when the Q-field is of the so called *freezing* kind, the friction term $3\mathsf{H}\dot{\mathrm{Q}}$ overcomes the $\ddot{\mathrm{Q}}$ term in Eq. (2.27), which means that the Q-field changes very little in the Hubble time, hence

$$|\beta| \ll 1, \tag{2.35}$$

and the parameter (2.33) becomes

$$\eta = \frac{3}{2}(1 + \omega_B) \ , \tag{2.36}$$

where ω_B is the state equation of the dominant cosmological fluid, or more precisely, an effective combination of the state equations of all the fluids that contribute to the total energy density.

It is important to note that, in freezing quintessence, the field will eventually reach the vacuum state in such a way that $\omega_{DE} \to -1$, therefore the DE field will match perfectly with the cosmological constant and so, the Universe expands accelerating forever.

Conversely, in the so called *thawing* quintessence, the friction term does not dominate over the first one in the equation of motion (2.27), further, they are comparable, thus, as shown in Appendix A.11, the parameters (2.34) and (2.33) respectively become

$$\beta = \frac{1 + \omega_B}{2} \ , \tag{2.37}$$

and

$$|\eta| \ll 1 \ . \tag{2.38}$$

Contrary to the freezing mode, in thawing quintessence, the field will move away from the vacuum state, such that $\omega_{DE} \to -1/3$, consequently, the Universe will undergo a phase transition in such a way that the accelerated expansion era would end.

As for the first slow-roll condition (2.30), notice that it is a necesary condition for both, freezing and thawing quintessence. In both cases, the field starts evolving with $\omega_{DE} \approx -1$, then, as said above, in late epochs, $\omega_{DE} \to -1$ for freezing, whereas $\omega_{DE} \to -1/3$ for thawing, leading to different Universe's fates.

Turning back to the Eq. (2.33), it is important to remark that it implies a deep consequence in the theroy. It turns out that, it puts a very strong bound on the mass of the scalar field. Let us show how it works.

As shown in Appendix A.11, for freezing quintessence the parameter η is of order unity, then, by Taylor expanding the potential arount its today value (Q_0) we get

$$m = \sqrt{V_{,\mathrm{QQ}}(\mathrm{Q}_0)} \ ,$$

then, by using Eqs. (2.36) and (2.33) we arrive to

$$m \sim \mathsf{H}_0 \ , \tag{2.39}$$

where H_0 is the nowadays value of the Hubble parameter. In the case of thawing quintessence, the parameter η is further constrained [see Eq. (2.38)], leading to

$$m \ll \mathsf{H}_0 \ .$$

The previous results imply that, in order to realize Q as DE, the value of the mass of Q should be as small as [6]

$$m \sim 10^{-33} \text{ eV} \ ,$$

which means that the quintessence potential is extremely flat. The smallness of this scale carries to the generic trouble that almost all quintessence models have to deal with. It turns out that, such a tiny mass is quite unstable under radiative corrections due to quadratic divergences that have to be added to the bare Q-mass, so that the flatness of the potential, and so the slow-roll condition could be wiped [52].

In order to solve this issue, supersymmetry and Goldstone symmetries have been used. By invoking supersymmetry, the quadratic divergences are exactly canceled by those of the superpartners, although it does not solve the problem totally because the corrections due to supersymmetry breaking could be still quite large [52, 53]. Then it is necessary to involve additional symmetries. It has been done, for instance, by assuming that Q is a pseudo-Goldstone boson that belongs to a higher dimensional space in which the supersymmetry scale is suppressed, therefore, after adding corrections, the boson appears as an effective boson in four dimensions preserving its stability [47, 53, 54].

These kind of corrections has to be taken into account when Q interacts with other fields, see for instance [55, 56] for DE and Dark Matter (DM) interactions, (a review on DE and DM can be consulted in [57]). Other examples in which Q is coupled with ordinary matter can be seen in [58]. To name only a few, studies on Q as an axionic particle or its connection with higher energy theories like string, superstring or M theory see [59, 60, 61].

The scalar nature, together with the similarities between the dynamics of inflation and Quintessence, have motivated many proposals intended to unify them, see for instance [46, 62, 63, 64].

In this work, we present a model intended for such unification, but unlike others, our approach is based on symmetry principles, thus, we unify the cosmic acceleration sectors in the field theory sense. To this end, we invoke a symmetry group to guide the building of our Lagrangian, as it is done for instance in the standard model of particle physics (although, unlike that our theory is not local but global).

As shown in detail in Chapter 3, we suggest that both, the quintessence field (Q), and the inflaton (which we call ξ), emerge from the complex fields ϕ and φ which belong to a fundamental doublet representation transforming under the two-dimensional representation of the $SO(1,1)$ group as

$$\Phi = \begin{pmatrix} \phi \\ \varphi \end{pmatrix}, \quad \Phi \longrightarrow g_\alpha \Phi, \quad \text{with} \quad g_\alpha \in SO(1,1) \ .$$

The unification of inflation and DE using this symmetry was first proposed in [65], but with phantom instead of Q as DE.

As it will be deeply explained later, the protective symmetry allows us to introduce a fundamental fermion doublet representation and a singlet, transforming under the $SO(1,1)$ as[4]

$$\Psi = \begin{pmatrix} \mathtt{N}_1^{\dot{a}} \\ \mathtt{N}_2^{\dot{a}} \end{pmatrix}, \quad \Psi \longrightarrow g_\alpha \Psi, \qquad \text{and} \qquad \mathtt{N}_0^{\dot{a}} \longrightarrow \mathtt{N}_0^{\dot{a}} \ ,$$

where the fields $\mathtt{N}_i$ with $i = 0, 1, 2$ are massless. This is a feature dictated by imposing extra symmetries, although it is also dictated in part by the protective symmetry.

We identify such fermion fields with right-handed neutrinos which under the

[4]The conventions concerning the notation are defined in Appendix B.1.

rules dictated by the custodial symmetry, couple to the fundamental scalar fields, in such a way that the physics of the unified cosmic scalar fields and their couplings to fermions is well determined at the Lagrangian level.

Next, after diagonalizing the scalar sector of the fundamental Lagrangian we obtain the effective fields

$$\Phi = \begin{pmatrix} \phi \\ \varphi \end{pmatrix} \quad \longrightarrow \quad \boldsymbol{\varphi} = \begin{pmatrix} \mathrm{Q} \\ \xi \end{pmatrix} \ ,$$

wherein, as said above, we identify the complex fields Q and ξ respectively as the quintessence and the inflaton.

Since the diagonalization of the scalar sector also rotates the interaction Lagrangian, there will arise the rotated neutrino fields, namely, $\mathtt{N}_0^{\dot{a}} \rightarrow \mathtt{N}_0^{\dot{a}}$ and

$$\Psi = \begin{pmatrix} \mathtt{N}_1^{\dot{a}} \\ \mathtt{N}_2^{\dot{a}} \end{pmatrix} \quad \longrightarrow \quad \mathbf{F} = \begin{pmatrix} \mathtt{F}_1^{\dot{a}} \\ \mathtt{F}_2^{\dot{a}} \end{pmatrix} \ ,$$

which are still masless and keep coupled to both, Q and ξ, for instance, for the DE sector we have

$$g_1 \mathtt{N}_{0\dot{a}} \mathrm{Q} \mathtt{F}_1^{\dot{a}} \quad \text{and} \quad g_2 \mathtt{N}_{0\dot{a}} \mathrm{Q}^* \mathtt{F}_2^{\dot{a}} \ ,$$

where g_i with $i = 1, 2$ are Yukawa couplings.

We argue that, by using the false vacuum state in which Q is trapped, these couplings generate Majorana mass terms for two right-handed neutrinos $\mathtt{K}_i$, $i = 1, 2$, to obtain

$$-\mathcal{L}_m = \frac{1}{2} m_k \mathtt{K}_{i\dot{a}} \mathtt{K}_i^{\dot{a}} + h.c. \ ,$$

where

$$m_k = \frac{a_c \langle \mathrm{Q} \rangle}{\sqrt{2}}, \quad \text{with} \quad a_c = \sqrt{|g_1|^2 + |g_2|^2} \ . \tag{2.40}$$

All this will be explained extensively throughout this work.

It is quite remarkable that this mass is in the range of the one needed to implement the seesaw type I mechanism.

In order to better explain this, let us now change the subject.

Neutrino oscillations experiments suggest that neutrinos are actually massive [66, 67, 68]. Such outcomes, that have been awarded with the Nobel Prize in Physics 2015, yield a very strong evidence in favor of the existence of physics beyond standard model (SM). See also [69, 70, 71, 72] (a review can be consulted in [73]).

The discovery of massive neutrinos conflicts with SM, in which neutrinos are massless, this is so, due to the chirality of the weak interactions which only involve left-handed Weyl neutrinos as can be seen in Appendix B.4, where we have made a very short introduction to the SM (for one generation). As shown there, electroweak interactions do not involve the right-handed Weyl fields, further, since those have not been observed in any experiment, it is assumed they do not take place into the SM, therefore, there is not possible to write Dirac mass terms for the neutrinos.

Nevertheless, since right-handed neutrinos are singlets under the symmetries of the electroweak interactions, they can be added to the Yukawa sector setting the so-called, minimal extension of SM.

As shown in Appendix B.5, during the electroweak phase transition, the neutrinos can obtain mass (m_D) through their coupling with the Higgs vacuum expectation value [6]

$$\langle H \rangle = 246 \text{ GeV} , \tag{2.41}$$

to get

$$m_D^\nu = \frac{y^\nu \langle H \rangle}{\sqrt{2}} , \tag{2.42}$$

where y^ν is the Yukawa coupling, which should be extremely small to explain the magnitude of the observed neutrino masses, which are around the sub-eV scale, for instance, the mass of the heaviest is bounded by [6, 74, 75]

$$5 \times 10^{-2} \text{ eV} \lesssim m^\nu_{\text{obs}} \lesssim \text{few} \times 10^{-1} \text{ eV} . \tag{2.43}$$

The smallness of the coupling y^ν seems quite unnatural since its value should be expected to be of the same order as the coupling of the charged leptons, so we are before a problem of hierarchy.

The search for solutions to this problem has motivated the introduction of the type I seesaw mechanism [76, 77, 78, 79, 80, 81, 82]. As shown in appendix B.6 (in which we explain in detail the basic idea of the type I seesaw mechanism), it is assumed that $m_D^\nu \sim \langle H \rangle$ and that the observed neutrino mass (which we will rename as m_ν) is given by

$$m_\nu = \frac{(m_D^\nu)^2}{m_R} \ ,$$

where m_R is the (Majorana) mass of the righ-handed neutrino, which, to realize the seesaw mechanism, has to be around $m_R \sim (10^{13} - 10^{15})$ GeV.

Comming back to our main result [Eq. (2.40)], notice that this is just the mass that we suggest can be generated by the coupling between the vacuum energy of the quintessence field ⟨Q⟩ and the fermions, coupling that is naturally contained under the protective symmetry of our model.

It is important to notice that, understanding the smallness of the neutrino masses with the seesaw mechanism translates into understanding of the origin of m_R (which from now on we will call m_k). In this work, we put on the table a possible solution to this issue, by suggesting that this mass has a cosmological origin.

Let us mention that other works have explored the connection between DE and neutrinos (but with active neutrinos instead of sterile), see for instance [83, 84, 85, 86, 87], these works have based the so-called mass-varying neutrinos models. Also, the Yukawa couplings between DE and fermionic DM and their cosmological evolution was addressed in [88]. On the other hand, the possible connection among sterile Majorana neutrino masses and ultra-light bosons that could be Q were first presented in [89], although no reference to any governing principle of symmetry for that was given there.

As said in the introductory chapter, we have to review, as far as possible here, the phenomenology that our model involves, namely, the decay of the inflaton into heavy neutrinos, then the decay and annihilation of these into SM particles. The aim of this is to verify that no heavy relics that could be observed today are left behind, as well as verifying that production and thermalization of SM fields in the early Universe is efficient enough.

Also, because neutrinos annihilate into quintessence quanta, it is neccesary to check if such a process is whether or not compatible with the initial conditions of big bang nuelceosyntesis (BBN). Let us explain this a little better.

In order to do study the impact our model could have on this sector, we consider standard BBN (SBBN) [3, 4, 5] (for a recent review see [90]), in which all of the input parameters, namely, the number of relativistic degrees of freedom in equilibrium (g_*), the neutron lifetime, the cross-sections of the involved nuclear processes, the mass difference between neutrons and protons and the strength of both the weak force and gravity, are in accordance with the standard model of particle physics and Einstein gravity. In SBBN all of those parameters are well determined. The unique input free parameter is the baryon to photon ratio, which determines the primordial abundances of the four light nuclei, namely ^{4}He, ^{3}He, H or D and ^{7}Li. None of them is modified directly in our model, apart, perhaps, from g_*.

Since SBBN assumes a Friedmann-Lemaître-Robertson-Walker (FLRW) universe and it occurs during the radiation domination age, any increment on g_* increases the value of the Hubble parameter, H, consequently, the value of the freeze-out temperature of the neutron-to-proton ratio also increases, which in turn implies an increment on the final primordial helium abundance. The same is accomplished if there is some net increase in the total radiation energy density due to any process beyond thermal equilibrium. That is just the kind of process of neutrino pair annihilation, thus, in order to quantify the impact our model could have on initial conditions of SBBN, we have to carefuly check the production of the relativistic energy density due to $\mathcal{X}$ quanta.

This process, together with the others of interest, will be carefully studied in Chapter 4. Such a phenomenology will allow us to set the validity of our proposal, but before, in the following chapter, we present the Lagrangian of our model.

Chapter 3

The Lagrangian of the $SO(1,1)$ Model

Following the motivations of the $SO(1,1)$ model as presented in Ref [65], we consider the scalar doublet

$$\Phi = \begin{pmatrix} \phi \\ \varphi \end{pmatrix}, \quad \text{with} \quad \phi,\, \varphi \quad \text{complex scalar fields.} \tag{3.1}$$

It transforms under the $SO(1,1)$ symmetry as

$$\Phi \longrightarrow g_\alpha \Phi \ .$$

In the previous equation g_α stands for an arbitrary element in $SO(1,1)$, whose exponential mapping is given by

$$g_\alpha = e^{i\alpha\sigma_1} \ , \tag{3.2}$$

with $\alpha \in \mathbb{R}$ and σ_1 the first Pauli matrix.

3.1 Invariants under the $SO(1,1)$ symmetry group

The indefinite orthogonal group $O(1,1,\mathbb{C})$ (see for instance [91, 92, 93]), is defined as the set of complex 2×2 matrices $\mathcal{O}$ that preserves the metric $\sigma_3 = \mathrm{diag}(1,-1)$, such that,

$$\sigma_3 \mathcal{O}^T \sigma_3 = \mathcal{O}^{-1} \quad \longrightarrow \quad \mathcal{O}^T \sigma_3 \mathcal{O} = \sigma_3 \ , \tag{3.3}$$

(note that in general $\mathcal{O}^T \neq \mathcal{O}^{-1}$), hence, this group has one invariant tensor of rank two, which is indeed the third Pauli matrix σ_3, such that the unique invariant bilinear built out of the complex scalar doublet (3.1), is of the form

$$\begin{aligned}\Phi^T \sigma_3 \Phi &= (\Phi^T \mathcal{O}^T)\,(\mathcal{O}^T)^{-1}\,\sigma_3 \mathcal{O}^{-1}\,(\mathcal{O}\Phi)\\ &= \Phi'^T \sigma_3 \Phi' \ ,\end{aligned} \tag{3.4}$$

as can be checked directly from (3.3).

Unlike this, the special indefinite orthogonal group $SO(1,1,\mathbb{C}) \subset O(1,1,\mathbb{C})$, whose elements can be written by mean of the exponential map as

$$A_z = e^{z\sigma_1}, \quad z \in \mathbb{C}, \quad A_z^T = A_z \ ,$$

fulfills,

$$i\sigma_2 A_z i\sigma_2 = -A_z^{-1} \quad \longrightarrow \quad A_z i\sigma_2 A_z = i\sigma_2 \ , \tag{3.5}$$

with σ_2 the second Pauli matrix, hence, $i\sigma_2$ is an invariant tensor, thus $SO(1,1,\mathbb{C})$ has an extra bilinear invariant, given by

$$\begin{aligned}\Phi^T i\sigma_2 \Phi &= (\Phi^T A_z^T)\,A_z^{-1} i\sigma_2 A_z^{-1}\,(A_z\Phi)\\ &= \Phi'^T i\sigma_2 \Phi' \ ,\end{aligned} \tag{3.6}$$

as can be checked directly from (3.5).

Note that by writing $z = \beta + i\alpha$, with $\alpha, \beta \in \mathbb{R}$, A_z can be separated into

$$A_z = h_\beta g_\alpha \ ,$$

where

$$h_\beta = e^{\beta\sigma_1} \in SO^R(1,1) \subset SO(1,1,\mathbb{C}) \ , \tag{3.7}$$

and

$$g_\alpha = e^{i\alpha\sigma_1} \in SO(1,1) \subset SO(1,1,\mathbb{C}) \ , \tag{3.8}$$

both of them are Lorentz like groups, the former is the non-compact group of the standard real boosts, and the latter is the compact group of the complex rotations.

As said above, the complex rotations group $SO(1,1)$ is the group we have chosen to build our model.

Unlike the group of the real boosts, which only has the invariants (3.4) and (3.6) inherited from $SO(1,1,\mathbb{C})$, the complex rotations group has two additional second rank tensor invariants, as it is shown below, by noticing that

$$g_\alpha^\dagger = g_\alpha^{-1} \quad \longrightarrow \quad g_\alpha g_\alpha^\dagger = \mathbb{I} \ , \tag{3.9}$$

then we have the third invariant (notice that $g_\alpha^\dagger \neq g_\alpha^T$)

$$\begin{aligned} \Phi^\dagger \Phi &= \left(\Phi^\dagger g_\alpha^\dagger\right)\left(g_\alpha \Phi\right) \\ &= \Phi'^\dagger \Phi' \ . \end{aligned} \tag{3.10}$$

Notice that (3.8) fulfills

$$\sigma_1 g_\alpha \sigma_1 = g_\alpha \quad \longrightarrow \quad g_\alpha^\dagger \sigma_1 g_\alpha = \sigma_1 \ , \tag{3.11}$$

where we have used Eq. (3.9), thus we have the fourth invariant by writing

$$\begin{aligned} \Phi^\dagger \sigma_1 \Phi &= \left(\Phi^\dagger g_\alpha^\dagger\right) g_\alpha \sigma_1 g_\alpha^\dagger \left(g_\alpha \Phi\right) \\ &= \Phi'^\dagger \sigma_1 \Phi' \ , \end{aligned} \tag{3.12}$$

as can be checked directly from (3.11).

Notice that the tensor invariants (3.5), (3.9), and (3.11), are correspondingly associated with the antisymmetric traceless part, the trace, and the symmetric traceless part of a general second rank tensor, such that both of its two indices are saturated as expected under an invariant Lorentz transformation.

By using all of those together with (3.3), the $SO(1,1)$ bilinear invariants are written. For clarity purposes we collect all of them here

$$\begin{aligned} \Phi^\dagger \Phi &= |\phi|^2 + |\varphi|^2, & \Phi^\dagger \sigma_1 \Phi &= \phi^* \varphi + \varphi^* \phi, \\ \Phi^T i\sigma_2 \Phi &= \phi\varphi - \varphi\phi, & \Phi^T \sigma_3 \Phi &= \phi^2 - \varphi^2 \ . \end{aligned} \tag{3.13}$$

We use these invariants to built the scalar potential. As shown later, by introducing the fermionic content of the model, the same kind of invariants allows us to build the interaction terms.

3.2 Scalar sector

We start by writing the kinetic term as $\partial_\mu \Phi^\dagger \partial^\mu \Phi$. Notice that it belongs to the first class of invariants (3.13), so that, the scalar sector is written as

$$\mathcal{L}_\Phi = \partial^\mu \Phi^\dagger \partial_\mu \Phi - V(\Phi) \ , \tag{3.14}$$

with the potential formed from the most general linear combination of the non trivial invariants

$$V(\Phi) = \Phi^\dagger \left(\alpha_0 \mathbb{I} + \alpha_1 \sigma_1\right) \Phi + \alpha_3 \Phi^T \sigma_3 \Phi + h.c. \tag{3.15}$$

where $\alpha_{i=0,1,3}$ are dimensionful quantities which in general can be complex.

It is worth noticing that these terms still allow for some diversity on the possible cosmological potentials one may consider. For instance, in the case of real field representations, first and fourth invariants can be combined together to provide for a whole class of systems where the fields have an independent evolution, by writing

$$\phi^2 = \Phi^\dagger \Phi + \Phi^T \sigma_3 \Phi, \quad \text{and} \quad \varphi^2 = \Phi^\dagger \Phi - \Phi^T \sigma_3 \Phi \ .$$

In such a case, the potentials $U(\phi^2)$ and $V(\varphi^2)$ written in terms of such combinations would always have a quadratic dependence on the fields.

Notice that in such a scenario as used in Ref. [65], the invariant $\Phi^\dagger \sigma_1 \Phi$ has been removed, this could be done by noticing that, unlike the other invariants, such a term is actually a pseudoscalar bilinear under the parity transformation defined as $\Phi \to \sigma_3 \Phi$, such that

$$\Phi^\dagger \sigma_1 \Phi \quad \longrightarrow \quad -\Phi^\dagger \sigma_1 \Phi \ .$$

Such a construction, however, ignores the most general complex nature of the cosmological field Φ and we will avoid it here.

3.3 Spinorial sector: adding fermions

The minimal fermionic matter content of the model is accounted by introducing a total of three spinorial fields, $\mathtt{N}^{\dot a}_{i=0,1,2}$, two of then arranged into a doublet

$$\Psi = \begin{pmatrix} \mathtt{N}^{\dot a}_1 \\ \mathtt{N}^{\dot a}_2 \end{pmatrix} \ , \tag{3.16}$$

and the remaining one treated as a singlet.

We choose fermions to be (two-component) right handed Weyl fields, such that they can be identified with those usually introduced in extensions of the standard model of particle physics in order to have massive neutrinos through the seesaw mechanism.

As shown in Appendix B.1, in which we specify our notation, the Dirac matrices are written as

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu}{}_{a\dot{c}} \\ \bar{\sigma}^{\mu\dot{a}c} & 0 \end{pmatrix} \ , \tag{3.17}$$

with,

$$\sigma^{\mu}{}_{a\dot{a}} = (\mathbb{I},\ \sigma)\,, \quad \bar{\sigma}^{\mu\dot{a}a} = (\mathbb{I},\ -\sigma)\,, \quad \bar{\sigma}^{\mu\dot{a}a} = \epsilon^{\dot{a}\dot{b}}\epsilon^{ab}\sigma^{\mu}{}_{b\dot{b}}\ ,$$

and $\sigma = (\sigma_1,\ \sigma_2,\ \sigma_3)$. In this notation the charge conjugation matrix and the β matrix (which is numerically equal to γ^0 but carrying different index structure), are respectively given by

$$C = \begin{pmatrix} \epsilon_{ac} & 0 \\ 0 & \epsilon^{\dot{a}\dot{c}} \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & \delta^{\dot{a}}{}_{\dot{c}} \\ \delta_{a}{}^{c} & 0 \end{pmatrix} \ . \tag{3.18}$$

From each Weyl field that appears in (3.16) a four-component $(a, \dot{a} = 1, 2)$ sterile Majorana neutrino $\psi_{i=1,2}$, is built by writing

$$\psi_i = \begin{pmatrix} \mathtt{N}^{\dagger}_{ia} \\ \mathtt{N}^{\dot{a}}_{i} \end{pmatrix} \ , \tag{3.19}$$

where $\mathtt{N}^{\dagger}_{ia}$ is the charge conjugate of the right-handed Weyl field (denoted by the upper index $^{\mathcal{C}}$), given by

$$\mathtt{N}^{\dagger}_{ia} = (\mathtt{N}^{\dot{a}}_{i})^{\mathcal{C}} \ . \tag{3.20}$$

The previous can be seen from (3.19) and $\psi^{\mathcal{C}} = C\bar{\psi}^{T}$ with the application of (3.18). The doublet (3.16) trasforms under $g_\alpha \in SO(1,1)$ as

$$\begin{pmatrix} \mathtt{N}^{\dot{a}}_{1} \\ \mathtt{N}^{\dot{a}}_{2} \end{pmatrix} \xrightarrow{g_\alpha} e^{i\alpha\sigma_1} \begin{pmatrix} \mathtt{N}^{\dot{a}}_{1} \\ \mathtt{N}^{\dot{a}}_{2} \end{pmatrix} = \begin{pmatrix} \mathtt{N}'^{\dot{a}}_{1} \\ \mathtt{N}'^{\dot{a}}_{2} \end{pmatrix} \ , \tag{3.21}$$

with the new Weyl fields arising from combinations and global phase changes of the previous ones. It is important to note that since the Weyl fields admit global phase transformations, it will be always possible to build a new four-component sterile Majorana neutrino

$$\psi'_i = \begin{pmatrix} \mathtt{N}'^{\dagger}_{ia} \\ \mathtt{N}'^{\dot{a}}_{i} \end{pmatrix} , \qquad \text{such that} \qquad \psi_i = \psi_i^{\mathcal{C}} \xrightarrow{g_\alpha} \psi'_i = \psi'^{\mathcal{C}}_i \ ,$$

therefore the transformation of the field ψ_i induced by the rotation (3.21) does not violate the Majorana condition. It wouldn't be the case, for instance, if we define the spinorial doublet directly in terms of ψ_i as

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \ ,$$

since a Majorana neutrino does not admit a global phase transformation, the $SO(1,1)$ rotation wouldn't be well defined.

Let's go back to the equation (3.19), over there we have chosen the Hermitian conjugate to denote the left-handed Weyl field. With this convention the kinetic term for the field (3.19) becomes (see Appendix B.3)

$$\frac{1}{2}\bar{\psi}_i i\gamma^\mu \partial_\mu \psi_i = \mathtt{N}_i^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu \mathtt{N}_i^{\dot{c}} \ ,$$

this lets us write the kinetic terms for $\psi_{i=1,2}$ in terms of (3.16) in a clearly $SO(1,1)$ invariant form, as

$$\Psi^\dagger i\sigma^\mu \partial_\mu \Psi = \mathtt{N}_1^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu \mathtt{N}_1^{\dot{c}} + \mathtt{N}_2^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu \mathtt{N}_2^{\dot{c}} \ . \tag{3.22}$$

Aside from (3.16), we define the right-handed Weyl field $\mathtt{N}_0^{\dot{a}}$ which transforms as a singlet under a $SO(1,1)$ rotation [65], so that the kinetic terms for all the fermions are

$$\mathcal{L}_\Psi = \mathtt{N}_0^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu \mathtt{N}_0^{\dot{c}} + \Psi^\dagger i\sigma^\mu \partial_\mu \Psi \ . \tag{3.23}$$

According to (3.20), the charge conjugate field of $\mathtt{N}_0^{\dot{a}}$ is $\mathtt{N}^\dagger_{0a}$, by taking the Hermitian conjugate of this last and the doublets (3.1) and (3.16) we can build the interaction terms from the linear combination of the invariants as:

$$-\mathcal{L}_I = \mathtt{N}_{0\dot{a}} \big\{ a_0 \Phi^\dagger \Psi + a_1 \Phi^\dagger \sigma_1 \Psi + a_2 \Phi^T i\sigma_2 \Psi + a_3 \Phi^T \sigma_3 \Psi \big\} + h.c. \tag{3.24}$$

where $a_{i=0,\ldots,3}$ are complex dimensionless couplings.

Notice that analogous to the invariant terms which appear in (3.13), there exist invariants formed from the doublet (3.16) taken with itself,

$$\Psi^\dagger\Psi, \quad \Psi^\dagger\sigma_1\Psi, \quad \Psi^T i\sigma_2\Psi \quad \text{and} \quad \Psi^T\sigma_3\Psi \ ,$$

all of them are allowed by the $SO(1,1)$ symmetry, however, they are not Lorentz invariant objects, therefore we take them off from the Lagrangian.

It is also worth asking if there are allowed mass terms for the fermions. We note that such a terms can be built by defining an additional doublet formed from the charge conjugate fields of $\mathtt{N}^{\dot{a}}_{i=1,2}$, as

$$\Psi^{\mathcal{C}} = \begin{pmatrix} \mathtt{N}^\dagger_{1a} \\ \mathtt{N}^\dagger_{2a} \end{pmatrix} \ . \tag{3.25}$$

The following product, which is a Lorentz-invariant scalar

$$\Psi^{\mathcal{C}\dagger}\Psi + h.c. = \mathtt{N}_{1\dot{a}}\mathtt{N}^{\dot{a}}_1 + \mathtt{N}_{2\dot{a}}\mathtt{N}^{\dot{a}}_2 + h.c. \tag{3.26}$$

clearly produces Majorana mass terms $\left(\psi_i^T C^\dagger \psi_i\right)$ for the fields $\psi_{i=1,2}$, however, in order to get a consistent transformation of $\Psi^{\mathcal{C}}$ under the symmetry, it is necessary to impose the condition that

$$\mathtt{N}'^\dagger_{ia} = (\mathtt{N}'^{\dot{a}}_i)^{\mathcal{C}} \ ,$$

which means that the components of the charge conjugate rotated doublet $\Psi'^{\mathcal{C}}$ have to be equal to the charge conjugate components of the rotated doublet Ψ'. In order to achieve this, the doublet (3.25) has to transform with the Hermitian conjugate $g^\dagger_\alpha$, as can be checked by means of the two-dimensional matrix representation of (3.2) acting on (3.16), and using (3.20). Consequently the term (3.26) is not invariant under a $SO(1,1)$ rotation and we remove it from the Lagrangian. The same occurs for all the terms formed from (3.25) and (3.16).

On the other hand, the $SO(1,1)$ symmetry allows a mass term for $\mathtt{N}^{\dot{a}}_0$ because it transforms as a singlet, however, we note that the interaction sector (3.24) is invariant under the following $U(1)$ transformation

$$\Psi \longrightarrow e^{iq}\Psi, \qquad \mathtt{N}^{\dot{a}}_0 \longrightarrow e^{iq_0}\mathtt{N}^{\dot{a}}_0 \ , \tag{3.27}$$

as long as $q = -q_0$, so that, the fields $\mathtt{N}^{\dot{a}}_{i=1,2}$ transform with the same charge and $\mathtt{N}^{\dot{a}}_{0}$ does it with the opposite. Thereby, by imposing invariance under $U(1)$ in the fermion sector, which implies lepton number conservation, we remove the singlet's mass term. We note that the same argument can be invoked in order to forbid mass terms for the fermions $\psi_{i=1,2}$, but this only confirms what the $SO(1,1)$ symmetry suggests.

Finally, the complete lagrangian we have left with, is

$$\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_\Psi + \mathcal{L}_I \ , \tag{3.28}$$

where the three sectors are respectively given by (3.14), (3.23) and (3.24).

The above Lagrangian is the most general one that can be written with $SO(1,1)$ bilinear invariant terms, it is also Lorentz invariant, P (as long as both scalar fields transform with the same parity phase) and CP invariant. As mentioned above the fermionic sector is $U(1)$ invariant, similarly, there is $U(1)$ invariance in the scalar sector, as long as both ϕ and φ transform with the same charge and the couplings α_i and a_i be complex.

3.4 Diagonalization of the Lagrangian.

We now procede to diagonalize the Lagrangian, this in order to get separated dynamics for the fields that we will identify with the inflaton and Q. (The details can be seen in appendix C.1). After diagonalization, the Lagrangian (3.28) becomes

$$\mathcal{L} = \mathcal{L}_{\boldsymbol{\varphi}} + \mathcal{L}_F + \mathcal{L}_I \ , \tag{3.29}$$

where the first term on the right hand side (RHS) corresponds to the scalar sector, which is now written as

$$\mathcal{L}_{\boldsymbol{\varphi}} = \partial^\mu \boldsymbol{\varphi}^\dagger \partial_\mu \boldsymbol{\varphi} - \boldsymbol{\varphi}^\dagger \mathbb{M} \boldsymbol{\varphi} \ , \tag{3.30}$$

where $\boldsymbol{\varphi}$ [see equation C.5], which is given by

$$\boldsymbol{\varphi} = \begin{pmatrix} \mathrm{Q} \\ \xi \end{pmatrix} \ , \tag{3.31}$$

is the doublet formed from the complex scalar fields Q and ξ which we have identified with the sources for DE and inflation respectively. The corresponding masses are the eigenvalues that appear in the matrix [see (C.7)]

$$\mathbb{M} = \begin{pmatrix} m^2 & 0 \\ 0 & M^2 \end{pmatrix} \ .$$

As stated in Appendix C.1, above masses, written in terms of α_i, are expressed as

$$M^2 = \mu_0^2 + \mu^2 \ , \quad \text{whereas} \quad m^2 = \mu_0^2 - \mu^2 \ , \tag{3.32}$$

where $\mu_0^2 = 2Re\ \alpha_0$ and $\mu^2 = 2\sqrt{(Re\ \alpha_1)^2 + |\alpha_3|^2}$.

It is important to remark that, by assuming that all involved scales are naturally about the same order, namely $\mu_0^2 \sim \mu^2$, a fine-tuning in the masses can be incorporated just to have

$$M^2 \gg m^2 \ ,$$

thus, our model naturally allows a marked hierarchy of the masses that allows us to identify ξ as the inflaton and Q as the quintessence field.

Notice that even though in Eq. (3.14) we started with a coupled system of complex fields, after field rotation we have ended with a new description where Q and ξ degrees of freedom have been decoupled. However, we should also notice that, even though this is a more suitable way of writing the potential, it is on the cost of hiding the $SO(1,1)$ symmetry, which now is not explicit in the Lagrangian.

Turning back to the second term in the RHS of (3.29), it corresponds to the kinetic energy terms of the spinorial sector [see equation (C.18)], it is given by

$$\mathcal{L}_F = \mathtt{N}_0^{\dagger a} i\sigma^{\mu}_{a\dot{c}} \partial_\mu \mathtt{N}_0^{\dot{c}} + \mathbf{F}^\dagger i\sigma^\mu \partial_\mu \mathbf{F} \ . \tag{3.33}$$

In the previous, the quantity $\mathbf{F}$ is given by [see (C.14)]

$$\mathbf{F} = \begin{pmatrix} \mathtt{F}_1^{\dot{a}} \\ \mathtt{F}_2^{\dot{a}} \end{pmatrix} . \tag{3.34}$$

It corresponds to the doublet arising from (3.16) after a $SO(2)$ rotation as can be seen in equation (C.15). This is a consequence of the diagonalization of the scalar sector.

The third term in the RHS of (3.29) corresponds to the scalar to fermion interactions [see equation (C.16)], it is given by

$$-\mathcal{L}_I = N_{0\dot{a}}\{\boldsymbol{\varphi}^\dagger \mathbb{G}_1 \mathbf{F} + \boldsymbol{\varphi}^T \mathbb{G}_2 \mathbf{F}\} + h.c. \ , \tag{3.35}$$

where the new coupling constants, which are just simple linear combinations of the original a_i constants written in Eq. (3.24), are contained in the matrices [see Eq. (C.17)]

$$\mathbb{G}_1 = \begin{pmatrix} 0 & g_2 \\ h_1 & 0 \end{pmatrix} \qquad \mathbb{G}_2 = \begin{pmatrix} g_1 & 0 \\ 0 & -h_2 \end{pmatrix} \ .$$

Former couplings can be written in a more useful way, as

$$-\ \mathcal{L}_I \ = \ \mathtt{N}_{0\dot{a}}\{g_1 \mathrm{Q}\mathtt{F}_1^{\dot{a}} \ + \ g_2 \mathrm{Q}^* \mathtt{F}_2^{\dot{a}} \ + \ h_1 \xi^* \mathtt{F}_1^{\dot{a}} \ - \ h_2 \xi \mathtt{F}_2^{\dot{a}}\} \ + \ h.c. \tag{3.36}$$

The last two terms of Eq. (3.36) provide the inflaton decay channels, $\xi \to \mathtt{N}_0\mathtt{F}_i$, that are required for reheating after inflation. The sudden evaporation of inflaton energy would inject entropy to the emptied Universe by inflation. Assuming that such a process is efficient enough, the reheating temperature should be $T_r \sim 6 \times 10^{-3}\ max\{|h_1|, |h_2|\}\ M_{\rm pl}$ (more of this in section 4.3).

Since the fermions on final states are assumed to be right handed neutrinos they should provide the portal, through the standard couplings $\bar{L}\widetilde{H}\mathtt{N}_0$ and $\bar{L}\widetilde{H}\mathtt{F}_i$, to produce all types of SM fields, which in turn should thermalize producing the primordial plasma (more of this in section D.6).

The Lagrangian (3.29) is invariant under the $U(1)$ symmetry in the fermion sector as long as both, the singlet and the doublet (3.34) transform with opposite charges, [this is guaranteed by (3.27) together with (C.15)]. However, after the diagonalization, as for the scalar sector, the $SO(1,1)$ symmetry is not explicit in the Yukawa Lagrangian anymore, instead of it the total $U(1)$ symmetry has emerged in the scalar sector, i.e., now it is not necessary the requirement that both scalars fields transform with the same charge.

3.5 Sourcing neutrino mass with DE.

At the end of inflation the ξ field suddenly evaporates completely, such that its energy density becomes null, sitting the inflaton field at its zero value which makes its couplings of no further relevance for thermal history. On the other hand, as we have already discussed in the part of Chapter 2 devoted to Quintessence, the Q field would remain trapped on its initial homogeneous configuration all along the Universe evolution, perhaps changing quite slowly until recent times, when it is still slow-rolling down its almost flat potential while causing the Universe accelerated expansion.

By inserting the Q false vacuum, conveniently defined as $\langle \mathrm{Q} \rangle/\sqrt{2}$, back in Eq. (3.36), one immediately realizes that due to the couplings provided by the $SO(1,1)$ model, DE naturally generates masses for the right handed neutrinos, given as

$$\mathcal{L}_m = m_1 \mathtt{N}_{0\dot{a}} \mathtt{F}_1^{\dot{a}} + m_2 \mathtt{N}_{0\dot{a}} \mathtt{F}_2^{\dot{a}} + h.c. \; , \tag{3.37}$$

where $m_i = g_i \langle \mathrm{Q} \rangle/\sqrt{2}$. These mass terms, as discussed in detail in Appendix C.2.4, give rise to two degenerate massive Majorana neutrinos, $\nu_{1,2}$ [see definition (C.52)], for which one can write

$$- \mathcal{L}_m = \frac{1}{2} m_k \left(\bar{\nu}_1 \nu_1 + \bar{\nu}_2 \nu_2 \right) \; . \tag{3.38}$$

This is a striking result, which connects the seesaw mechanism, and thus the origin of standard neutrino mass, to the origin of DE.

Here, we have implicitly written the Majorana condition, namely $\bar{\nu} = \nu^T C$, with C the charge conjugation matrix [see equation (3.18)]. Likewise, the mass m_k appearing in Eq. (3.38), as defined in (C.50), is given by

$$m_k = \frac{a_c \langle \mathrm{Q} \rangle}{\sqrt{2}}, \tag{3.39}$$

where the effective coupling a_c [see Appendix C.2 definition (C.37)], is a free parameter coming from the original couplings,

$$a_c = \sqrt{|g_1|^2 + |g_2|^2} \; .$$

We note that by choosing a_c in the interval

$$10^{-5} \; \lesssim \; a_c \; \lesssim \; 10^{-3} \; , \tag{3.40}$$

which seems reasonable, we can get right handed neutrino masses in the range of 10^{13} GeV $\lesssim m_k \lesssim 10^{15}$ GeV, which are values around those needed to implement the standard seesaw mechanism (more on this in section 4.2).

Another immediate outcome of the present model is the alignment to mass terms of couplings among quintessence quantum excitations, $\mathcal{X}$, and neutrinos. Setting in the excitations over the false vacuum, by redefining

$$\mathrm{Q} = \frac{(\langle \mathrm{Q} \rangle + \mathcal{X})}{\sqrt{2}} \ ,$$

it is clear that after diagonalizing fermion masses, one gets

$$-\mathcal{L}_{I\mathcal{X}} = \frac{a_c}{2\sqrt{2}} \mathcal{X} \left(\bar{\nu}_1 \nu_1 + \bar{\nu}_2 \nu_2 \right) \ . \tag{3.41}$$

As will be said later, this coupling is relevant to thermal history.

3.6 The complete Lagrangian summarized

In this section, we summarize the transformations applied to the Lagrangian. The mathematical manipulations are explained in detail in appendices C.1 and C.2.

The original Lagrangian we start with is

$$\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_\Psi + \mathcal{L}_I \ ,$$

where

$$\begin{aligned} \mathcal{L}_\Phi &= \partial^\mu \Phi^\dagger \partial_\mu \Phi - V(\Phi) \ , \\ V(\Phi) &= \Phi^\dagger \left(\alpha_0 \mathbb{I} + \alpha_1 \sigma_1 \right) \Phi + \alpha_3 \Phi^T \sigma_3 \Phi + h.c. \ , \end{aligned}$$

and

$$\begin{aligned} \mathcal{L}_\Psi &= \mathtt{N}_0^{\dagger a} i \sigma^\mu_{a\dot{c}} \partial_\mu \mathtt{N}_0^{\dot{c}} + \Psi^\dagger i \sigma^\mu \partial_\mu \Psi, \\ -\mathcal{L}_I &= \mathtt{N}_{0\dot{a}} \left\{ a_0 \Phi^\dagger \Psi + a_1 \Phi^\dagger \sigma_1 \Psi + a_2 \Phi^T i \sigma_2 \Psi + a_3 \Phi^T \sigma_3 \Psi \right\} + h.c. \end{aligned}$$

After diagonalization we are left with

$$\mathcal{L} = \mathcal{L}_\varphi + \mathcal{L}_F + \mathcal{L}_I \ ,$$

where

$$
\begin{aligned}
\mathcal{L}_{\boldsymbol{\varphi}} &= \partial^\mu \boldsymbol{\varphi}^\dagger \partial_\mu \boldsymbol{\varphi} - \boldsymbol{\varphi}^\dagger \mathbb{M} \boldsymbol{\varphi} \ , \\
\mathcal{L}_F &= \mathtt{N}_0^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu \mathtt{N}_0^{\dot{c}} + \mathbf{F}^\dagger i\sigma^\mu \partial_\mu \mathbf{F} \ , \\
-\mathcal{L}_I &= \mathtt{N}_{0\dot{a}} \{ \boldsymbol{\varphi}^\dagger \mathbb{G}_1 \mathbf{F} + \boldsymbol{\varphi}^T \mathbb{G}_2 \mathbf{F} \} + h.c.
\end{aligned}
$$

After perfoming the parametrization (C.20) the three previous terms become

$$
\begin{aligned}
\mathcal{L}_{\boldsymbol{\varphi}} &= \partial^\mu \boldsymbol{\varphi}_R^T \partial_\mu \boldsymbol{\varphi}_R + \boldsymbol{\varphi}_R^T \mathbb{M} \boldsymbol{\varphi}_R + \mathcal{T}(\boldsymbol{\varphi}_R, \mathbb{P}), \\
-\mathcal{L}_I &= \mathtt{N}_{0\dot{a}} \boldsymbol{\varphi}_R^T \mathbb{G} \mathbf{F}' + h.c. \\
\mathcal{L}_F &= \mathtt{N}_0^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu \mathtt{N}_0^{\dot{c}} + \mathbf{F}'^\dagger i\sigma^\mu \partial_\mu \mathbf{F}' + \frac{\partial_\mu \vartheta}{\langle \mathrm{Q} \rangle} \mathbf{F}'^\dagger \sigma^\mu \sigma_3 \mathbf{F}' \ ,
\end{aligned}
$$

Next, in order to transform to the massive neutrino base, we use the intermediate transformation by means of the definition (C.35) to get

$$
\begin{aligned}
-\mathcal{L}_I &= \mathtt{N}_{0\dot{a}} \boldsymbol{\varphi}_R^T \mathbb{G}' \boldsymbol{\eta} + h.c. \ , \\
\mathcal{L}_F &= \mathtt{N}_0^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu \mathtt{N}_0^{\dot{c}} + \boldsymbol{\eta}^\dagger i\sigma^\mu \partial_\mu \boldsymbol{\eta} + \frac{\partial_\mu \vartheta}{\langle \mathrm{Q} \rangle} \boldsymbol{\eta}^\dagger \sigma^\mu \mathbb{Y} \boldsymbol{\eta} \ ,
\end{aligned}
$$

[for details see Eqs. (C.38) and (C.41)].

Finally, by using equations (C.42) and (C.56) we separate the interaction Lagrangian into

$$
\mathcal{L}_I = \mathcal{L}_g + \mathcal{L}_m + \mathcal{L}_{I\mathfrak{X}},
$$

where

$$
-\mathcal{L}_g = \frac{1}{4} C_1(\theta, \vartheta) |\xi| \left(\mathtt{K}_{1\dot{a}} \mathtt{K}_1^{\dot{a}} + \mathtt{K}_{2\dot{a}} \mathtt{K}_2^{\dot{a}} \right) + \frac{1}{2\sqrt{2}} C_2(\theta, \vartheta) |\xi| \left(\mathtt{K}_{1\dot{a}} - i \mathtt{K}_{2\dot{a}} \right) \mathtt{K}_3^{\dot{a}} + h.c. \tag{3.42}
$$

[where the couplings are given in (C.40)], and

$$
-\mathcal{L}_m = \frac{1}{2} m_k \left(\mathtt{K}_{1\dot{a}} \mathtt{K}_1^{\dot{a}} + \mathtt{K}_{2\dot{a}} \mathtt{K}_2^{\dot{a}} \right) + h.c. \tag{3.43}
$$

$$
-\mathcal{L}_{I\mathfrak{X}} = \frac{a_c}{2\sqrt{2}} \mathfrak{X} \left(\mathtt{K}_{1\dot{a}} \mathtt{K}_1^{\dot{a}} + \mathtt{K}_{2\dot{a}} \mathtt{K}_2^{\dot{a}} \right) + h.c. \tag{3.44}
$$

The previous equations are equivalent to Eqs. (3.38) and (3.41).

Eq. (3.42) provides the channel for production of heavy neutrinos by sudden decay of ξ at the end of inflation. As said above, because of inflaton's energy density nulls, this equation does not play any role for thermal history.

Conversely, Eqs. (3.43) and (3.44) are relevant to the thermal history and deserve careful study. They show that two neutrinos have acquired mass and couple to the $\mathcal{X}$ field (the third neutrino remains masless and decoupled).

Because the heavy neutrinos enter as singlets in the minimal extension of SM, they couple with the Higgs and SM leptons, consequently, there exist the portal to complete the reheating and the production of all the particles of the standard model through the Higgs portal.

As for Eq. (3.44), it allows out-of-equilibrium processes leading to the production of $\mathcal{X}$ quanta, which contributes to the total ultra-relativistic energy density. As said in the previous chapter, such production could impact the initial conditions of Big Bang Nucleosynthesis, so it also has to be studied.

Finally, because of the seesaw mechanism, Eqs. (3.43) and (3.44) will produce a mixing between heavy (sterile) and light (active) neutrinos, which together with photons of the CMB, permeate the background Universe in the form of radiation. Consequently, because of the temperature of the cosmological neutrino background, the potential $V(\mathcal{X})$ has to be thermally corrected. Such corrections could increase the scalar mass in such a way that the flatness of the potential could we wiped, leading to a violation of the slow-roll condition.

In the next chapter, we will proceed with the analysis of all this phenomenology. The outcomes will establish the consistency and viability of our model.

Chapter 4

Phenomenology

4.1 Introduction

At the end of the inflationary age, the inflaton leaves the slow-roll regime and decays suddenly into heavy right-handed neutrinos through the couplings defined in (3.42) [or alternatively in Eq. (3.36)] reheating the universe, such that its energy density becomes zero. On the other hand, the Q-field remains trapped in its false vacuum state, which, as mentioned in the previous chapter, we have conveniently defined as $\langle \mathrm{Q} \rangle / \sqrt{2}$.

As explained in the Appendix C.2, we have used the false vacuum to write the Q-field on a polar base as

$$\mathrm{Q} = \frac{(\langle \mathrm{Q} \rangle + \mathcal{X})}{\sqrt{2}} e^{i\vartheta/\nu}, \tag{4.1}$$

wherein, both degrees of freedom of the complex field Q are now described by the real scalar fields $\mathcal{X}$ and ϑ.

The new field $\mathcal{X}$ has a null vacuum state[1], in addition, we assumed that it evolves under the slow-roll regime, therefore, it remains close to its zero value for almost the entire life of the Universe, so that the DE density can be written as

$$\rho_{DE} = m^2 \mathrm{Q}\mathrm{Q}^* = \frac{1}{2} m^2 \langle \mathrm{Q} \rangle^2 \ . \tag{4.2}$$

[1]Since the Q field is released from its fixed point at late time, in this parameterization, is the $\mathcal{X}$ field which gets released from its zero value at late time.

In quintessence models, this value equals the observed DE density

$$\rho_{DE} = M_{\rm pl}^2 \Lambda,$$

consequently

$$m^2 = 2\frac{M_{\rm pl}^2}{\langle \mathrm{Q} \rangle^2}\Lambda \ .$$

Then, by assuming

$$\langle \mathrm{Q} \rangle \sim m_{pl} \ , \tag{4.3}$$

where m_{pl} is the Planck mass, and by using the definition of the reduced Plank mass, $M_{\rm pl} = m_{pl}/\sqrt{8\pi}$, we can write

$$m^2 = \frac{2}{\sqrt{8\pi}}\Lambda \ ,$$

then by knowing that [see Eq. (D.21)]

$$\Lambda \approx 4.261 \times 10^{-84} \ \mathrm{GeV}^2 \ ,$$

we arrive to the known result which asserts that the mass of Q should be as small as

$$m \approx 5.8 \times 10^{-34} \ \mathrm{eV} \ . \tag{4.4}$$

As stated in the previous chapter (and deeply developed in Appendix C.2), we use $\langle \mathrm{Q} \rangle$ to generate neutrino masses, as a consequence of this, new interactions of relevance for thermal history arise among the massive neutrinos and the field $\mathcal{X}$, as it is described by Eq. (3.44) [or alternatively (3.41)]. By writing the part of this equation, that corresponds to only one of the neutrinos, one has

$$-\mathcal{L}_{I\mathcal{X}_i} = \frac{a_c}{2\sqrt{2}}\mathcal{X}\bar{\nu}_i\nu_i. \tag{4.5}$$

The annhilation amplitude for this Lagrangian is shown in Fig. 4.1.

Also, because of the standard model minimal extension, right-handed neutrinos interact with both, the Higgs and SM leptons [see Eq. (B.52)], for instance, for one neutrino this coupling (in two-component notation) has the form

$$-\mathcal{L}_{yuk} = y^{\nu}\ell_{\dot{a}}^{\dagger}\tilde{\Phi}\mathrm{K}^{\dot{a}} + h.c.$$

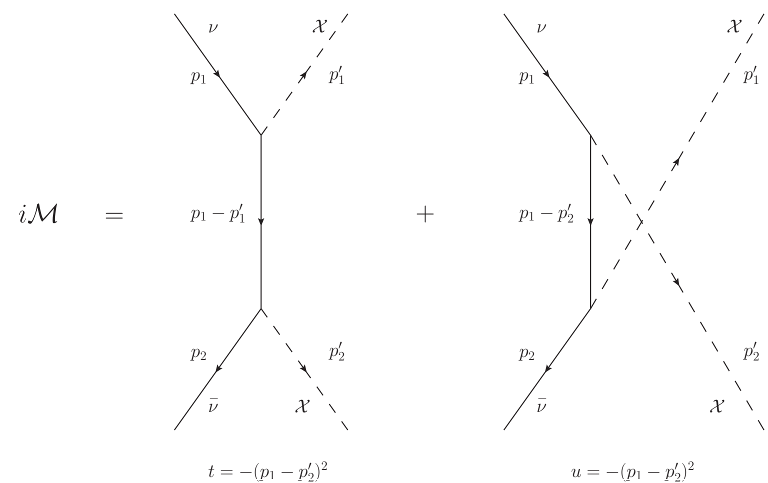

Figure 4.1: The annihilation channels allowed by (4.5), where p_1 and p_2 are respectively the 4-momenta of the incoming particles ν and $\bar{\nu}$, and where p'_1 and p'_2 are the 4-momenta of the outgoing particles $\mathcal{X}\,\mathcal{X}$.

where ℓ_a is the lepton doublet [for definition see Eq. (B.44)] and $\tilde{\Phi}$ is the conjugated Higgs doubled [see Eq. (B.47)]. In terms of the Weyl fields the previous Lagrangian reads

$$-\mathcal{L}_{yuk} = y^{\nu} h^{0\dagger} \mathtt{U}_{\dot{a}} \mathtt{K}^{\dot{a}} - y^{\nu} h^{-} \chi_{\dot{a}} \mathtt{K}^{\dot{a}} + h.c. \tag{4.6}$$

where $\mathtt{U}_{\dot{a}}$ and $\chi_{\dot{a}}$ are respectively the active neutrinos and the charged leptons and $h^{0\dagger}$ and h^{-} are the (conjugated) neutral and charged Higgs fields.

Therefore, there exist channels for decay of heavy neutrinos into Higgs and leptons

$$\mathtt{K}^{\dot{a}} \longrightarrow \mathtt{U}^{\dagger}_{a} + h^{0+} \qquad \text{and} \qquad \mathtt{K}^{\dot{a}} \longrightarrow \chi^{\dagger}_{a} - h^{-} \ , \tag{4.7}$$

and channels for coannihilation of heavy neutrinos into pair of Higgses

$$\overline{\nu} + \nu \longrightarrow h^{+} + h^{-} \quad \text{and} \quad \overline{\nu} + \nu \longrightarrow h^{0} + h^{0\dagger} \ , \tag{4.8}$$

as shown for the last in Fig. 4.2.

The previous processes are of great interest in studying the phenomenology of our model and have to be seriously taken into account to study the thermal history of the Universe. Furthermore, because of the mass m_k [which appears

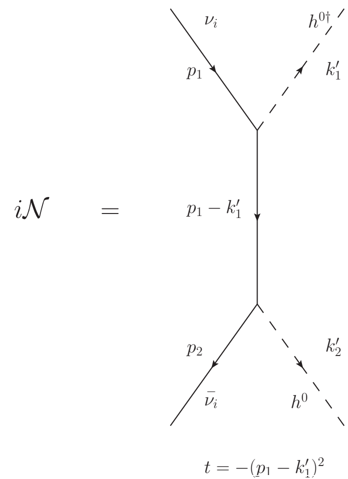

Figure 4.2: The annihilation channel allowed by (4.6), where p_1 and p_2 are respectively the 4-momenta of the incoming particles ν and $\bar{\nu}$, and where k_1' and k_2' are the 4-momenta of the outgoing particles h^0 and $h^{0\dagger}$.

in Eq. (3.43)] feeds the seesaw mechanism, heavy neutrinos couple to the Higgs field and light (observed) neutrinos that emerge from seesaw, yielding effective couplings of the form

$$-\mathcal{L}_{I\mathfrak{X}} = \frac{a_c}{2\sqrt{2}}\mathfrak{X}\left[-\left(\frac{m_D^\nu}{m_k}\right)^2 \mathtt{N}_{\dot{a}}\mathtt{N}^{\dot{a}}\right] + h.c.$$

where $\mathtt{N}^{\dot{a}}$ are the observed (active) light neutrinos. As said above, there are corrections to the mass given in Eq. (4.4), that is something that must be revised (more on this later).

Finally, some phenomenology related to the phases of the complex scalar fields is revised.

Exploring the phenomenology of the $SO(1,1)$ model constitutes the principal goal of this work, thus, we will adress all of these issues through this chapter. Our results have been reported in Ref. [94].

4.2 Energy scales

As said before, the mass of heavy neutrinos is

$$m_k = \frac{1}{\sqrt{2}} a_c \langle \mathrm{Q} \rangle, \tag{4.9}$$

where the Yukawa-like parameter a_c was defined in Eq. (C.37), which, in order to realize a perturbative model it is requiered to be less than or equal to unity.

The mass of Dirac neutrinos, by the Higgs mechanism [see Appendix B.5], is

$$m_D^\nu = \frac{1}{\sqrt{2}} y^\nu \langle H \rangle \ , \tag{4.10}$$

where y^ν is the Yukawa coupling and where the vacuum expectation value of the Higgs is [6]

$$\langle H \rangle = 246 \text{ GeV}. \tag{4.11}$$

Notice that with (D.2), (4.3) y (4.11) we get

$$\frac{\langle H \rangle}{\langle \mathrm{Q} \rangle} = 2.015 \times 10^{-17}, \tag{4.12}$$

also

$$\frac{\langle H \rangle^2}{\langle \mathrm{Q} \rangle} = 4.957 \times 10^{-6} \text{ eV}. \tag{4.13}$$

From the seesaw mechanism, the observed neutrino mass is

$$m_\nu = \frac{(m_D^\nu)^2}{m_k}. \tag{4.14}$$

According to the data [6], the observed mass is

$$5 \times 10^{-2} \text{ eV} \ \lesssim \ m_\nu \ \lesssim \ 10^{-1} \text{ eV}. \tag{4.15}$$

By substituting, (4.9) and (4.10) into (4.14) we get

$$m_\nu = \frac{\sqrt{2}}{2} \frac{(y^\nu)^2}{a_c} \frac{\langle H \rangle^2}{\langle \mathrm{Q} \rangle} \ ,$$

with the previous equation and the inequality (4.15) we obtain

$$5\times\frac{2}{\sqrt{2}}\frac{\langle \mathrm{Q}\rangle}{\langle H\rangle^2}\times 10^{-2}\ \mathrm{eV}\ \lesssim\ \frac{(y^\nu)^2}{a_c}\ \lesssim\ \frac{2}{\sqrt{2}}\frac{\langle \mathrm{Q}\rangle}{\langle H\rangle^2}\times 10^{-1}\ \mathrm{eV}, \tag{4.16}$$

and because of Eq. (4.13) we arrive to

$$5\times 2.853\times 10^3\ \lesssim\ \frac{(y^\nu)^2}{a_c}\ \lesssim\ 2.853\times 10^4\ , \tag{4.17}$$

which is the same as

$$3.505\times 10^{-5}\ \lesssim\ \frac{a_c}{(y^\nu)^2}\ \lesssim\ 7.01\times 10^{-5}\ . \tag{4.18}$$

By taking the mean value of this we can write

$$\frac{a_c}{(y^\nu)^2}\approx 5.257\times 10^{-5}.$$

Thus, the Yukawa depending on a_c becomes

$$y^\nu\approx\sqrt{a_c/(5.257\times 10^{-5})},$$

from which we can write

$$\begin{array}{lcl} a_c = 10^{-3} & \rightarrow & y^\nu = 4.361 \\ a_c = 10^{-4} & \rightarrow & y^\nu = 1.379 \\ a_c = 10^{-5} & \rightarrow & y^\nu = 0.436 \end{array} \tag{4.19}$$

4.3 Reheating

The reheating temperature (T_r) is the maximun temperature the Universe reaches just after the inflationary age ends.

We assume here the sudden decay scenario, in which it is argued that the whole energy density contained in the inflaton field convert instantaneusly into radiation. It occurs when the value of the Hubble parameter (H) is around to that of the decay width of the inflaton for the relevant channel.

In the $SO(1,1)$ model, the channel of disintegration of the inflaton into matter is given by the Lagrangian (3.36)

$$-\mathcal{L}_I = N_{0\dot{a}}\{g_1 \mathrm{Q} F_1^{\dot{a}} + g_2 \mathrm{Q}^* F_2^{\dot{a}} + h_1 \xi^* F_1^{\dot{a}} - h_2 \xi F_2^{\dot{a}}\} + h.c. \tag{4.20}$$

which provide the channels

$$\xi \longrightarrow N_0 + F_i \ , \tag{4.21}$$

where F_i, with $i = 1, 2$ are the fields defined in (3.34). Alternatively, in the massive neutrino base, the same channels are described by the lagrangian (3.42), with the couplings defined in Eq. (C.40).

A rough estimation can be easily obtained (see for instance [15, 95]). The reheating happens when

$$\mathsf{H} \approx \Gamma_\xi \ , \tag{4.22}$$

where Γ_ξ is the decay width for the proccess (4.21), which is

$$\Gamma_\xi = \frac{h_i^2 M}{8\pi} \ ,$$

where h_i are the couplings appearing in (4.20), and where M is the inflaton mass. By assuming that the plasma goes into thermal equilibrium quickly after the sudden decay, when $T = T_r$, the radiation energy density can be written as

$$\rho_r = g_* \frac{\pi^2}{30} T_r^4 \ ,$$

where g_* are the effective relativistic degrees of freedom in energy density in equilibrium. Next, by using (4.22) into the first Friedmann equation (for a flat Universe) given in Eq. (2.6) , we get

$$\rho_r = 3\Gamma_\xi^2 M_{\mathrm{pl}}^2 \ ,$$

then, by using all three previous equations we arrive to

$$T_r = \left(\frac{90}{64 g_*}\right)^{1/4} \frac{|h_i|}{\pi} 10^{-2} M_{\mathrm{pl}}^{1/2} M^{1/2} \ ,$$

next, by considering $M \sim m_{pl}$, and by using Eq. (D.4) we obtain

$$T_r = \left(\frac{90}{512\pi g_*}\right)^{1/4} \frac{|h_i|}{\pi} m_{pl} \ ,$$

Since the final states are three Majorana neutrinos, the effective degrees of freedom equal

$$g_* = \frac{7}{8} \times 6 = 5.25 ,$$

then we get

$$T_r = 10^{-1}|h_i| \ m_{pl} ,$$

finally, by assuming that[2] $|h_i| \sim (10^{-5} - 10^{-3})$, the reheating temperature sould be around

$$T_r \sim (10^{13} - 10^{15}) \text{ GeV} .$$

This is the temperature of the primordial neutrino plasma, just after the disintegration of the inflaton. Because of the coupling among neutrinos and DE field they acquire mass. When the temperature drops due to expansion, neutrinos decay and annihilate into SM particles as well as quintessence quanta, as we will explain next.

4.4 Quintessence quanta and SM particles production

Because $\mathcal{X}$-particles have the same mass associated with Q they are ultra-relativistic, and thus, right-handed neutrinos pair annihilation constitutes a source that can inject an extra degree of freedom into the cosmological plasma during the radiation dominated age. Hence, as explained at the end of Chapter 2, it is necessary to check whether the presence of such radiation is compatible or not with the initial conditions of SBBN such it keeps its predictions.

Due to the annihilation process shown in Fig. 4.1, the system formed by the neutrinos and $\mathcal{X}$-particles can go out of equilibrium, such that the energy density of the latter becomes relevant, otherwise, in equilibrium, the pair annihilation can be reversed yielding to a net increment of zero in the total radiation energy density.

[2] Notice these values are the same assumed for a_c in Eq. (3.40), thus, the couplings g_i and h_i appearing in Eq. (4.20) are all of the same order.

As the whole process is controlled solely by the coupling a_c, and thus by the scale of right handed neutrino masses, the analysis of such a process should constrain this parameter in order to avoid perturbing the predictions of SBBN through an excess of injected $\mathcal{X}$.

Therefore, to evaluate the total impact on the Hubble parameter, it is necessary to determine the out-of-equilibrium radiation production along with the one in equilibrium. In order to do that, we evolve the Boltzmann equation for the *radiation number density*, $n_{\mathcal{X}}$, as a function of the temperature in an FLRW Universe. Let us then introduce the Boltzmann equation.

4.4.1 Introducing The Boltzmann Equation

The Boltzmann equation is the standard tool to study the evolution of energy densities when processes beyond thermal equilibrium are involved. Due to the relevance of this equation, we have included in Appendix E.1 a deeper review of it. Based on that, in this section, we set the needed Boltzmann equations for our analysis.

As explained in Appendix E.1, for a process

$$1 + 2 \leftrightarrow 3 + 4 \ ,$$

the evolution of the number density of species 1 is governed by the equation [see Eq. (E.12)]

$$\frac{1}{a^3}\frac{d(a^3 n_1)}{dt} = -A n_1 n_2 + B n_3 n_4 \ , \tag{4.23}$$

where

$- A n_1 n_2$: describes the annihilation of $1 + 2$ (production of $3 + 4$),

$+ B n_3 n_4$: describes the annihilation of $3 + 4$ (production of $1 + 2$).

For a process like the one shown in Fig. 4.1, namely

$$\bar{\nu} + \nu \leftrightarrow \mathcal{X} + \mathcal{X} \tag{4.24}$$

we can study the production of specie $\mathcal{X}$ by means of

$$\frac{1}{a^3}\frac{d(a^3 n_{\mathcal{X}})}{dt} = -A_{\mathcal{X}} (n_{\mathcal{X}})^2 + B_{\mathcal{X}} (n_\nu)^2 \ , \tag{4.25}$$

wherein we have used the fact that $n_{\bar{\nu}} = n_\nu$.

Notice that, in equilibrium, the density $n_{\mathcal{X}}$ is constant, it only dilutes due to the Universe expansion, so that, the right-hand side of (4.25) equals zero, thus, each of the coefficients $A_{\mathcal{X}}$ and $B_{\mathcal{X}}$ can be expressed in terms of the other as follows

$$- A_{\mathcal{X}}(n_{\mathcal{X}})^2_{eq} + B_{\mathcal{X}}(n_\nu)^2_{eq} = 0 \quad \Longrightarrow \quad A_{\mathcal{X}} = B_{\mathcal{X}} \frac{(n_\nu)^2_{eq}}{(n_{\mathcal{X}})^2_{eq}} \,, \tag{4.26}$$

by substituting this into the equation (4.25), the production Boltzmann equation for $n_{\mathcal{X}}$, becomes

$$\frac{1}{a^3}\frac{d(a^3 n_{\mathcal{X}})}{dt} = \langle \sigma_{\mathcal{X}} v_r \rangle \left[n_\nu^2 - n_{\mathcal{X}}^2 \frac{(n_\nu)^2_{eq}}{(n_{\mathcal{X}})^2_{eq}} \right], \tag{4.27}$$

wherein we have used (E.13) to write

$$B_{\mathcal{X}} = \langle\, \sigma_{\mathcal{X}} v_r \rangle \,, \tag{4.28}$$

which is the thermally averaged cross-section (TACS), for the annihilation process $\bar{\nu} + \nu$ into pairs $\mathcal{X} + \mathcal{X}$, as it was defined in appendix E.2, and which we will calculate below.

Notice that the Boltzmann equation given in Eq. (4.27) involves the neutrino number density n_ν as source, therefore, it is neccesary writting the corresponding Boltzmann equation for this last, which in turn involves its own collision term raising from all the processes for the Higgs channel. Let us proceed next with the detailed analysis.

By starting our calculation with the process (4.24), we can write the neutrino annihilation Boltzmann equation, such as

$$\frac{1}{a^3}\frac{d(a^3 n_\nu)}{dt} = -B_{\mathcal{X}}(n_\nu)^2 + A_{\mathcal{X}}(n_{\mathcal{X}})^2,$$

next, by noticing that the condition (4.26) also holds for the previous equation, it becomes

$$\frac{1}{a^3}\frac{d(a^3 n_\nu)}{dt} = \langle\, \sigma_{\mathcal{X}} v_r \rangle \left[(n_\nu)^2_{eq} \frac{n_{\mathcal{X}}^2}{(n_{\mathcal{X}})^2_{eq}} - n_\nu^2 \right], \tag{4.29}$$

however, it is not enough to study the evolution of the density n_ν, because, as said above, there are other channels involving annihilation of neutrino pairs, therefore it is necessary to write down the corresponding Boltzmann equations, let us do it.

For the annihilation channel $\bar{\nu}_i + \nu_j \longrightarrow h^0 + h^{0\dagger}$, allowed by equation (4.6), we have

$$\frac{1}{a^3}\frac{d(a^3 n_\nu)}{dt} = \langle\, \sigma_{H} v_r \rangle \left[(n_\nu)^2_{eq} \frac{n_H^2}{(n_H)^2_{eq}} - n_\nu^2 \right],$$

where n_H $[(n_H)_{eq}]$, is the Higgs number density [in equilibrium], and where $\langle\, \sigma_H v_r \rangle$ is the termally averaged cross-section for the channel, with,

$$\sigma_H = \sigma_{\bar{\nu}_i \nu_j \to h^0 h^{0\dagger}}, \tag{4.30}$$

which we will calculate below.

Because of the Higgs field is strongly coupled to the other SM fields, we can assume that $n_H \approx (n_H)_{eq}$, consequently, the previous equation becomes

$$\frac{1}{a^3}\frac{d(a^3 n_\nu)}{dt} = \langle\, \sigma_H v_r \rangle \left[(n_\nu)^2_{eq} - n_\nu^2 \right]. \tag{4.31}$$

As shown in Eq. (4.8), there is a second channel allowed by Eq. (4.6), namely, $\bar{\nu}_i + \nu_j \longrightarrow h^+ + h^-$. It turns out that, because of the $SU(2)$ symmetry, the cross-sections for the Higgs channel fulfill that

$$\sigma_{\bar{\nu}_i \nu_j \to h^+ h^-} = \sigma_{\bar{\nu}_i \nu_j \to h^0 h^{0\dagger}} = \sigma_H, \tag{4.32}$$

therefore, the annihilation Boltzmann equation for the second channel coincides with (4.31).

It is important to note that, we also have to include the decay channels given in (4.7), let us define the total decay width for this processes as Γ_d, which will be calculated below. The corresponding decay Boltzmann equation for these channels is writen as

$$\frac{1}{a^3}\frac{d(a^3 n_\nu)}{dt} = -\Gamma_d n_\nu\ . \tag{4.33}$$

Finally, the evolution of the number density n_ν, is described by the summation of all the contributions given in (4.29), (4.31), and (4.33).

Then, the total annihilation and decay Boltzmann equation for the number density n_ν, casts

$$\frac{1}{a^3}\frac{d(a^3 n_\nu)}{dt} = C[T] \ , \tag{4.34}$$

where, the total collission term $C[T]$ is given by

$$C[T] = \frac{1}{4}\left\{2\langle\,\sigma_{\mathfrak{X}} v_r\rangle\left[(n_\nu)^2_{eq}\frac{n^2_{\mathfrak{X}}}{(n_{\mathfrak{X}})^2_{eq}} - n^2_\nu\right] + 2\langle\,\sigma_{H} v_r\rangle\left[(n_\nu)^2_{eq} - n^2_\nu\right] - \Gamma_d n_\nu\right\} , \tag{4.35}$$

wherein the factor of 2 in the $\mathfrak{X}$-term accounts for the two Majorana neutrinos involved [see Eq. (3.41)], and the second factor of 2 is there because of Eq. (4.32).

In order to evolve the Boltzmann equation (4.34), we have to explicitly calculate the previous collision term. This is done in the next secction.

4.4.2 The collision term for neutrino annihilation

First of all, we write the explicit thermally averaged cross-section (TACS) for each annihilation channel, as well as the decay width, namely,

$$\langle\,\sigma_{\mathfrak{X}} v_r\rangle, \qquad \langle\,\sigma_{H} v_r\rangle \qquad \text{and} \qquad \Gamma_d \ .$$

The detailed calculations of these three objects are contained in Appendix D. As can be found there, for the $\mathfrak{X}$ channel we obtained (see Appendix D.4)

$$\langle\sigma_{\mathfrak{X}} v_r\rangle = \frac{a_c^4}{4096\,\pi m_k T\, K_2^2(m_k/T)}\,\mathcal{I}_{\mathfrak{X}}(m_k;T) \ , \tag{4.36}$$

where K_2 is the modified Bessel function of the second kind of order 2, and where we have defined the integral

$$\mathcal{I}_{\mathfrak{X}}(m_k;T) \equiv \int_0^1 dx\,\frac{g_{\mathfrak{X}}(x)}{x\sqrt{x}}K_1\left(\frac{2m_k}{T\sqrt{x}}\right) , \tag{4.37}$$

with K_1 the modified Bessel function of the second kind of order 1, and where

$$g_{\mathfrak{X}}(x) = \left(\frac{1}{x} + 4 - 2x\right)\log\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - 2\left(\frac{1}{x}+2\right)\sqrt{1-x} \ .$$

Similarly, for the Higgs channel we arrive to (see Appendix D.5)

$$\langle \sigma_H v_r \rangle = \frac{3}{64\pi} \frac{y^4}{m_k T K_2^2\,(m_k/T)} \mathcal{I}_H(m_k;T) \;, \tag{4.38}$$

where

$$\mathcal{I}_H(m_k;T) = \int_0^1 dx \frac{g_H(x)}{x\sqrt{x}} K_1\left(\frac{2m_k}{T\sqrt{x}}\right) \;, \tag{4.39}$$

and where

$$g_H(x) = \frac{2-x}{x}\sqrt{1-x} + \frac{3}{4}\log\frac{1-\frac{x}{2}+\sqrt{1-x}}{1-\frac{x}{2}-\sqrt{1-x}} \;.$$

As for the decay width we obtained (see Appendix D.6)

$$\Gamma_d = \frac{3}{32\pi} y^2 m_k \;. \tag{4.40}$$

Notice that the collision term given in Eq. (4.35) can be simplified by knowing that the channel of annihilation into SM particles is preferential over the channel of annihilation into $\mathcal{X}$ quanta. This can be seen from the relation among the coupling a_c and the Yukawa coupling, as shown in Eq. (4.19), then, we expected that the termally averaged cross section of the SM channel dominates greatly over the $\mathcal{X}$ channel, as shown in Fig. 4.3.

This means that, the heavy Majorana neutrinos annihilate mostly into Higgs and charged leptons than into DE particles, so that, we can say in advance, that the production of $\mathcal{X}$-quanta will be highly suppressed, such that their number density does not deviate from the initial condition (the equilibrium distribution), thus the collision term given in Eq. (4.35) can be written as

$$C[T] = \frac{1}{4}\left\{2\left[\langle\,\sigma_{\mathcal{X}} v_r\rangle + \langle\,\sigma_H v_r\rangle\right]\left[(n_\nu)^2_{eq} - n_\nu^2\right] - \Gamma_d n_\nu\right\}. \tag{4.41}$$

The last term to be defined in the collision term is the neutrino number density in equilibrium which is given by

$$(n_\nu)_{eq} = 4\pi m_k^2 T K_2(m_k/T) \;. \tag{4.42}$$

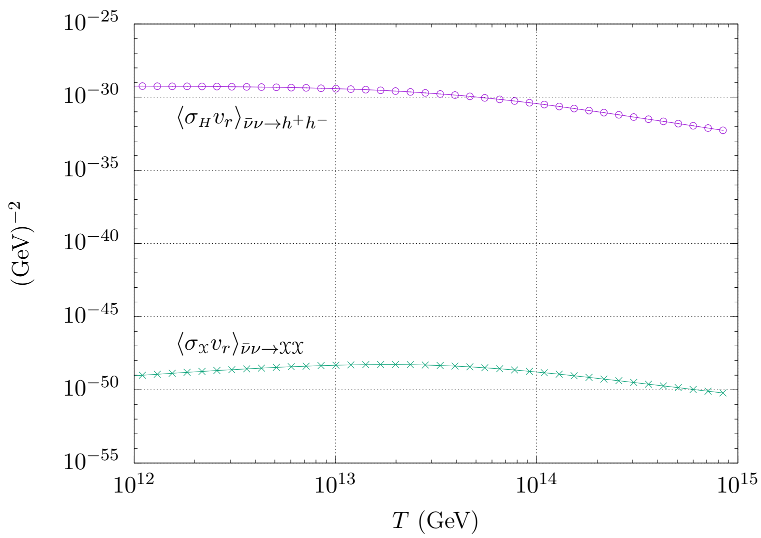

Figure 4.3: Comparison between the thermally averaged cross sections given in Eqs. (4.36) and (4.38). As can be seen, the annihilation channel into SM particles dominates over the channel that produces $\mathcal{X}$ quanta, hence, we expect no large contribution of this last to the total relativistic energy density of the Universe.

4.4.3 Boltzmann equation for SM particles production

Coming back to the Boltzmann equation (4.34), we can rewrite it by changing time evolution in favor of the temperature (which is possible to do during the radiation dominated age, see for instance [13, 95]), then we get

$$\frac{d}{dT}(a^3 n_\nu) = -\frac{M_{\rm pl}}{\pi}\left(\frac{90}{g_*(T)}\right)^{1/2}\frac{a^3}{T}C[T]\ , \tag{4.43}$$

where $g_*(T)$ is the number of relativistic degrees of freedom in energy density in equilibrium, and $C[T]$ is given in Eq. (4.41). By means of this equation, we now procede to calculate the evolution of the number density n_ν.

Notice that, for small values of the parameter a_c, the Yukawa coupling does not deviate significantly from unity, as can be seen in Eq. (4.19), hence, without losing generality, we can set $y = 1$ from now on.

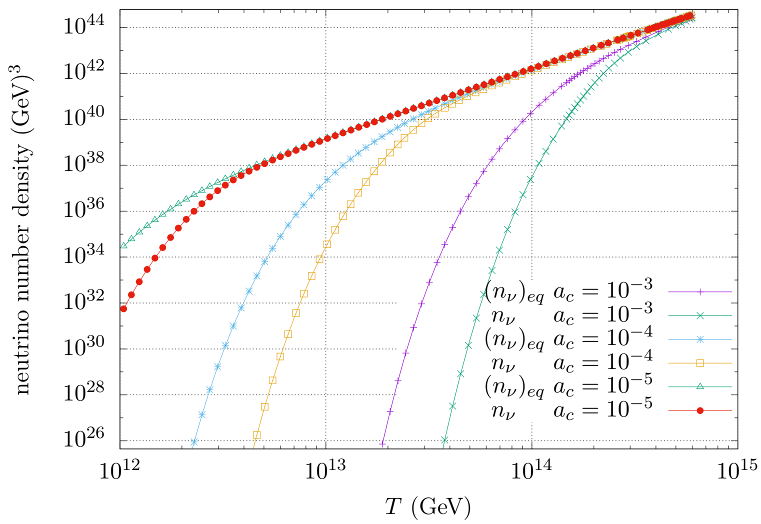

Figure 4.4: The neutrino number density n_ν varying on temperature driven by equation (4.43) and the equilibrium number density given in (4.42). Notice that the system goes out-of-equilibrium at early times but the number density gets suppressed strongly due to the decay term proportional to (4.40).

Thus, by numerical evolving equation (4.43), we found that the number density n_ν never overrides that of equilibrium $(n_\nu)_{eq}$, as it is shown in figure 4.4, in accordance to which, the system evolves in thermal equilibrium at early times and then leaves equilibrium to get highly suppressed due to the decay channel characterized by (4.40).

In figure 4.5 we plot, for a few values of the coupling a_c, the out-of-equilibrium condition

$$\Gamma_{H} \equiv 2 \times \langle \sigma_{H} v_r \rangle \times n_\nu \lesssim \mathsf{H}, \tag{4.44}$$

where Γ_{H} is the neutrino interaction rate for the Higgs channel and H is the Hubble parameter. As it is shown there, because of the decay of neutrinos into Higgs and leptons, the greater the coupling a_c (and so the mass m_k), the earlier the out-of-equilibrium epoch. This also shows that, as expected, neutrino into SM fields coannihilation is efficient enough at higher temperatures as to thermalize the heavy neutrinos.

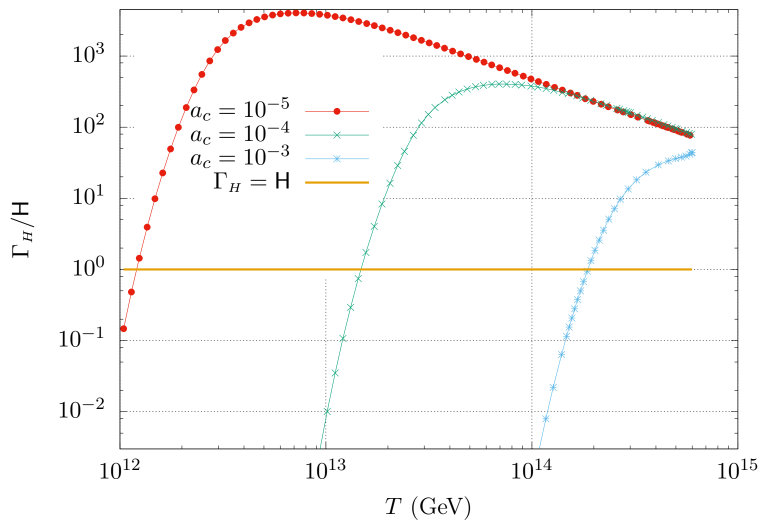

Figure 4.5: The out-of-equilibrium condition given in Eq. (4.44) for some values of the parameter a_c. As stated in the text, the greater the mass m_k the earlier the beginning of the out-of-equilibrium epoch, due to the decay processes into SM particles.

In figure 4.6 we illustrate the behavior of the system in the space of the temperature versus the parameter a_c. As said before, inflaton decays into neutrinos ν and reheats the Universe at temperature $T_r \sim 10^{15}$ GeV, below this temperature and above the upper line that stands for the value of the mass m_k, the population of neutrinos behave like pure radiation in thermal equilibrium and stays that way until the temperature drops into the region below the line m_k and above the line $\Gamma_H \approx \mathsf{H}$, in which the system becomes non-relativistic but still keeps in thermal equilibrium. Below the bottom line the system goes out-of-equilibrium, and the population of neutrinos decreases due to the co-annihilation into Quintessence and Higgs pairs, as well as the decay into Higgs and leptons.

4.4.4 The collision term for $\mathcal{X}$ quanta production

Turning back to the Boltzmann equation (4.27), notice that, because we are interested in maximizing the production of $\mathcal{X}$-quanta, which states the worst

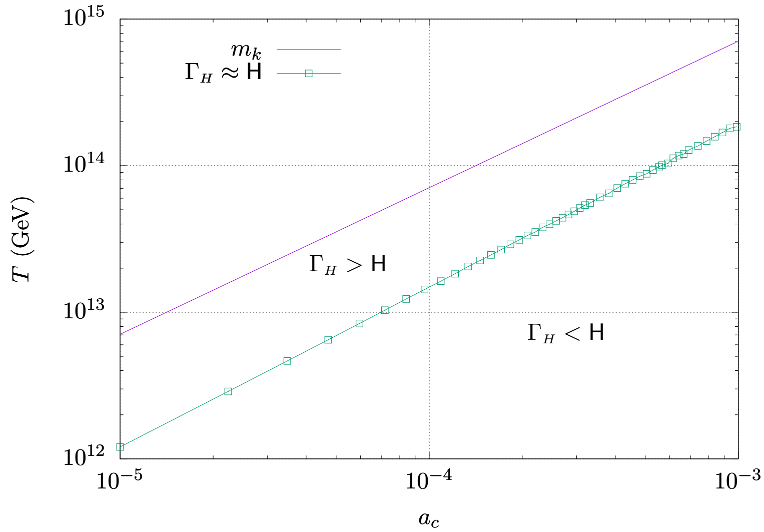

Figure 4.6: The behaviour of the population of Majorana sterile neutrinos as a function of the parameter a_c at a given temperature. As stated in the text, between $T_r \sim 10^{15}$ GeV and $T \approx m_k$, the number density of neutrinos corresponds to that of radiation in thermal equilibrium, between $T \approx m_k$ and the temperature of equality $\Gamma_H \approx \mathsf{H}$, the system becomes non-relativistic but still keeps in thermal equilibrium. Below this line the system goes out-of-equilibrium and the population of neutrinos gets suppressed due to the processes of annihilation and decay populing the universe with SM particles.

possible scenario for the model, and because $(n_\nu)_{eq} \gtrsim n_\nu$ [see fig. 4.4], we can choose $(n_\nu)_{eq}$ instead of n_ν in the collision term, and we can neglect the ratio $n_{\mathcal{X}}^2/(n_{\mathcal{X}})_{eq}^2$ which accounts for a tiny fraction of $n_{\mathcal{X}}$ produced due to the inverse proccess $\mathcal{X}\mathcal{X} \to \bar{\nu}\nu$, in this way we overestimate the production of $\mathcal{X}$-quanta. Then the collision term in Eq. (4.27) is simplified as

$$\langle\sigma_{\mathcal{X}} v_r\rangle \left[n_\nu^2 - n_{\mathcal{X}}^2 \frac{(n_\nu)_{eq}^2}{(n_{\mathcal{X}})_{eq}^2}\right] \longrightarrow \langle\sigma_{\mathcal{X}} v_r\rangle (n_\nu)_{eq}^2 \ . \tag{4.45}$$

As we will shown next, this approximation will be enough to stablish the cosmological consistency of the model, because the result does not conflict with the requirements of Big Bang Nucleosynthesis.

4.4.5 Boltzmann equations for $\mathcal{X}$ quanta production

By substituting the reduced collision term given in Eq. (4.45) into Eq. (4.27) we arrive to

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_{\mathcal{X}}) = \langle\sigma_{\mathcal{X}} v_r\rangle (n_\nu)^2_{eq} \ . \tag{4.46}$$

By using Eqs. (4.36) and (4.42), the Boltzmann equation (4.46) becomes

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_{\mathcal{X}}) = \frac{\pi}{256} m_k^3 a_c^4 T\, \mathcal{I}_{\mathcal{X}}(m_k;T) \ . \tag{4.47}$$

As before, after changing time evolution in favor of the temperature, the Boltzmann equation (4.47) becomes

$$\frac{d}{dT}(a^3 n_{\mathcal{X}}) = -\frac{M_{pl}}{256}\left(\frac{90}{g_*(T)}\right)^{1/2} m_k^3 a_c^4 \frac{a^3}{T^2}\mathcal{I}_{\mathcal{X}}(m_k;T) \ . \tag{4.48}$$

Since the Universe is cooling, we perform the integration at both sides backward in T, from T_{out} to a certain temperature $T' < T_{out}$, so, we have

$$\int_{(a^3 n_{\mathcal{X}})(T_{out})}^{(a^3 n_{\mathcal{X}})(T')} d(a^3 n_{\mathcal{X}}) = \\ -\frac{M_{\rm pl}}{256}\sqrt{90} m_k^3 a_c^4 \int_{T_{out}}^{T'} dT \frac{a^3(T)}{\sqrt{g_*(T)}T^2}\mathcal{I}_{\mathcal{X}}(m_k;T) \ , \tag{4.49}$$

where in the RHS, we have written explicitly the universal scale factor dependence on T, such a dependence, during the radiation dominated age, is given by

$$a(T) = \frac{b_0}{g_{*s}^{1/3}(T) T} \ , \tag{4.50}$$

where b_0 is a constant and $g_{*s}(T)$ is the number of relativistic degrees of freedom in entropy density in equilibrium.

When the cooling Universe reaches the temperature T_{out} the density $n_{\mathcal{X}}$ starts to increase, i.e. the system $\bar{\nu}\nu \leftrightarrow \mathcal{X}\mathcal{X}$ goes out of equilibrium, which is true whenever

$$\Gamma_{\mathcal{X}} \equiv 2 \times \langle\sigma_{\mathcal{X}} v_r\rangle \times n_\nu \lesssim \mathsf{H}, \tag{4.51}$$

where $\Gamma_{\mathcal{X}}$ is the neutrino interaction rate of the channel, which can be calculated by using (4.36) and the numerical output of (4.43). It turns out that

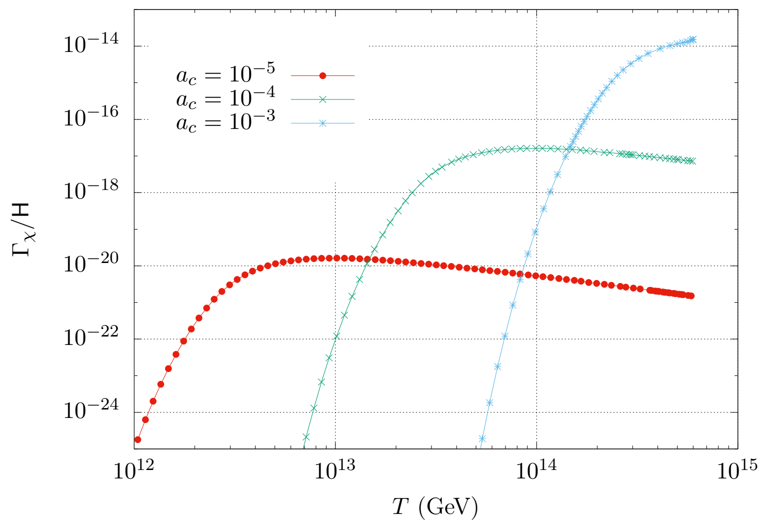

Figure 4.7: The out-of-equilibrium condition given in Eq. (4.51) for some values of the parameter a_c. As stated in the text, the integral (D.38) is very suppressed, hence the system $\bar{\nu}\nu \leftrightarrow \chi\chi$ is always out of equilibrium, even for temperatures as high as of the one for reheating.

for any value of $T \lesssim T_r$, the integral (4.37) is very suppressed and so is the rate $\Gamma_\mathcal{X}$ as it is shown in the figure 4.7. Then the inequality (4.51) is always fulfilled and we can use the temperature $T_{out} \sim T_r$ as the lower limit to obtain a good estimate of the integral that appears in the RHS of Eq. (4.49).

Furthermore, as the initial state of the $\mathcal{X}$-field is one of pure vacuum, and this is not coupled to the inflaton, there are not initial quanta, consequently, we can impose the condition

$$(a^3 n_\mathcal{X})(T_{out}) = 0 \ ,$$

which jointly to Eq. (4.50) allows expressing the integral in Eq. (4.49) as

$$n_\mathcal{X}(T') = N a_c^7 g_{*s}(T') T'^3 \int_{T'}^{T_r} \frac{dT}{T^5} \frac{\mathcal{I}_\mathcal{X}(m_k; T)}{g_{*s}(T)\sqrt{g_*(T)}} \ , \tag{4.52}$$

where N is a constant factor given by

$$N = 2 \times \frac{M_{\rm pl}}{512} \sqrt{45} \langle \mathrm{Q} \rangle^3 \ , \tag{4.53}$$

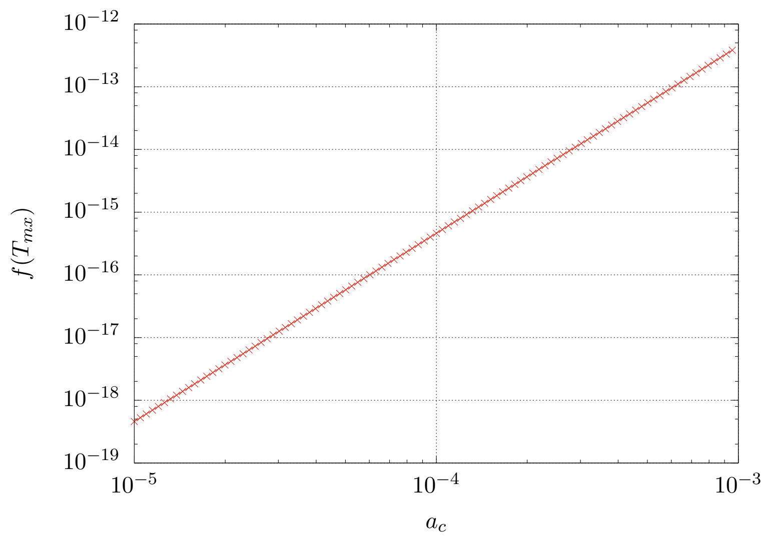

Figure 4.8: The maximums of the function $f(T)$ given in (4.56) for different values of the parameter a_c. Notice that the integral is always less than the unit and so the increase in n_{TOT} given in Eq. (4.57) is negligible.

and where we have multiplied it by 2 because there are two Majorana neutrinos involved [see Eq. (3.41)].

By considering $g_{*s} \sim g_{*n}$, whit g_{*n} the relativistic degrees of freedom in number density in equilibrium, the integral (4.52) can be written as

$$n_\chi(T') = n_r(T') \times f(T') \ , \tag{4.54}$$

where $n_r(T')$ is the relativistic number density in equilibrium, given by

$$n_r(T') = \frac{\zeta(3)}{\pi^2} g_{*n}(T') T'^3 \ , \tag{4.55}$$

with $\zeta(3)$ the Apéry's constant given in Eq. (D.23), and where

$$f(T') = N a_c^7 \frac{\pi^2}{\zeta(3)} \int_{T'}^{T_r} \frac{dT}{T^5} \frac{\mathcal{I}_\chi(m_k; T)}{g_{*s}(T)\sqrt{g_*(T)}} \ . \tag{4.56}$$

By means of equation (4.54) we write the total relativistic number density of our model n_{TOT}, in terms of (4.56) as

$$n_{\text{TOT}}(T') = n_r(T')(1 + f(T')) \ . \tag{4.57}$$

The integral (4.56) can be calculated numerically for different values of the a_c parameter, with the result that for each value of the latter, the integral depends smoothly on the temperature and it is easy to maximize.

Since $n_r(T)$ is a growing monotonic function, it is enough to know whether, for certain a_c, the value of $f(T_{mx})$ exceeds that of $n_r(T_{mx})$, where T_{mx} is the temperature that maximizes the integral (4.56). What we found is that $f(T_{mx})$ is always several orders of magnitude below one for any value of a_c within the range we are interested on, as shown in figure 4.8, so the increase in the total relativistic number density of $\mathcal{X}$ particles due to the co-annihilation of right handed neutrinos is of no cosmological consequences.

Clearly, once neutrino decay into SM fields is switched on, the actual $\mathcal{X}$ would be much smaller that the value we have just calculated. The model, to this extent, appears consistent with the cosmological constraints.

4.5 Thermal corrections to quintessence mass

As explained in Appendix B.6, the type I seesaw mechanism mixes the heavy (sterile) neutrinos $\mathtt{K}^{\dot{a}}$ that belong to our model and the (active) neutrinos $\mathtt{X}^{\dot{a}}$ which belong to SM, to yield

$$\mathtt{N}_1^{\dot{a}} = -i\mathtt{X}^{\dot{a}} + i\frac{m_D^\nu}{m_k}\mathtt{K}^{\dot{a}} \qquad \text{and} \qquad \mathtt{N}_2^{\dot{a}} = \frac{m_D^\nu}{m_k}\mathtt{X}^{\dot{a}} + \mathtt{K}^{\dot{a}} \ , \tag{4.58}$$

where $\mathtt{N}_1^{\dot{a}}$ and $\mathtt{N}_2^{\dot{a}}$ are respectively, the light and the heavy neutrino fields which emerge from the seesaw, as shown in Eq. (B.105).

The Eq. (4.58) can be rewritten as

$$\begin{pmatrix} \mathtt{N}_1^{\dot{a}} \\ \mathtt{N}_2^{\dot{a}} \end{pmatrix} = \begin{pmatrix} -i & i\frac{m_D^\nu}{m_k} \\ \frac{m_D^\nu}{m_k} & 1 \end{pmatrix} \begin{pmatrix} \mathtt{X}^{\dot{a}} \\ \mathtt{K}^{\dot{a}} \end{pmatrix} \ , \tag{4.59}$$

by inverting this system we have

$$\begin{pmatrix} \mathtt{X}^{\dot{a}} \\ \mathtt{K}^{\dot{a}} \end{pmatrix} = \frac{(m_k)^2}{(m_k)^2 + (m_D^\nu)^2} \begin{pmatrix} i & \frac{m_D^\nu}{m_k} \\ -i\frac{m_D^\nu}{m_k} & 1 \end{pmatrix} \begin{pmatrix} \mathtt{N}_1^{\dot{a}} \\ \mathtt{N}_2^{\dot{a}} \end{pmatrix} . \tag{4.60}$$

The coefficient that accompanies the matrix is of order one, so we have

$$\mathtt{X}^{\dot{a}} \approx i\mathtt{N}_1^{\dot{a}} + \frac{m_D^\nu}{m_k}\mathtt{N}_2^{\dot{a}}, \qquad \mathtt{K}^{\dot{a}} \approx -i\frac{m_D^\nu}{m_k}\mathtt{N}_1^{\dot{a}} + \mathtt{N}_2^{\dot{a}} \ . \tag{4.61}$$

Let us write once again the part of Eq. (3.44) that corresponds solely to one neutrino, it reads

$$-\mathcal{L}_{I\mathcal{X}} = \frac{a_c}{2\sqrt{2}}\mathcal{X}\mathtt{K}_{\dot{a}}\mathtt{K}^{\dot{a}} + h.c. \tag{4.62}$$

next, by virtue of Eq. (4.61) the previous lagrangian becomes

$$-\mathcal{L}_{I\mathcal{X}} = \frac{a_c}{2\sqrt{2}}\mathcal{X}\left[-\left(\frac{m_D^\nu}{m_k}\right)^2\mathtt{N}_{1\dot{a}}\mathtt{N}_1^{\dot{a}} - i\frac{m_D^\nu}{m_k}\mathtt{N}_{1\dot{a}}\mathtt{N}_2^{\dot{a}} - i\frac{m_D^\nu}{m_k}\mathtt{N}_{2\dot{a}}\mathtt{N}_1^{\dot{a}} + \mathtt{N}_{2\dot{a}}\mathtt{N}_2^{\dot{a}}\right] + h.c.$$

From the previous equation, we can see that the scalar field $\mathcal{X}$ couples to the (Majorana) neutrinos which emerge from the seesaw mechanism. Accordingly to Eq. (4.58), the active neutrino is $\mathtt{N}_1^{\dot{a}}$ (notice that the active part on $\mathtt{N}_2^{\dot{a}}$ is highly suppressed), furthermore, in the context of our model, as we have shown, the heavy neutrino $\mathtt{N}_2^{\dot{a}}$ disappear at early times, therefore there is left only the light (active) neutrino coupled to the scalar field $\mathcal{X}$, such that the previous equation reduces to

$$-\mathcal{L}_{I\mathcal{X}} = \frac{a_c}{2\sqrt{2}}\mathcal{X}\left[-\left(\frac{m_D^\nu}{m_k}\right)^2\mathtt{N}_{1\dot{a}}\mathtt{N}_1^{\dot{a}}\right] + h.c. \tag{4.63}$$

The previous result is of great relevance for DE dynamics, because it implies an effective coupling between the quintessence field and the active neutrinos, which permeate the background Universe together with the photons of CMB. The cosmological neutrino background has temperature T_ν, consequently, the effective coupling generates thermal corrections to scalar mass given in Eq. (4.4). As said in Chapter 2 [see Eq. (2.39)], the quintessence mass should be less than or at least equal to the Hubble parameter, otherwise, the slow-roll condition is violated.

As we will show, the slow-roll condition stays through the Universe life, such that the scalar field behaves like DE in spite of thermal corrections. Next, we calculate such corrections, to this end, let us write the evolution of the neutrino bath temperature depending on the cosmological scale factor.

4.5.1 Neutrino background temperature

The temperature of the neutrino background is related to the temperature of the photons by (see for instance Refs. [95, 96])

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \ . \tag{4.64}$$

In turn, the photon temperature evolves on the scale factor as

$$T_\gamma = T_{\gamma,0} \left(\frac{a_0}{a}\right) ,$$

where a_0 is the today value of the scale factor. By using Eq. (4.64) the neutrino bath temperature evolves on the scale factor as

$$T_\nu = T_{\nu,0} \left(\frac{a_0}{a}\right) , \tag{4.65}$$

where

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma,0} \ ,$$

is the today's neutrino temperature, which, by using Eq. (D.1) becomes

$$T_{\nu,0} = 1.676 \times 10^{-4} \text{ eV} \ . \tag{4.66}$$

4.5.2 Coupling between $\mathcal{X}$ and active neutrinos

From Eq. (4.63) we see that the effective coupling between $\mathcal{X}$ and neutrinos is given by

$$\lambda := \frac{a_c}{2\sqrt{2}} \left(\frac{m_D^\nu}{m_k}\right)^2 . \tag{4.67}$$

By using Eqs. (4.9) and (4.10) this coupling becomes

$$\begin{aligned}\lambda &= \frac{a_c}{2\sqrt{2}} \left(\frac{y^\nu \langle H \rangle}{a_c \langle \mathrm{Q} \rangle}\right)^2 \\ &= \frac{1}{2\sqrt{2}} \frac{(y^\nu)^2}{a_c} \left(\frac{\langle H \rangle}{\langle \mathrm{Q} \rangle}\right)^2 ,\end{aligned}$$

from here we get

$$\frac{(y^\nu)^2}{a_c} = 2\lambda\sqrt{2}\left(\frac{\langle H\rangle}{\langle \mathrm{Q}\rangle}\right)^{-2} , \tag{4.68}$$

then, by using Eq. (4.17) we obtain

$$5\times\frac{2.853}{2\sqrt{2}}\left(\frac{\langle H\rangle}{\langle \mathrm{Q}\rangle}\right)^{2}\times 10^3 \ \lesssim\ \lambda\ \lesssim\ \frac{2.853}{2\sqrt{2}}\left(\frac{\langle H\rangle}{\langle \mathrm{Q}\rangle}\right)^{2}\times 10^4,$$

and by using Eq. (4.12) we arrive to

$$5\times 4.095\times 10^{-31}\ \lesssim\ \lambda\ \lesssim\ 4.095\times 10^{-30}\ . \tag{4.69}$$

This result means that, in our model, the effective coupling among the light neutrinos and the $\mathcal{X}$ field is determined by the ratio of the energy scales.

4.5.3 Mass correction due to thermal bath

The contribution to the scalar potential comming from the thermal bath is given by (see Refs. [97, 98])

$$V^T(\mathrm{Q},T) = \frac{g}{48}m^2_{(\mathrm{Q})}T^2\ , \tag{4.70}$$

where g accounts for the internal degrees of freedom of the Majorana neutrinos coupled to the scalar field.

We can identify the mass $m_{(\mathrm{Q})}$ by defining the effective Yukawa coupling that appears in Eq. (4.63) as

$$y^{\mathrm{eff}} := \frac{a_c}{2}\left(\frac{m^\nu_{\scriptscriptstyle D}}{m_k}\right)^2\ , \tag{4.71}$$

such that we can define

$$m_{(\mathrm{Q})} := \frac{1}{\sqrt{2}}y^{\mathrm{eff}}\left\langle \mathrm{Q}\right\rangle, \tag{4.72}$$

thus, in virtue of Eq. (4.67) we have that

$$m_{(\mathrm{Q})} := \lambda\left\langle \mathrm{Q}\right\rangle, \tag{4.73}$$

then, by subtituting this into Eq. (4.70) it becomes

$$V^{T}(\mathrm{Q},T) = \frac{g}{48}\lambda^2 T^2 \langle \mathrm{Q} \rangle^2 \ . \tag{4.74}$$

The effective potential is defined by adding this potential to that of the scalar field

$$V^{\text{eff}}(\mathrm{Q},T) = V(\mathrm{Q}) + V^{T}(\mathrm{Q},T),$$

thus, by using the density (4.2) together with (4.74) we obtain

$$\begin{aligned} V^{\text{eff}}(\mathrm{Q},T) &= \frac{1}{2}m^2\langle \mathrm{Q} \rangle^2 + \frac{g}{48}\lambda^2 T^2 \langle \mathrm{Q} \rangle^2 \ , \\ &= \frac{1}{2}\left[m^2 + \frac{g}{24}\lambda^2 T^2\right]\langle \mathrm{Q} \rangle^2 \ , \\ &= \frac{1}{2}\mathcal{M}^2_{\text{eff}}\langle \mathrm{Q} \rangle^2 \ , \end{aligned}$$

where the effective mass has been defined as

$$\mathcal{M}^2_{\text{eff}} = m^2 + m_T^2 \ , \tag{4.75}$$

wherein the thermal mass (con $T = T_\nu$) is given by

$$m_T = \sqrt{\frac{g}{24}}\ \lambda T_\nu \ . \tag{4.76}$$

4.5.4 Evolution of m_T on the scale factor a

As said above, we have to compare the thermal mass with the Hubble parameter, to do this, we first calculate the evolution of m_T on the scale factor.

By substitution of T_ν as given in Eq. (4.65) into (4.76), it becomes

$$m_T = \sqrt{\frac{g}{24}}\ \lambda T_{\nu,0}\ \left(\frac{a_0}{a}\right), \tag{4.77}$$

from which we can write

$$m_T = m_{T,0}\ \left(\frac{a_0}{a}\right), \tag{4.78}$$

where we have defined the thermal mass today as

$$m_{T,0} = \sqrt{\frac{g}{24}}\ \lambda T_{\nu,0} \ . \tag{4.79}$$

For each massive Majorana neutrino there are two internal degrees of freedom, since there are three active light Majorana neutrinos, we have

$$g = 2 \times 3 = 6 \ ,$$

with this value, together with Eqs. (4.66) and (4.69) into (4.79) we arrive to

$$1.716 \times 10^{-34} \text{ eV} \ \lesssim \ m_{T,0} \ \lesssim \ 3.432 \times 10^{-34} \text{ eV}. \tag{4.80}$$

4.5.5 Comparing the thermal mass with the Hubble parameter

4.5.5.1 Radiation dominated age

During the radiation dominated age, the Hubble parameter evolves as

$$\mathsf{H}^2 = \mathsf{H}_0^2 \Omega_{R,0} \left(\frac{a}{a_0} \right)^{-4} \ . \tag{4.81}$$

Therefore, by using Eq. (4.78), during such a era, the quotient between the thermal mass and the Hubble parameter is

$$\frac{m_T}{\mathsf{H}} = \frac{m_{T,0}\,(a_0/a)}{\mathsf{H}_0 \sqrt{\Omega_{R,0}}\,(a/a_0)^{-2}} \ ,$$

which is the same that

$$\frac{m_T}{\mathsf{H}} = \frac{m_{T,0}}{\mathsf{H}_0 \sqrt{\Omega_{R,0}}} \times \left(\frac{a}{a_0} \right) . \tag{4.82}$$

By evaluating the quantities given in Eqs. (4.80), (D.6) and (D.15), it becomes

$$3.189 \left(\frac{a}{a_0} \right) \ \lesssim \ \left. \frac{m_T}{\mathsf{H}} \right|_{RAD} \ \lesssim \ 6.379 \left(\frac{a}{a_0} \right) , \quad a \leq a_{eq} \ , \tag{4.83}$$

wherein a_{eq} is the scale factor at the equality of matter radiation epoch.

When $a = a_{eq}$ it is fulfilled that (see [13, 15])

$$\left(\frac{a_{eq}}{a_0} \right) = \left(\frac{\Omega_{R,0}}{\Omega_{DM,0} + \Omega_{b,0}} \right) .$$

By using Eqs. (D.11), (D.12) and (D.15) into the previous equation, we get

$$\left(\frac{a_{eq}}{a_0}\right) = 4.454 \times 10^{-3} \ . \tag{4.84}$$

By using this value into Eq. (4.83) we have that, at the end of the radiation age, it is fulfilled that

$$1.421 \times 10^{-2} \ \lesssim \ \frac{m_T}{\mathsf{H}}\bigg|_{RAD} \ \lesssim \ 2.841 \times 10^{-2} \ , \qquad a = a_{eq} \ . \tag{4.85}$$

From the previous result, we see that the slow-roll condition is accomplished during the radiation dominated age.

4.5.5.2 Matter dominated age

During the matter dominanted age, the Hubble parameter evolves as

$$\mathsf{H}^2 = \mathsf{H}_0^2 \Omega_{M,0} \left(\frac{a}{a_0}\right)^{-3} \ , \tag{4.86}$$

where $\Omega_{M,0}$ is given in Eq. (D.13). During this age, the quotient between the thermal mass and the Hubble parameter is

$$\frac{m_T}{\mathsf{H}} = \frac{m_{T,0}\,(a_0/a)}{\mathsf{H}_0\sqrt{\Omega_{M,0}}\,(a/a_0)^{-3/2}} \ ,$$

which is the same that

$$\frac{m_T}{\mathsf{H}} = \frac{m_{T,0}}{\mathsf{H}_0\sqrt{\Omega_{M,0}}} \left(\frac{a}{a_0}\right)^{1/2} \ , \tag{4.87}$$

by evaluating Eqs. (4.80), (D.6), and (D.13) into the previous equation, we obtain

$$2.128 \times 10^{-1} \left(\frac{a}{a_0}\right)^{1/2} \ \lesssim \ \frac{m_T}{\mathsf{H}}\bigg|_{MAT} \ \lesssim \ 4.257 \times 10^{-1} \left(\frac{a}{a_0}\right)^{1/2} \ , \quad a_{eq} \leq a \ , \tag{4.88}$$

It is not difficult to verify by using Eq. (4.84) that, provided $a = a_{eq}$, the previous inequality coincides with (4.85).

And at the epoch of the transition to dominant DE, which in our model [see Eq. (4.93)] occurs when

$$\left(\frac{a_{DE}}{a_0}\right) \sim 0.45 \ ,$$

we have that

$$1.428 \times 10^{-1} \ \lesssim \ \frac{m_T}{\mathsf{H}}\bigg|_{MAT} \ \lesssim \ 2.856 \times 10^{-1}, \qquad a = a_{DE} \ . \tag{4.89}$$

This means that, during the era of matter domination, the slow-roll condition is also accomplished.

4.5.5.3 At present day ($a = a_0$)

Nowadays

$$\frac{m_T}{\mathsf{H}}\bigg|_{a=a_0} = \frac{m_{T,0}}{\mathsf{H}_0} \ .$$

By virtue of Eqs. (4.80) and (D.6) we have

$$0.193 \ \lesssim \ \frac{m_{T,0}}{\mathsf{H}_0} \ \lesssim \ 0.239 \ .$$

This means that, nowadays, the slow-roll condition is also accomplished.

4.5.6 Effective DE density

First of all, notice that with the definition for the effective mass given in Eq. (4.75) and by using Eqs. (4.4) and (4.80) we have

$$6.064 \times 10^{-34} \ \lesssim \ \mathcal{M}_{\text{eff}}\bigg|_{a=a_0} = \ \lesssim \ 6.753 \times 10^{-34} \ ,$$

so that, the value of the effective mass is narrowly near to that given in Eq. (4.4). This is a quite interesting result that allows us to put a bound on the active neutrino mass. By assuming that the non-corrected scalar mass is actually less than the thermal contribution, such that

$$\mathcal{M}_{\text{eff}} \approx m_T \ ,$$

we note that the largest mass that light active neutrinos can have is

$$m_\nu = 1.694 \times 10^{-1} \text{ eV} ,$$

then, the upper limit of the interval (4.15) should be corrected by this factor, such that the upper limit of the inteval (4.80) becomes

$$\begin{aligned} m'_{T,0} &= (1.694) \times 3.432 \times 10^{-34} \text{ eV} \\ &= 5.816 \times 10^{-34} \text{ eV}, \end{aligned} \tag{4.90}$$

which is justly the required value [see Eq. (4.4)] in order to reproduce the observed DE density, as well as the Hubble parameter nowadays.

Let us define the effective DE density as

$$\rho^{\text{eff}}_{DE} := \frac{1}{2}\mathcal{M}^2_{\text{eff},1.7} \langle \mathrm{Q} \rangle^2 , \tag{4.91}$$

where

$$\mathcal{M}_{\text{eff},1.7} = m'_{T,0} \left(\frac{a_0}{a}\right) . \tag{4.92}$$

With this, we can write the effective DE density parameter as

$$\Omega^{\text{eff}}_{DE} := \frac{\rho^{\text{eff}}_{DE}}{\rho_{\text{crit}}} ,$$

then, by using Eqs. (4.91) and (4.92), it becomes

$$\Omega^{\text{eff}}_{DE} = \Omega^{\text{eff}}_{DE,0} \left(\frac{a_0}{a}\right)^2 , \tag{4.93}$$

where

$$\Omega^{\text{eff}}_{DE,0} = \frac{1}{2}\frac{(m'_{T,0})^2 \langle \mathrm{Q} \rangle^2}{\rho_{\text{crit}}} ,$$

finally, in virtue of Eqs. (4.90), (4.3) and (D.7) we arrive to

$$\Omega^{\text{eff}}_{DE,0} = 6.845 \times 10^{-1} , \tag{4.94}$$

which coincides with the value given in Eq. (D.8) as expected under our construction.

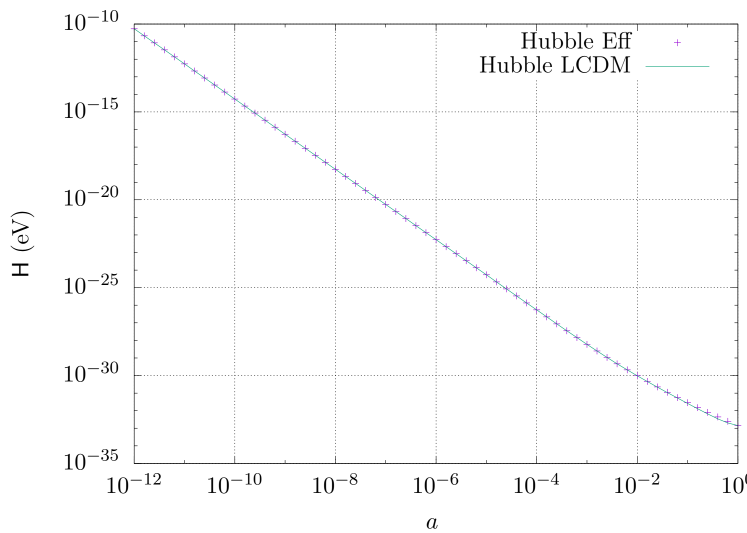

Figure 4.9: The Hubble parameter given in Eq. (4.95) compared with that of Λ-CDM, notice that thermal corrections do not alter its cosmic evolution.

4.5.7 Hubble parameter with thermal correction and slow-roll condition

By virtue of Eqs. (D.13), (D.15) and (4.93), the effective Hubble parameter is given by

$$\mathsf{H}_{\rm eff} = \mathsf{H}_0\sqrt{0.314\left(\frac{a_0}{a}\right)^3 + 1.4\times 10^{-3}\left(\frac{a_0}{a}\right)^4 + 0.685\left(\frac{a_0}{a}\right)^2}. \tag{4.95}$$

The plot of the effective Hubble parameter compared with that of Λ-CDM is shown in Fig. 4.9. Notice that, because of the effective DE redshifts with the smaller power, the evolution of the Hubble parameter through the Universe life is not altered.

Finally, in order to plot the quotient between the effective mass and the Hubble parameter, we define the ratio

$$R = \frac{\mathcal{M}_{\rm eff,1.7}}{\mathsf{H}}\ , \tag{4.96}$$

where $\mathcal{M}_{\rm eff,1.7}$ was defined in Eq. (4.92). In Fig. 4.10 we plot the evolution of this condition. As said before, this condition guaranties the slow-roll

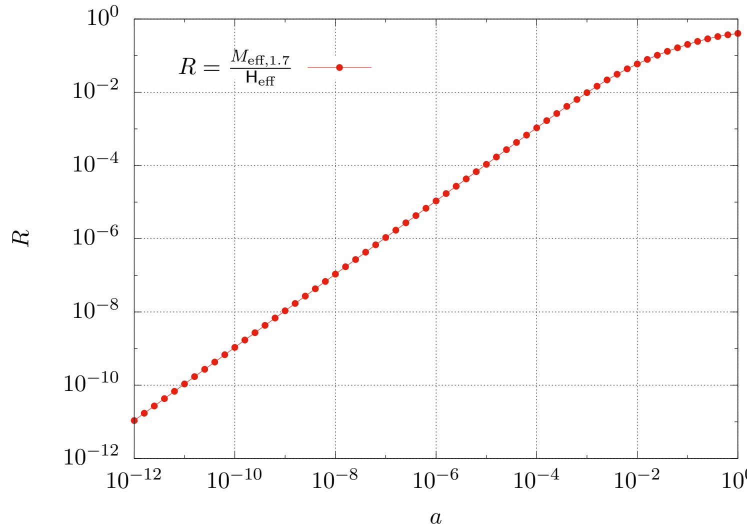

Figure 4.10: The ratio of thermal mass to Hubble parameter defined in Eq. (4.96). The requirement that $R \lesssim 1$ is fulfilled through the whole life of the universe, this guarantees that the scalar field behaves as DE.

condition is always accomplished, therefore the scalar field behaves as DE through the whole life of the Universe.

4.6 Considerations about the phase fields

In the previous sections, we have studied the phenomenology of our model, by focusing mainly on the real scalar field $\mathcal{X}$. Nevertheless, the scalar fields ξ and Q (coming from the fundamental ones which we first used to build our Lagrangian), are complex scalar fields. Therefore, there are two additional degrees-of-freedom, which correspond to the phase of the inflaton (θ) and the phase of the quintessence field (ϑ), as we have explicitly written in Eq. (C.20).

Since the inflaton field evaporates completely at the end of inflation, we do not expect its phase has effects on the thermal history, although, during the inflaton disintegration, the phase field θ (together with the field ϑ) could control the rate at which neutrinos are produced [see Eq. (3.42)], nevertheless, after the evaporation of inflaton, it is reasonable assuming there will be no extra consequences, at least at the level of the homogeneous background. At the perturbative level, on the other hand, this phase could generate some observable effect. That is something that is beyond this work and could be studied elsewhere.

Conversly, the phase ϑ is present in the DE and fermionic sectors, as can be seen respectively in Eqs. (C.28) and (C.34), notice however that, in the latter the term including the phase is highly suppressed due to the energy scale of $\langle \mathrm{Q} \rangle$, such that we expect this does not have observable consequences.

As for the former, it deserves to be more carefully checked. As explained in detail in Appendix C.2.6, the Eq. (C.28) leads to the slow-roll condition [see Eq. (C.74)]

$$\frac{1}{2}\dot{\mathcal{X}}^2 + \frac{1}{2}\left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 \dot{\vartheta}^2 \ll \frac{1}{2} m^2 (\langle \mathrm{Q} \rangle + \mathcal{X})^2 \ , \tag{4.97}$$

from which, we can see that the velocity of the phase ϑ can control the accomplishment of the slow-roll condition, thus, the phase can act directly on the DE nature of the field. The behavior of the field $\dot{\vartheta}$ and that of the previous condition can be known by evolving the dynamic system (see the obtention of this system in Appendix C.2.6)

$$\begin{aligned}
&\mathsf{H}^2 = \frac{1}{3M_{pl}^2} V(\mathcal{X}), \\
&\ddot{\mathcal{X}} + 3\mathsf{H}\dot{\mathcal{X}} + V(\mathcal{X})_{,\mathcal{X}} = 0, \\
&\ddot{\vartheta} + 3\mathsf{H}\dot{\vartheta} = 0, \\
&\dot{\mathsf{H}} = \frac{-1}{2M_{pl}^2}\left(\rho_{DM} + \rho_b + \frac{4}{3}\rho_\gamma + \frac{4}{3}\rho_n\right), \\
&\dot{\rho}_{DM,b} + 3\mathsf{H}\rho_{DM,b} = 0, \\
&\dot{\rho}_{\gamma,n} + 4\mathsf{H}\rho_{\gamma,n} = 0 \ ,
\end{aligned} \tag{4.98}$$

in which, by completness we also have included the densities of Dark Matter (ρ_{DM}), baryons (b), light active neutrinos (n), and photons (γ), as components of the background.

Let us write the first slow-roll condition (4.97) as the ratio

$$\mathrm{FSRC} := \frac{\frac{1}{2}\dot{\mathcal{X}}^2 + \frac{1}{2}\left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 \dot{\vartheta}^2}{\frac{1}{2} m^2 (\langle \mathrm{Q} \rangle + \mathcal{X})^2} \ll 1 \ , \tag{4.99}$$

then, by using the numerical outputs of the system (4.98) to evaluate it, we obtain the results showed in the Fig. 4.11. There we plot the ratio given in the previous equation for some initial values of the velocity $\dot{\vartheta}$. As shown in Fig. 4.11, the smaller the initial condition of $\dot{\vartheta}$, the earlier the system starts to evolve consistently with condition (4.99).

In particular, for the trivial $\dot{\vartheta} = 0$, which is plotted in the bottom line of Fig. 4.11, the system evolves in the slow-roll regime during all the Universe life, and as expected for thawing systems, it will leave the regime in the future. In conclusion, during the DE domination age, the system is compatible with a wide range of initial conditions provided that $\dot{\vartheta} \ll 1$, in particular, for the simplest $\dot{\vartheta} = 0$, the condition is always fulfilled.

As we have shown, by choosing the trivial initial condition the phase does not play any role, thus in this work, we have chosen it without losing generality. Our model, nonetheless, is completely compatible with different values, as shown in Appendices C.1 and C.2, and although it is not more developed here, we believe that a deeper analysis of the initial conditions could be related to the studies on the problem of coincidence, as well as to effects beyond the homogeneous limit.

As an important remark, the previous results were gotten by using the constant value for the quintessence mass as given in Eq. (4.4). This is possible to do because, as explained in Section 4.5, today value of the scalar mass including thermal corrections is about the same as that given in Eq. (4.4), thus, we expect that using the constant value is an acceptable assumption.

However, a deeper analysis, in which the variable mass coming from thermal effects is included, should be addressed. This is something we have not considered here and should be done in the future.

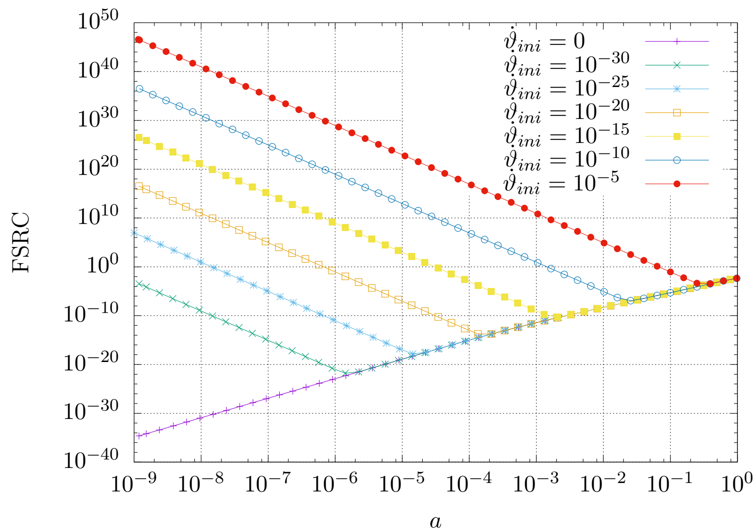

Figure 4.11: The FSRC given in Eq. (4.99) vs the universal scale factor. As stated in the text, the smaller the initial condition of $\dot{\vartheta}$, the earlier the system starts to evolve consistently with condition (4.99). In particular for $\dot{\vartheta} = 0$, the condition is fulfilled during the whole life of the Universe.

Chapter 5

Summary and Concluding Remarks

In this work, we have presented the $SO(1,1)$ cosmological model, in which, both the early accelerated expansion of the Universe (commonly called cosmic inflation), and the also accelerated expansion currently observed, have been unified under a governing symmetry.

The cosmic inflation and the present accelerating expansion are respectively driven by the inflaton and the quintessence, which, in our model, are complex scalar fields. We have shown how such fields are naturally unified since they emerge from the fundamental doublet-field representation (Φ), which transforms under the $SO(1,1)$ custodial symmetry. We have linearly combined all the bilinear invariants that can be formed with the doublet Φ, to build the Lagrangian for the scalar sector, from which quadratic potentials having a marked hierarchy of masses arise, thus we have naturally identified the emerging fields with the inflaton and quintessence.

We have also involved three fermionic matter fields in our model, which were introduced under a doublet representation (Ψ) and a singlet ($\mathtt{N}$) of $SO(1,1)$, then, we linearly combined all the invariants formed with these representations and the scalar doublet Φ, into our interaction Lagrangian. Under such a construction, the model naturally provides Yukawa-like couplings between the inflaton and fermions and couplings between quintessence and fermions.

Since the cosmological doublet representation does not belong to the Standard Model particle sector, the new fermions do not belong either, thus we naturally identified them as right-handed neutrinos.

The couplings between the inflaton and the neutrinos provide the disintegration channel of inflaton leading to the reheating of the early Universe whereas in the quintessence sector the neutrinos keep coupled to the false vacuum state in which the quintessence field is trapped. We have shown in detail, how the neutrino fields acquire a large mass due to the couplings with the false vacuum state.

This is a remarkable outcome of our model, without any further assumption, beyond the use of symmetries, it introduces a way to naturally understand the existence of large sterile Majorana neutrino masses as sourced by DE. It is quite interesting that, by assuming all the Yukawa couplings about the same values, our model yields to values of such neutrino masses in the range needed for the seesaw mechanism to work $(10^{13} - 10^{15})$ GeV, as well as the typical reheating temperature of the primordial neutrino plasma which is in the same range.

Since the primordial neutrinos are coupled to the Higgs and Standard Model leptons through couplings of the form $\bar{L}\widetilde{H}\mathtt{N}$, our model includes the decay and co-annihilation channels of neutrinos for the creation of the plasma involving all the SM particles. We have studied these processes and we have found they are efficient enough to wipe all the heavy neutrinos allowing that posterior thermal history proceeds as usual.

On the other hand, the same mechanism of our model that generate neutrino masses, implies possible contributions to relativistic number density in the form of $\mathcal{X}$ quanta, due to out-of-equilibrium right-handed neutrino co-annihilation processes of the form $\bar{\nu}\nu \to \mathcal{X}\mathcal{X}$, which, unlike the SM channel, could impact the thermal history of the Universe.

We have estimated this effect by numerical integration of the Boltzmann equations for a reasonable range of the neutrino masses. We have found that the $\mathcal{X}$ quanta production is so suppressed that the total amount of injected relativistic number density is negligible. This clearly indicates that, without any further constraints or assumptions, our model remains consistent with the conditions required for a successful Big Bang Nucleosynthesis.

Also, because the seesaw mechanism induces effective couplings between the scalar field and the light active neutrinos, which have a temperature, we have calculated the precise increase in the quintessence mass due to thermal corrections induced by the effective couplings.

What we have found is that such corrections do not spoil the flatness of the potential, hence, the scalar field preserves its DE nature despite the redshift of its effective mass. This redshift, nevertheless, could be important regarding the problem of coincidence, that is something that would be interesting to look at. As a surprising additional result, we found that the maximum allowable thermal correction matches the known bounds on light neutrino masses.

Our model requires complex scalars to realize the symmetry, and thus it involves dynamical phases. We have shown that, provided the trivial initial conditions, the phases stay fix to zero, consequently, they are not potentially relevant for the phenomenology studied here, neither for the after-inflation evolution of the Universe. However, other initial conditions would lead to new phenomenology that we have not studied yet.

Throughout this work, we have exposed in detail the building of our model as well as its main phenomenological features. Our proposal and findings have been reported in Ref. [94], in which the condensed version of this work can be consulted.

Appendix A

Complementary Notes on Cosmology

A.1 Evolution of the Hubble parameter on the scale factor, density parameter and matter-radiation equality

Let us assume that the content of the background universe can be described by a set of fluids each of them with a constant equation of state (EoS) of the form

$$\omega_i = P_i/\rho_i \ , \tag{A.1}$$

when $i = \{r, m, \kappa, {\scriptstyle DE}\}$ stands for radiation ($\omega_r = 1/3$), matter ($\omega_m = 0$), spatial curvature ($\omega_\kappa = -1/3$) and DE ($\omega_{DE} = -1$), these two last considered as effective fluids. Let us also assume that the different species are decoupled such that the continuity equation is fullfilled for each one separately

$$\dot{\rho}_i + 3\mathsf{H}(\rho_i + P_i) = 0 \ , \tag{A.2}$$

so that, with (A.1) and (2.8) it becomes

$$\frac{d\rho_i}{dt} + \frac{3}{a}\frac{da}{dt}(1+\omega_i)\rho_i = 0 \ ,$$

then

$$\ln \rho_i = -3(1+\omega_i)\ln a + \ln \mathcal{K}, \quad \rightarrow \quad \rho_i = \mathcal{K} a^{-3(1+\omega_i)} \ ,$$

where $\mathcal{K}$ is a constant that can be determined by the initial conditions such that when $a = a_0$ then $\rho = \rho_{i,0}$, where the subscript $_0$ is used to indicate the value of the density and the scale factor today, then

$$\mathcal{K} = \frac{\rho_{i,0}}{a_0^{-3(1+\omega_i)}} \ ,$$

therefore the density ρ_i is given by

$$\rho_i = \rho_{i,0} \left(\frac{a}{a_0} \right)^{-3(1+\omega_i)} \ . \tag{A.3}$$

The total energy density is the sumatory

$$\rho = \sum_i \rho_i \ ,$$

so, the first Friedmann equation (2.12), can be written as

$$\mathsf{H}^2 = \sum_i \frac{1}{3M_{\rm pl}^2} \rho_{i,0} \left(\frac{a}{a_0} \right)^{-3(1+\omega_i)} \ . \tag{A.4}$$

The previous equation can be put in a more standard form by definition of the *density parameter*

$$\Omega := \frac{\rho}{\rho_c} \ , \tag{A.5}$$

where ρ_c is the *critic density*, which is commonly defined in terms of the value of the Hubble parameter today as

$$\rho_c = 3M_{\rm pl}^2 \mathsf{H}_0^2 \ . \tag{A.6}$$

Thus, by using equation (A.3) we can write the density parameter for each matter component as

$$\Omega_i = \frac{\rho_i}{\rho_c} \quad \longrightarrow \quad \Omega_i = \Omega_{i,0} \left(\frac{a}{a_0} \right)^{-3(1+\omega_i)} \ , \tag{A.7}$$

where $\Omega_{i,0}$ is the today's value of the density parameter for each matter component, i.e.

$$\Omega_{i,0} := \frac{\rho_{i,0}}{3M_{\rm pl}^2 \mathsf{H}_0^2} \ . \tag{A.8}$$

With the definitions (A.7) and (A.8), the equation (A.4) becomes

$$\mathsf{H}^2 = \mathsf{H}_0^2 \sum_i \Omega_{i,0} \left(\frac{a}{a_o}\right)^{-3(1+\omega_i)} \quad . \tag{A.9}$$

The previous equation accounts for all the different kind of species forming the content of the universe (even curvature), nevertheles, for a given epoch, we can consider only the contribution coming from the dominant one, such that, without losing of generality we can write

$$\mathsf{H} = \mathsf{H}_0 \Omega_{i,0}^{1/2} \left(\frac{a}{a_0}\right)^{-3(1+\omega_i)/2} \quad . \tag{A.10}$$

Comming back to the equation (A.7), notice that it allows finding the value of the scale factor at which the matter energy-density equals that of radiation one, it is given by

$$\left(\frac{a_{eq}}{a_0}\right) = \frac{\Omega_{r,0}}{\Omega_{m,0}} \ , \tag{A.11}$$

by subtitution of this into (A.9) leads to

$$\mathsf{H}_{eq} = \frac{\Omega_{m,0}^2}{\Omega_{r,0}^{3/2}} \mathsf{H}_0 \sqrt{2} \ , \tag{A.12}$$

which is the value of the Hubble parameter at the time of matter-radiation equality.

A.2 The Horizon

Among others, the references used for this section are [95, 96, 99, 100, 101].

Remember that the physical distance at the time t traveled by a particle moving with the light velocity in radial direction ($d\theta^2 = d\phi^2 = 0$), is the measure over the spatial hypersurface ($dt = 0$). If such a particle starts from the origin, by using (2.5) the physical distance traveled is

$$s(t) = a(t) \int_0^{r'} \frac{dr}{\sqrt{1 - kr^2}} \ . \tag{A.13}$$

So that, the metric (2.5) allows to define the *horizon*, which is the physical distance between two particles at time t_2, which started to separate each other in opposite directions at the time t_1. Since the particles travel with the light velocity, the horizon defines the *causally connected region*.

By means of the previous equation, the horizon is defined as

$$d_H(t_1, t_2) := 2a(t_2) \int_0^{r_2} \frac{dr}{\sqrt{1 - kr^2}} \ . \tag{A.14}$$

On the other hand, for a light ray $ds = 0$. If it starts from the origin in radial direction and by using the metric (2.5) is is obtained

$$dt = a(t) \frac{dr}{\sqrt{1 - kr^2}} \ ,$$

by integration between t_1 and t_2 it becomes

$$\eta := \int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_0^{r_2} \frac{dr}{\sqrt{1 - kr^2}} \ . \tag{A.15}$$

By sustituting this into (A.14) a most commonly used definition of the horizon is obtained, it is

$$d_H(t_1, t_2) = 2a(t_2) \int_{t_1}^{t_2} \frac{dt}{a(t)} \ . \tag{A.16}$$

This defines the causally connected region, since any event at the origin at time t_1 would have effects inside the volume of the sphere of diametre d_H at time t_2.

In addition to the horizon, there is another importan quantity called the *conformal time* η, as defined in equation (A.15), which, accordingly to equation (A.13) equals the physical distance divided by the scale factor, and similarly to the physical distance, it allows to define the *commoving horizon* as

$$X_H(t_i, t_f) := \int_{t_i}^{t_f} \frac{dt}{a(t)} \ , \tag{A.17}$$

such that

$$d_H(t_i, t_f) = a(t_f) X_H(t_i, t_f) \ . \tag{A.18}$$

A.3 Evolution of the horizon, strong energy condition (SEC), and Hubble radius

By using equation (2.8), the horizon (A.16) can be rewriten as

$$d_H(a_1, a_2) = 2a_2 \int_{a_1}^{a_2} \frac{da}{a^2 \mathsf{H}(a)} \ . \tag{A.19}$$

The evolution of H is given by (A.10), by substitution of this into (A.19), it is obtained the integral

$$d_H(a_1, a_2) = 2\frac{a_2 a_0^{-3(1+\omega_i)/2}}{\mathsf{H}_0 \Omega_{i,0}^{1/2}} \int_{a_1}^{a_2} \frac{da}{a^{(1-3\omega_i)/2}} \ , \tag{A.20}$$

by performing the integral the horizon becomes

$$d_H(a_1, a_2) = \frac{4}{\mathsf{H}_0 \Omega_{i,0}^{1/2}} \frac{a_2 a_0^{-3(1+\omega_i)/2}}{(1+3\omega_i)} \left\{ a_2^{(1+3\omega_i)/2} - a_1^{(1+3\omega_i)/2} \right\} \ . \tag{A.21}$$

The inequality

$$1 + 3\omega_i > 0 \ , \tag{A.22}$$

defines the *strong energy condition* (SEC). When this is fullfilled, which happens during both, the radiation ($\omega_r = 1/3$) and matter ($\omega_m = 0$) dominated ages, the integral is convergent for $a_1 \to 0$, and the horizon grows on a. Further, if $a_2 \gg a_1$ the evaluation (A.21) does not depend on a_1 and simplifies to

$$d_H(a_1, a_2) = \frac{4}{\mathsf{H}_0 \Omega_{i,0}^{1/2}} \frac{a_0^{-3(1+\omega_i)/2} a_2^{3(1+\omega_i)/2}}{(1+3\omega_i)} \ , \tag{A.23}$$

which can be simplified even more by virtue of equation (A.10) to yield

$$d_H(a_1, a_2) = \frac{4}{\mathsf{H}(a_2)(1+3\omega_i)} \ . \tag{A.24}$$

When the SEC (A.22) is violated, which happens during the DE dominated age, the integral does not converge for $a_1 \to 0$. For example, for cosmological constant domination $\omega_\Lambda = -1$, the equation (A.21) becomes

$$d_H(a_1, a_2) = \frac{2}{\mathsf{H}_0 \Omega_{\Lambda,0}^{1/2}} \left\{ \frac{a_2}{a_1} - 1 \right\} \ .$$

In the case when Λ dominates totally, the quantity of $\mathsf{H}_0\Omega_{\Lambda,0}^{1/2}$ can be replaced by either, H_1 or H_2, where H_1 is the Hubble parameter when Λ started to dominate. Remember that during this era H keeps constant and the scale factor grows exponentially, then, when $a_2 \gg a_1$, the previous equation becomes

$$d_H(a_1, a_2) = \frac{2}{\mathsf{H}_2}\frac{a_2}{a_1} \ , \tag{A.25}$$

where a_1 is the value of the scale factor at which Λ begins to dominate.

The quantity H^{-1} appearing in the previous equations, is defined as the *Hubble radius* R_H,

$$R_H(a) := \frac{1}{\mathsf{H}(a)} \ , \tag{A.26}$$

so that, the horizon is often written in terms of R_H, as for example, in accordance with (A.24), during the radiation dominated age one has that

$$d_H^{rad}(a_1, a_2) = 2R_H(a_2) \ , \tag{A.27}$$

and during the matter dominated age one has that

$$d_H^{mat}(a_1, a_2) = 4R_H(a_2) \ , \tag{A.28}$$

similarly, accordingly to (A.25), during the Λ-domination, the horizon is

$$d_H^{\Lambda}(a_1, a_2) = 2R_H(a_2)\frac{a_2}{a_1} \ . \tag{A.29}$$

A.4 The flatness and the horizon problems

The main references for this section are [99, 100].

Accordig to (A.3), the radiation energy density ($\omega_r = 1/3$), evolves as

$$\rho_r = \rho_{r,0}\left(\frac{a}{a_0}\right)^{-4} \ , \tag{A.30}$$

and the curvature density ($\omega_\kappa = -1/3$), as showed in (2.20) evolves as

$$\rho_\kappa = \rho_{\kappa,0}\left(\frac{a}{a_0}\right)^{-2} \ . \tag{A.31}$$

If the scale factor today is a_0 and at the beginning of the universe is a_i, then, the ratio of the radiation energy density today to the one at the beginning is

$$\frac{\rho_r(a_0)}{\rho_r(a_i)} = \left(\frac{a_i}{a_0}\right)^4 . \tag{A.32}$$

Let us also write the ratio of the initial curvature energy density to the initial radiation energy density

$$\frac{\rho_\kappa(a_i)}{\rho_r(a_i)} = \frac{\rho_{\kappa,0} a_i^{-2}}{\rho_{r,0} a_i^{-4}} .$$

As showed in section (D.1),

$$\Omega_r := \Omega_\gamma + \Omega_\nu \sim \Omega_\kappa \sim 10^{-3} ,$$

thus in the previous equation we can choose $\rho_{\kappa,0} \sim \rho_{r,0}$ to get

$$\frac{\rho_\kappa(a_i)}{\rho_r(a_i)} = a_i^2 . \tag{A.33}$$

Now, from the quantities given in section (D.1.4) together with (D.7) it is known that the value of the radiation energy density today is

$$\rho_r(a_0) = (10^{-4}\ \mathrm{eV})^4 . \tag{A.34}$$

On the other hand, when $a = a_i$, it is supposed that the universe just had emerged from the Plank era, so that it is reasonable to assume that the radiation energy density, which then dominated, was in the Planck scale, such that

$$\rho_r(a_i) = (10^{19}\ \mathrm{GeV})^4 = (10^{28}\ \mathrm{eV})^4 , \tag{A.35}$$

from the previous quantities it is calculated the quotient

$$\frac{\rho_r(a_0)}{\rho_r(a_i)} = (10^{-32})^4 , \tag{A.36}$$

which together with (A.32) and assuming by convention that $a_0 = 1$, leads to the value of the universal scale factor at the end of Planck era,

$$a_i = 10^{-32} \qquad \longrightarrow \qquad a_i^2 = 10^{-64} . \tag{A.37}$$

From this result and the equation (A.33) it is obtainded the initial condition

$$\rho_\kappa(a_i) = \rho_r(a_i) \times 10^{-64} . \tag{A.38}$$

The previous condition puts a very strong constraint to the initial value of the curvature energy density, which must be fine tunned 64 orders of magnitude below the corresponding to the radiation energy density to account for the value observed today. This fine tunning is known as *the flatness problem.*

Let us now calculate the angular size of the causally connected regions of the sky as them should appear in the cosmic microwave background (CMB) detected on the Earth nowadays.

Let t_{dec} be the cosmological time at which the CMB was produced, i.e., the time when the Compton scattering stopped due to the proton-electron bounding to form Hidrogen atoms.

As showed in equation (A.17), the diameter of the commoving causally connected region which could have thermalized at time t_{dec}, is given by the commoving horizon $X_H(t_i, t_{dec})$. It corresponds to the sky's arc sector away from us by a commoving distance $X_H(t_{dec}, t_0)/2$. If θ is the angle subtended by the region, then

$$\theta = \frac{2X_H(t_i, t_{dec})}{X_H(t_{dec}, t_0)} \ ,$$

which can be put in terms of the (physical) horizon by means of equation (A.18) to get

$$\theta = \frac{2a_0 d_H^{rad}(t_i, t_{eq})}{a_{eq} d_H^{mat}(t_{eq}, t_0)} \ .$$

In the previous equation we have assumed that $t_{dec} \approx t_{eq}$, where t_{eq} is the time at which the equality matter-radiaton was achieved, so we can replace (A.27) and (A.28) into it to get

$$\theta = \frac{a_0}{a_{ec}} \frac{H_0}{H_{ec}} \ .$$

Finally, by substitution of equations (A.11) and (A.12) into the previous one, it is gotten

$$\theta = \frac{\Omega_{r,0}^{1/2}}{\Omega_{m,0}} \sqrt{2} \ .$$

The result, as calculated with the values given in (D.1.4) gives an angle of around 10 degrees, which strongly contradicts with the observations of the CMB showing a quite smooth distribution of temperatures covering the whole sky, which differs from its mean value by one part in 10^{-6}, as if it would have been emitted completely thermalized, i.e., as if the causaly connected region had been larger than the horizon at the time of decoupling. This inconsistency is known as *the horizon problem*.

A.5 Cosmic inflation as a solution to the flatness and horizon problems

The flatness and horizon problems can be sorted out by the supposition that previous to the radiation dominated age, there was an stage of cosmic accelerated expansion. Such an stage is called *cosmic inflation*, which can be realized if the SEC is violated the enough time.

Let us show how it works. Let be a_i and a_f the scale factor at the beginning and at the end of inflation respectively, and let be a_0 the scale factor today.

Let us assume that when $a = a_i$ the universe was dominated totally by spatial curvature, i.e., $|\Omega_k(a_i)| \sim 1$, this in order to estimate the *minimum* inflation required, then

$$\rho_T(a_i) = \rho_\kappa(a_i) \ . \tag{A.39}$$

During inflation, the parameter H is constant so is the total energy density, then

$$\rho_T(a_i) = \rho_T(a_f) \ . \tag{A.40}$$

Notice that if the expansion is accelerated, then $\Omega_\kappa \sim 1/\dot{a}^2$, which means that it is diluted because $\dot{a}$ grows. This let us to impose the condition that when $a = a_f$ at the end of inflation only the radiation dominates, then

$$\rho_T(a_f) = \rho_r(a_f) \ . \tag{A.41}$$

From (A.39), (A.40), and (A.41) it is gotten

$$\rho_\kappa(a_i) = \rho_r(a_f) \ .$$

This let us comparing with the today's radiation density to get

$$\frac{\rho_r(a_0)}{\rho_\kappa(a_i)} = \frac{\rho_r(a_0)}{\rho_r(a_f)} \ . \tag{A.42}$$

Let us also assume, an universe compossed only by curvature and radiation, then in accordance with (2.6) when $a = a_0$, it becomes

$$\Omega_r(a_0) - \Omega_\kappa(a_0) = 1 \ ,$$

which in virtue of (A.5) is the same as

$$\rho_r(a_0) - \rho_\kappa(a_0) = \rho_c \ . \tag{A.43}$$

By writing the today's curvature density as a fraction of the critic density

$$\rho_\kappa(a_0) = x\rho_c \ , \qquad 0 < x \leq 1 \ , \tag{A.44}$$

and by combination of this with (A.43) it is obtained

$$\rho_r(a_0) = \frac{(1+x)}{x}\rho_\kappa(a_0) \ .$$

By subtituting the previous equation into (A.42) it becomes

$$\frac{(1+x)}{x}\frac{\rho_\kappa(a_0)}{\rho_\kappa(a_i)} = \frac{\rho_r(a_0)}{\rho_r(a_f)} \ ,$$

next, by (A.30) and (A.31) the previous equation becomes

$$\frac{(1+x)}{x}\left(\frac{a_0}{a_i}\right)^{-2} = \left(\frac{a_0}{a_f}\right)^{-4} \ ,$$

which is the same that

$$\sqrt{\frac{(1+x)}{x}}\frac{a_0}{a_f} = \frac{a_f}{a_i} \ ,$$

Finally, with the value of $x > 0$ as defined in (A.44), we arrive to the condition

$$\frac{a_0}{a_f} \leq \frac{a_f}{a_i} \ . \tag{A.45}$$

It means, that in order to flatten the universe, starting from a highly spatial curvature as large as $|\Omega_\kappa| \sim 1$, it is necessary that previously to the beginning of the radiation dominated age, the universe had expanded at least as it has expanded since the end of the inflation age until today.

Let us now move to the horizon problem. In order to account for the CMB as we see it today, thermalized completely in all directions on the sky, it is necessary that, at the time when it was emitted, the causally connected region has been at least of the size of the today's horizon. Except for a factor of π, a reasonable approximation of such a condition, in commoving coordinates can be written as

$$X_H(a_f, a_0) \leq X_H(a_i, a_f) \ .$$

By using equation (A.18) into the previous we have

$$\frac{d_H^{rad}(a_f, a_0)}{a_0} \leq \frac{d_H^{\Lambda}(a_i, a_f)}{a_f} \ , \tag{A.46}$$

where we have assumed again that the universe has been dominated by radiation since the end of inflation, and that it was dominated by a fluid violating the SEC during it, then with (A.27), (A.29) and (A.26), the previous equation becomes

$$\frac{a_f}{a_0}\frac{\mathsf{H}(a_f)}{\mathsf{H}(a_0)} \leq \frac{a_f}{a_i} \ ,$$

finally, with (A.10) we arrive to

$$\frac{a_0}{a_f} \leq \frac{a_f}{a_i} \ , \tag{A.47}$$

which is the same condition given in (A.45) found before, so that, the same is required in order to solve the horizon problem.

A.6 The e-folding number and amount of inflation

The e-folding number (N) is defined as

$$N := \log a \ , \tag{A.48}$$

so, it's the quantity for which the universe increases its size by a factor of e. Notice that, from the definition of the Hubble parameter (2.8) we have $da/a = \mathsf{H}dt$, from which it is straightforward to arrive to

$$\Delta N = N_f - N_i = \int_{t_i}^{t_f} \mathsf{H}dt \ . \tag{A.49}$$

Turning to the condition (A.47), we have that the minimun amount of inflation corresponds to the equality, thus, with the definition (A.48) we can write

$$\Delta N = N_f - N_i = \log \frac{a_0}{a_f} \ . \tag{A.50}$$

The quotient a_0/a_f can be estimated by means of Eq. (A.30), it leads to

$$\frac{\rho_r(a_0)}{\rho_r(a_f)} = \left(\frac{a_0}{a_f}\right)^{-4} \ . \tag{A.51}$$

By virtue of (A.35) and (A.40) we can write

$$\rho_r(a_f) = (10^{28} \text{ eV})^4 \ ,$$

on the other hand by equation (A.34) we have

$$\rho_r(a_0) = (10^{-4} \text{ eV})^4 \ ,$$

thus, with these into (A.51) we arrive to

$$\frac{a_0}{a_f} = 10^{32} \ ,$$

which, together with (A.50) leads to

$$\Delta N \approx 74 \ , \tag{A.52}$$

therefore, if the energy scale of the inflationary age was of the order of Planck energy, it is necessary that the universe had undergone ≈ 74 e-foldings in order to solve the flatness and the horizon problems.

A.7 The inflaton, performing inflation with a scalar field and the slow-roll conditions.

The inflationary age can be realized by means of a canonical scalar field whose energy density dominates the Universe completely during that epoch and which, in certain regimen, behaves like a fluid violating the SEC (A.22). Such a scalar fiel is called *the inflaton*. Its dynamics is explained below.

The Lagrangian density for a real scalar field φ, reads

$$\mathcal{L}(\varphi) = \frac{1}{2} g^{\mu\nu} \partial_\nu \varphi \partial_\mu \varphi - V(\varphi) \ , \tag{A.53}$$

where, in the context of standar cosmology, the (inverse) metric $g^{\mu\nu}$ appearing here, corresponds to the FLRW metric (2.5).

The action is

$$S[g^{\mu\nu}, \varphi] = \int dx^4 \sqrt{-g}\mathcal{L}(\varphi) \ . \tag{A.54}$$

By variation of this action with respect to φ, it is obtained the Klein-Gordon equation in a expanding universe, which by considerig φ homogeneous, takes the form

$$\ddot{\varphi} + 3\mathsf{H}\dot{\varphi} + V_{,\varphi}(\varphi) \ . \tag{A.55}$$

Also, by variation of the action (A.54), but this time, with respect to $g^{\mu\nu}$, it is obtained the energy-momentum tensor

$$T^{\mu}_{\ \nu}(\varphi) = 2g^{\mu\alpha}\frac{\delta\mathcal{L}(\varphi)}{\delta g^{\alpha\nu}} - \delta^{\mu}_{\ \nu}\mathcal{L}(\varphi) \ , \tag{A.56}$$

which, by involving the Lagrangian (A.53) takes the form

$$T^{\mu}_{\ \nu}(\varphi) = g^{\mu\beta}\partial_\beta\varphi\partial_\nu\varphi - \delta^{\mu}_{\ \nu}\mathcal{L}(\varphi) \ . \tag{A.57}$$

For a perfect fluid the energy-momentum tensor is of the form

$$\{T^{\mu}_{\ \nu}\} = \mathrm{diag}(\rho, -p, -p, -p) \ ,$$

thus we can calculate the energy density (ρ_φ) and the pressure (p_φ) in a FLRW universe from (A.57) and (A.53) as follows: the ${}^{0}_{\ 0}$ component is

$$T^{0}_{\ 0}(\varphi) = g^{00}(\partial_0\varphi)(\partial_0\varphi) - \delta^{0}_{\ 0}\left\{\frac{1}{2}\left[g^{00}(\partial_0\varphi)(\partial_0\varphi) + g^{kk}(\partial_k\varphi)(\partial_k\varphi)\right] - V(\varphi)\right\} \ ,$$

which, in the case of φ homogeneous and with $T^{0}_{\ 0}(\varphi) = \rho_\varphi$, leads to

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \ , \tag{A.58}$$

and similarly, the non zero spatial components of (A.57) are

$$T^i{}_j(\varphi) = g^{ii}(\partial_i\varphi)(\partial_i\varphi) - \delta^i{}_j \left\{ \frac{1}{2} \left[g^{00}(\partial_0\varphi)(\partial_0\varphi) + g^{kk}(\partial_k\varphi)(\partial_k\varphi) \right] - V(\varphi) \right\} \ ,$$

which again, in the case of φ homogeneous and with $-3p_\varphi = T^i{}_i(\varphi)$ leads to

$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \ . \tag{A.59}$$

During inflation the universe is supposed to be dominated totally by φ, there still was not ordinary matter, not even DE, (although the inflaton itself behaves like this, it does at an energy scale higher than which we observe today), then with no presence of Λ, the Einstein equations becomes

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{M_{\mathrm{pl}}^2}T_{\mu\nu}(\varphi) \ .$$

The only one energy density possibly present aside that of the inflaton is that of spatial curvature, but as said above it dilutes quickly due to inflation, thus instead equations (2.6) and (2.7), we have

$$\mathsf{H}^2 = \frac{\rho_\varphi}{3M_{\mathrm{pl}}^2} \ , \tag{A.60}$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\mathrm{pl}}^2}(\rho_\varphi + 3p_\varphi) \ . \tag{A.61}$$

The last equation can be rewritten by using the Eq. (2.8), which leads to

$$\mathsf{H} = \frac{\dot{a}}{a} \qquad \to \qquad \frac{\ddot{a}}{a} = \dot{\mathsf{H}} + \mathsf{H}^2 \ ,$$

with this, together with (A.60), the equation (A.61) becomes

$$\dot{\mathsf{H}} = -\frac{1}{2M_{\mathrm{pl}}^2}\left(\rho_\varphi + p_\varphi\right) \ , \tag{A.62}$$

which in turn, by virtue of (A.58) and (A.59) simplifies to

$$\dot{\mathsf{H}} = -\frac{\dot{\varphi}^2}{2M_{\mathrm{pl}}^2} \ . \tag{A.63}$$

However, all of this is not enough to drive inflation yet, the highly accelerated expansion does not occur unless the scalar field evolves under the so-called, *slow-roll* regime, in which is fullfilled the condition that

$$\frac{1}{2}\dot{\varphi}^2 \ll V(\varphi) \ , \tag{A.64}$$

which is known as the first slow-roll condition (FSRC), meaning that the inflaton rolls down the potential quite slowly. With this condition the energy density (A.58) and the pressure (A.59) become respectively

$$\rho_\varphi \approx V(\varphi) \ , \tag{A.65}$$

and

$$p_\varphi \approx -V(\varphi) \ , \tag{A.66}$$

from which, it can be seen that

$$\omega_\varphi = \frac{p_\varphi}{\rho_\varphi} \approx -1 \ ,$$

therefore, in the slow-roll regime, the SEC (A.22) is violated instantaneously. By using (A.65), the equation (A.60) becomes

$$\mathsf{H}^2 \approx \frac{V(\varphi)}{3M_{\rm pl}^2} \ , \tag{A.67}$$

wherein the potential is approximately constant, as can be seen from the equation (A.63) which under this regimen becomes

$$\dot{\mathsf{H}} \approx 0 \ , \tag{A.68}$$

consequently, during inflation the horizon evolves accordingly to (A.25).

On the other hand, since the FSRC applies instantaneously, there is neccesary a second slow roll condition (SSRC) in order to guarantee that inflation holds for the enough time. The SSRC is defined as the absolute value of the time derivative of the FSRC, then

$$|\dot{\varphi}\ddot{\varphi}| \ll \left|\dot{\varphi}\frac{\partial V}{\partial\varphi}\right| \quad \rightarrow \quad |\ddot{\varphi}| \ll \left|\frac{\partial V}{\partial\varphi}\right| . \tag{A.69}$$

An equivalent form of the previous, is gotten by combining (A.64) and (A.67) from where, we get

$$\frac{1}{2}\dot{\varphi}^2 \ll 3M_{\rm pl}^2\mathsf{H}^2 \ ,$$

by taking the derivative of this last, and by using (A.63), we arrive to

$$|\ddot{\varphi}| \ll 3\mathsf{H}|\dot{\varphi}| \ . \tag{A.70}$$

This condition says us that the second term in the left hand side member of (A.55) dominates over the first one, carrying the system into the so named freezing mode, in which the dynamic equation for the inflaton simplifies to

$$3\mathsf{H}\dot{\varphi} \approx -V_{,\varphi} \ . \tag{A.71}$$

A.8 Slow-roll conditions in terms of the potential and definition of the slow-roll parameters

From (A.71)

$$\dot{\varphi} \approx -\frac{V_{,\varphi}}{3\mathsf{H}} \ , \tag{A.72}$$

with this last and (A.67) it is obtained

$$\dot{\varphi}^2 = \frac{M_{pl}^2}{3}\frac{V_{,\varphi}^2}{V} \ ,$$

and again with the FSRC (A.64) it is possible to write itself in terms of the potential only

$$\frac{1}{6}M_{pl}^2\left(\frac{V_{,\varphi}}{V}\right)^2 \ll 1 \ . \tag{A.73}$$

By differentiating (A.72)

$$\ddot{\varphi} = -\frac{1}{3}\Big(\frac{\mathsf{H}\dot{V}_{,\varphi} - \dot{\mathsf{H}}V_{,\varphi}}{\mathsf{H}^2}\Big) \ . \tag{A.74}$$

By differentiating

$$\dot{V}_{,\varphi} = V_{,\varphi\varphi}\dot{\varphi} \ ,$$

with this last and by using again (A.72) we get

$$\dot{V}_{\varphi} = -\frac{V_{,\varphi\varphi}V_{,\varphi}}{3\mathsf{H}} \ , \tag{A.75}$$

with the last equation, together with (A.67) into equation (A.74) it is possible to arrive to

$$\ddot{\varphi} = -\frac{M_{pl}^2}{3}\Big(- \frac{V_{,\varphi\varphi}V_{,\varphi}}{V} + \frac{V_{,\varphi}^3}{2V^2}\Big) \ . \tag{A.76}$$

By substitution of this last in the dynamic equation (A.55), and using (A.72) we arrive to

$$\frac{M_{pl}^2}{3}\frac{V_{,\varphi\varphi}}{V} = \frac{M_{pl}^2}{6}\Big(\frac{V_{,\varphi}}{V}\Big)^2 \ ,$$

but, because of the FSRC it is concluded that

$$\frac{1}{3}M_{pl}^2\frac{V_{,\varphi\varphi}}{V} \ll 1 \ . \tag{A.77}$$

Liddle and Lyth introduced the so called, *slow-roll parameters* (see Refs. [14]), which come from (A.73) and (A.77), they are definded as

$$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{V_{,\varphi}}{V}\right)^2, \qquad \epsilon \ll 1 \ , \tag{A.78}$$

and

$$\eta = M_{pl}^2\left(\frac{V_{,\varphi\varphi}}{V}\right), \qquad |\eta| \ll 1 \ . \tag{A.79}$$

A.9 Slow Roll Conditions in terms of H

It is ilustrative to express the slow-roll conditions only in terms of H. We start from (A.74), by substitution of (A.75) into it, we arrive to

$$\ddot{\varphi} = \frac{1}{3}\Big(M_{pl}^2\frac{V_{,\varphi\varphi}V_{,\varphi}}{V} + \frac{\dot{\mathsf{H}}}{\mathsf{H}^2}V_{,\varphi}\Big) \ ,$$

with this last equation into (A.55) it is obtainded

$$-\frac{\dot{\mathsf{H}}}{3\mathsf{H}^2} = \frac{1}{3}M_{pl}^2\frac{V_{,\varphi\varphi}}{V} \ ,$$

but because of (A.77) the last equation implies

$$-\dot{\mathsf{H}} \ll 3\mathsf{H}^2 \,. \tag{A.80}$$

By differentiating the last equation, it becomes

$$|\ddot{\mathsf{H}}| \ll |6\mathsf{H}\dot{\mathsf{H}}| \,. \tag{A.81}$$

Let us make use of (A.63) and (A.72) to write

$$-6\mathsf{H}\dot{\mathsf{H}} = \frac{1}{M_{pl}^2}\frac{V_{,\varphi\varphi}^2}{3\mathsf{H}} \,, \tag{A.82}$$

let us also to write the FSRC (A.73) as

$$\frac{V_{,\varphi}^2}{3\mathsf{H}M_{pl}^2} \ll \frac{6V^2}{3\mathsf{H}M_{pl}^4} \,.$$

By substitution of both the last equation and equation (A.67) into (A.82) it is obtained

$$-6\mathsf{H}\dot{\mathsf{H}} \ll 18\mathsf{H}^3 \,. \tag{A.83}$$

Finally, both equation (A.81) and (A.83) can be written together as

$$|\ddot{\mathsf{H}}| \ll |6\mathsf{H}\dot{\mathsf{H}}| \ll 18\mathsf{H}^3 \,. \tag{A.84}$$

A.10 The initial value of the inflaton, a simple example

From equation (A.71) we get

$$dt = -\frac{3\mathsf{H}}{V_{,\varphi}}d\varphi \quad \longrightarrow \quad \mathsf{H}dt = -\frac{3\mathsf{H}^2}{V_{,\varphi}}d\varphi \,,$$

which, by using (A.67), becomes

$$\mathsf{H}dt = -\frac{1}{M_{\rm pl}^2}\frac{V}{V_{,\varphi}} \,.$$

With this into (A.49) we arrive to

$$\Delta N = -\frac{1}{M_{\rm pl}^2}\int_{\varphi_i}^{\varphi_f} \frac{V}{V_{,\varphi}} d\varphi \ . \tag{A.85}$$

The inflationary age ends when either, the FSRC or the SSRC stops being fulfilled, therefore, for a given potential, the upper limit of the previous integral can be know by approximation to the unity of either (A.73) or (A.77).

For example, in *chaotic* inflation the potential is given by

$$V = \frac{1}{2} m_\varphi^2 \varphi^2 \ , \tag{A.86}$$

then

$$\left(\frac{V_{,\varphi}}{V}\right)^2 = \frac{4}{\varphi^2} \ . \tag{A.87}$$

Then, with the previous and by setting the FSRC (A.73) equals one we get

$$\frac{1}{6} M_{pl}^2 \left(\frac{V_{,\varphi}}{V}\right)^2_{(\varphi=\varphi_f)} = 1 \quad \longrightarrow \quad \frac{4}{\varphi_f^2} = \frac{6}{M_{\rm pl}^2} \ ,$$

from which we arrive to

$$\varphi_f^2 = \frac{2}{3} M_{\rm pl}^2 \ . \tag{A.88}$$

Next, by substitution of the square root of the inverse of (A.87) it is obtained

$$\Delta N = -\frac{1}{4M_{\rm pl}^2}\left(\varphi_f^2 - \varphi_i^2\right) \ ,$$

which, by virtue of (A.88) yields to

$$\varphi_i = 2\left(\Delta N + \frac{1}{6}\right)^{1/2} M_{\rm pl} \ .$$

Finally, for a given number of e-foldings, as for example, those calculated in (A.52), the initial value of the inflaton field would be of

$$\varphi_i \approx 17 M_{\rm pl} \ .$$

A.11 Quintessence

The field of quintessence is denoted in this appendix by Q, which is a canonical real scalar field. The dynamics in a expanding universe is ruled by the Klein-Gordon equation

$$\ddot{Q} + 3\mathsf{H}\dot{Q} + V_{,Q} = 0 \ . \tag{A.89}$$

In the same way as in the inflation case, in order to effectively exert a force again the pull of gravity it is necesary that quintessece to violate the SEC. To realize this the field Q has to comply with the first slow roll condition, therefore the FSRC for quintessence is

$$\frac{1}{2}\dot{Q}^2 \ll V(Q) \ . \tag{A.90}$$

This implies, like to inflation, that

$$\rho_Q \approx V(Q) \quad \text{and} \quad p_Q \approx -V(Q) \ .$$

However, since there exist other material components aside quintessence, it is no longer possible, to write equations similar to (A.60) and (A.63), instead those, it is imperative to put

$$H^2 = \frac{\rho_T}{3M_{pl}^2} \ , \tag{A.91}$$

and

$$\dot{H} = -\frac{1}{2M_{pl}^2}(\rho_T + p_T) \ , \tag{A.92}$$

where ρ_T accounts for the summation of the densities of all the cosmic invenary, namely, photons, baryons, quintessence itself, etc. At this point, it is neccesary to distinguish between two kinds of quintessence, they are call freezing and thawing.

A.11.1 Freezing vs Thawing Quintessence

The main references for this section are Ref. [49, 50].

As said above, unlike the inflation case, there are other components aside of the field Q, it implies that the second term (the friction term) in the first member of (A.89), not always becomes dominant over the first one, so the system can enter into either, freezing evolution [like in Eq. (A.71)] or thawing evolution, in which both terms are comparable.

In order to take in account both scenarios it is acostumed to define the parameter β as follows

$$\beta \equiv \frac{\ddot{Q}}{3H\dot{Q}} \ . \tag{A.93}$$

In terms of β, the equation (A.89) becomes

$$\beta + 1 + \frac{V(Q)_{,Q}}{3H\dot{Q}} = 0 \ , \tag{A.94}$$

There are two cases, freezing and thawing. For freezing $\beta \approx 0$ and the second term dominates. For thawing $\beta \approx \mathcal{O}(1)$. Also it is considered that $\beta \approx const$, in the sense that it is accomplished that

$$|\dot{\beta}| \ll H|\beta| \ . \tag{A.95}$$

More rigorously, freezing and thawing are related with the accesibility of the minima of the potential, this is the fundamental difference between each other, this translates into different directions of the evolution of the equation of the state ω. As said in Chapter 2, in both cases, the field starts evolving with $\omega_{DE} \approx -1$, then, in late epochs, $\omega_{DE} \to -1$ for freezing, whereas $\omega_{DE} \to -1/3$ for thawing, leading to different Universe's fates.

A.11.2 Slow-Roll conditions for Quintessence

A.11.2.1 First Slow Roll Condition for Quintessence

As said before quintessence has to evolve under the first slow roll condition given in equation (A.90). From Eq. (A.94), the field $\dot{Q}$ can be cleared to get,

$$\dot{Q} = -\frac{V(Q)_{,Q}}{3H(\beta + 1)} \ . \tag{A.96}$$

By means of this last, the equation (A.90) becomes

$$\frac{1}{(\beta+1)^2}\frac{1}{18H^2}\frac{V(Q)_{,Q}^2}{V(Q)} \ll 1 \ . \tag{A.97}$$

As said before, β can be either zero or order unity, consequently it can be ommited from (A.97) to get

$$\frac{1}{18H^2}\frac{V(Q)_{,Q}^2}{V(Q)} \ll 1 \; , \tag{A.98}$$

this is the generic form for the FSRC for quintessence, and from it the first parameter of slow roll for quintessence can be defined as

$$\epsilon = \frac{1}{6H^2}\frac{V(Q)_{,Q}^2}{V(Q)}, \qquad \epsilon \ll 1 \; . \tag{A.99}$$

It is defined in such a way that it coincides with the first parameter of slow roll for inflation, provided that the scalar field dominates.

It is important to note that the FSRC for quintessence as given in (A.98) [and consequentelly the parameter given by (A.99)], is valid in both cases, freezing and thawing quintessence models.

Mass condition. As an example, the value of the mass of the scalar field Q has a constriction arising directly from the FSRC. For instance, when the potential is of the form

$$V(Q) = \frac{1}{2}m_Q^2 Q^2 \; ,$$

the mass, accordingly to (A.99), has to fulfill

$$m_Q \ll \sqrt{3}H \; . \tag{A.100}$$

A.11.2.2 Second Slow Roll Condition for Quintessence

Similary to inflation, there must be consistence between the FSRC and the equation of motion. From the time derivative of (A.96) it is gotten

$$\ddot{Q} = -\frac{\dot{V}(Q)_{,Q}}{3(1+\beta)H} + \frac{V(Q)_{,Q}\dot{H}}{3(1+\beta)H^2} + \frac{V(Q)_{,Q}\dot{\beta}}{3(1+\beta)^2 H} \; . \tag{A.101}$$

The time derivative of $V(Q)_{,Q}$ is

$$\dot{V}(Q)_{,Q} = \dot{Q}V(Q)_{,QQ} = -\frac{V(Q)_{,Q}V(Q)_{,QQ}}{3(1+\beta)H} \; , \tag{A.102}$$

where it has been used the equation (A.96).

By writing both, the total energy density and the total pressure, which apperar in (A.92), as the contribution of all componets, it's obtained

$$\rho_T = \sum_B \rho_B, \qquad p_T = \sum_B p_B = \sum_B \omega_B \rho_B \ ,$$

where $B = \{\gamma, \nu, b, DM, DE\}$ for photons, neutrinos, baryons, dark matter and dark energy respectively, and where $\omega_B = 1/3$ for photons and neutrinos, $\omega_B = 0$ for baryons and dark matter, and $\omega_B = 0$ for dark energy. Whit this (A.92) becomes

$$\dot{H} = -\frac{1}{2M_{pl}^2} \sum_B (1+\omega_B)\rho_B \ ,$$

and (A.91) becomes

$$H^2 = \frac{1}{3M_{pl}^2} \sum_B \rho_B \ .$$

By using the previous equations (for any fixed value of B), we get

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(1+\omega_B) \ . \tag{A.103}$$

By substitution of (A.102) and (A.103) into (A.101) we arrive to

$$\ddot{Q} = \frac{V(Q)_{,Q} V(Q)_{,QQ}}{9(1+\beta)^2 H^2} - \frac{V(Q)_{,Q}(1+\omega_B)}{2(1+\beta)} + \frac{V(Q)_{,Q}\dot{\beta}}{3(1+\beta)^2 H} \ . \tag{A.104}$$

On the other hand, from (A.93) and (A.96) it is gotten

$$\ddot{Q} = -\frac{\beta V(Q)_{,Q}}{(1+\beta)} \ , \tag{A.105}$$

by matching (A.104) with (A.105) it is obtained

$$\beta = -\frac{V(Q)_{,QQ}}{9(1+\beta)H^2} + \frac{(1+\omega_B)}{2} - \frac{\dot{\beta}}{3(1+\beta)H} \ .$$

Taking in account the inequality showed in (A.95), the last equation becomes

$$3\beta = -\frac{V(Q)_{,QQ}}{3(1+\beta)H^2} + \frac{3}{2}(1+\omega_B) \ . \tag{A.106}$$

Let us define the second parameter of slow roll for Q as (remember that β is either zero or $\mathcal{O}(1)$, therefore it can be ommited in the definition of η),

$$\eta \equiv \frac{V(Q)_{,QQ}}{3H^2} \ , \tag{A.107}$$

then, by substituting this into Eq. (A.106), we get

$$3\beta = -\eta + \frac{3}{2}(1 + \omega_B) \ . \tag{A.108}$$

From here, it is possible to know what is the value of the parameter η, according to wheter the system is freezing or thawing, in the following way.

Freezing Quintessence As said in section (A.11.1) for freezing models the parameter β becomes negligible,

$$|\beta| \ll 1 \ ,$$

therefore from (A.108), the second parameter of slow roll for quintessence becomes

$$\eta = \frac{3}{2}(1 + \omega_B) \ . \tag{A.109}$$

Thawing Quintessence As said in section (A.11.1) for thawing models the parameter β becomes of order one. This has the following consequence: according to (A.95), the left hand side member of equation (A.108) is slowly-varying in time, it can be considered almost constant, which means that is an time independent quantity. The same applies to the second term of the right hand side member. On the other hand, since the parameter η as given in (A.107) is in general a time dependent quantity, the equality (A.108) holds only if the parameter η is negligible.

$$|\eta| \ll 1 \ ,$$

and β becomes

$$\beta = \frac{1 + \omega_B}{2} \ .$$

Appendix B

Complementary Notes on High-Energy Physics

B.1 Two-component notation

Along this work we use the following two component notation.

A right-handed Weyl field is written as

$$\mathtt{N}^{\dot{a}}, \qquad \dot{a} = \{1, 2\} \ , \tag{B.1}$$

whereas a left-handed Weyl field is written as

$$\mathtt{V}^{\dagger}_{a}, \qquad a = \{1, 2\} \ . \tag{B.2}$$

The Dirac matrices are

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu}_{a\dot{c}} \\ \bar{\sigma}^{\mu\dot{a}c} & 0 \end{pmatrix} \ , \tag{B.3}$$

where,

$$\sigma^{\mu}{}_{a\dot{a}} = (\mathbb{I},\ \sigma)\,, \qquad \bar{\sigma}^{\mu\dot{a}a} = (\mathbb{I},\ -\sigma)\,, \qquad \bar{\sigma}^{\mu\dot{a}a} = \epsilon^{\dot{a}\dot{b}}\epsilon^{ab}\sigma^{\mu}{}_{b\dot{b}} \ , \tag{B.4}$$

wherein

$$\sigma = (\sigma_1,\ \sigma_2,\ \sigma_3)\,, \quad \text{with} \quad \sigma_i \quad \text{the Pauli matrices.}$$

The β-matrix, which is numerically equal to the γ^0 but carries a different index structure is

$$\beta = \begin{pmatrix} 0 & \delta^{\dot{a}}{}_{\dot{c}} \\ \delta_{a}{}^{c} & 0 \end{pmatrix} \ . \tag{B.5}$$

The charge conjugation matrix is

$$C = \begin{pmatrix} \epsilon_{ac} & 0 \\ 0 & \epsilon^{\dot{a}\dot{c}} \end{pmatrix} = \begin{pmatrix} -\epsilon^{ac} & 0 \\ 0 & -\epsilon_{\dot{a}\dot{c}} \end{pmatrix} \ , \tag{B.6}$$

where

$$\epsilon^{12} = \epsilon^{\dot{1}\dot{2}} = \epsilon_{21} = \epsilon_{\dot{2}\dot{1}} = +1 \ .$$

The Dirac conjugated is

$$\overline{\psi}_{_D} = \psi^\dagger_{_D}\beta \ . \tag{B.7}$$

The charge conjugated is

$$\psi^{\mathcal{C}}_{_D} = C\overline{\psi}^{T}_{_D} \ . \tag{B.8}$$

The γ_5 matrix is

$$\gamma_5 = \begin{pmatrix} -\delta_a{}^c & 0 \\ 0 & +\delta^{\dot{a}}{}_{\dot{c}} \end{pmatrix} \ , \tag{B.9}$$

therefore, the left and right projection matrices are

$$P_L = \frac{1-\gamma_5}{2} = \begin{pmatrix} \delta_a{}^c & 0 \\ 0 & 0 \end{pmatrix}, \qquad P_R = \frac{1+\gamma_5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & \delta^{\dot{a}}{}_{\dot{c}} \end{pmatrix} \ . \tag{B.10}$$

B.2 Dirac fields

From the Weyl fields (B.1) and (B.2), a four-component Dirac field is composed as

$$\psi_{_D} = \begin{pmatrix} \mathtt{V}^\dagger_a \\ \mathtt{N}^{\dot{a}} \end{pmatrix} . \tag{B.11}$$

Its Hermitian conjugated is $\psi^\dagger_{_D} = \begin{pmatrix} \mathtt{V}_{\dot{a}} & \mathtt{N}^{\dagger a} \end{pmatrix}$, thus, by using (B.5) its Dirac conjugated becomes,

$$\overline{\psi}_{_D} = \begin{pmatrix} \mathtt{N}^{\dagger c} & \mathtt{V}_{\dot{c}} \end{pmatrix} , \tag{B.12}$$

with the transposed of the previous and (B.6) the charge conjugated of (B.11) becomes

$$\psi^{\mathcal{C}}_{_D} = \begin{pmatrix} \mathtt{N}^\dagger_a \\ \mathtt{V}^{\dot{a}} \end{pmatrix} . \tag{B.13}$$

Let us take the left-chiral component and the right-chiral componet of (B.11) separately, by application of the chiral projection operators P_L and P_R, to write

$$P_L\psi_{_D} = \begin{pmatrix} \mathtt{V}^\dagger_a \\ 0 \end{pmatrix} , \qquad P_R\psi_{_D} = \begin{pmatrix} 0 \\ \mathtt{N}^{\dot{a}} \end{pmatrix} .$$

From equations (B.11) and (B.13), and by application of the definition (B.8) to the previous ones, it is clear that

$$(P_L\psi_D)^{\mathcal{C}} = P_R\psi_D^{\mathcal{C}} \quad \longleftrightarrow \quad \begin{pmatrix} \mathtt{V}_a^\dagger \\ 0 \end{pmatrix}^{\mathcal{C}} = \begin{pmatrix} 0 \\ \mathtt{V}^{\dot{a}} \end{pmatrix}, \tag{B.14}$$

and

$$(P_R\psi_D)^{\mathcal{C}} = P_L\psi_D^{\mathcal{C}} \quad \longleftrightarrow \quad \begin{pmatrix} 0 \\ \mathtt{N}^{\dot{a}} \end{pmatrix}^{\mathcal{C}} = \begin{pmatrix} \mathtt{N}_a^\dagger \\ 0 \end{pmatrix}, \tag{B.15}$$

so that, under charge conjugation, a left-handed field comes to be right-handed and a vice versa.

At the Weyl field level, we define

$$\left(\mathtt{N}^{\dot{a}}\right)^{\mathcal{C}} = \mathtt{N}_a^\dagger \quad \longleftrightarrow \quad \left(\mathtt{N}_a^\dagger\right)^{\mathcal{C}} = \mathtt{N}^{\dot{a}}. \tag{B.16}$$

The kinetic term for a Dirac field reads

$$\begin{aligned} i\overline{\psi}_D\gamma^\mu\partial_\mu\psi_D &= i\left(\mathtt{N}^{\dagger a} \ \ \mathtt{V}_{\dot{a}}\right)\begin{pmatrix} 0 & \sigma^\mu_{a\dot{c}} \\ \bar{\sigma}^{\mu\dot{a}c} & 0 \end{pmatrix}\partial_\mu\begin{pmatrix} \mathtt{V}_c^\dagger \\ \mathtt{N}^{\dot{c}} \end{pmatrix} \\ &= i\mathtt{N}^{\dagger a}\sigma^\mu_{a\dot{c}}\partial_\mu\mathtt{N}^{\dot{c}} + i\mathtt{V}_{\dot{a}}\bar{\sigma}^{\mu\dot{a}c}\partial_\mu\mathtt{V}_c^\dagger \ . \end{aligned} \tag{B.17}$$

Finally, the mass terms for a Dirac field reads,

$$\begin{aligned} \overline{\psi}_D\psi_D &= \left(\mathtt{N}^{\dagger a} \ \ \mathtt{V}_{\dot{a}}\right)\begin{pmatrix} \mathtt{V}_a^\dagger \\ \mathtt{N}^{\dot{a}} \end{pmatrix} \\ &= \mathtt{N}^{\dagger a}\mathtt{V}_a^\dagger + \mathtt{V}_{\dot{a}}\mathtt{N}^{\dot{a}} \qquad \text{(B.18)} \\ &= \mathtt{V}_{\dot{a}}\mathtt{N}^{\dot{a}} + h.c. \qquad \text{(B.19)} \end{aligned}$$

B.3 Majorana fields

Unlike the Dirac field (B.11) which is compossed by two different Weyl fields, a Majorana field is compossed solely by one, for instance,

$$\psi_M = \begin{pmatrix} \mathtt{N}_a^\dagger \\ \mathtt{N}^{\dot{a}} \end{pmatrix} \ . \tag{B.20}$$

From (B.15), we see that the left chiral componet of the previous is gotten by charge conjugation of its own right component, consequently, a Majorana field is its own antiparticle, hence it fulfills

$$\psi_{M}^{\mathcal{C}} = \psi_{M} \ , \tag{B.21}$$

therefore, it has two degrees of freedom less than a Dirac field.

The Hermitian conjugated of (B.20) is $\psi_{M}^{\dagger} = \begin{pmatrix}\mathtt{N}_{\dot{a}} & \mathtt{N}^{\dagger a}\end{pmatrix}$, thus, by using (B.5) its Dirac conjugated becomes

$$\overline{\psi}_{M} = \begin{pmatrix}\mathtt{N}^{\dagger a} & \mathtt{N}_{\dot{a}}\end{pmatrix} \ . \tag{B.22}$$

The kinetic term for a Majorana field reads

$$\begin{aligned}\frac{i}{2}\overline{\psi}_{M}\gamma^{\mu}\partial_{\mu}\psi_{M} &= \frac{i}{2}\begin{pmatrix}\mathtt{N}^{\dagger a} & \mathtt{N}_{\dot{a}}\end{pmatrix}\begin{pmatrix}0 & \sigma^{\mu}_{a\dot{c}} \\ \bar{\sigma}^{\mu\dot{a}c} & 0\end{pmatrix}\partial_{\mu}\begin{pmatrix}\mathtt{N}^{\dagger}_{c} \\ \mathtt{N}^{\dot{c}}\end{pmatrix} \\ &= \frac{i}{2}\mathtt{N}^{\dagger a}\sigma^{\mu}_{a\dot{c}}\partial_{\mu}\mathtt{N}^{\dot{c}} + \frac{i}{2}\mathtt{N}_{\dot{a}}\bar{\sigma}^{\mu\dot{a}c}\partial_{\mu}\mathtt{N}^{\dagger}_{c} \qquad \text{(B.23)} \\ &= i\mathtt{N}^{\dagger a}\sigma^{\mu}_{a\dot{c}}\partial_{\mu}\mathtt{N}^{\dot{c}} \ , \qquad \text{(B.24)}\end{aligned}$$

where, in (B.23) we have used the last of the Eqs. (B.4) to write

$$\begin{aligned}\mathtt{N}_{\dot{a}}\bar{\sigma}^{\mu\dot{a}c}\partial_{\mu}\mathtt{N}^{\dagger}_{c} &= \mathtt{N}_{\dot{a}}\epsilon^{\dot{a}\dot{c}}\epsilon^{ca}\sigma^{\mu}{}_{a\dot{c}}(\partial_{\mu}\mathtt{N}^{\dagger}_{c}) \\ &= \mathtt{N}^{\dot{c}}\sigma^{\mu}{}_{a\dot{c}}(\partial_{\mu}\mathtt{N}^{\dagger a}) \\ &= -(\partial_{\mu}\mathtt{N}^{\dagger a})\sigma^{\mu}{}_{a\dot{c}}\mathtt{N}^{\dot{c}} \ ,\end{aligned}$$

then, by using

$$\partial_{\mu}\left[\mathtt{N}^{\dagger a}(\sigma^{\mu}{}_{a\dot{c}}\mathtt{N}^{\dot{c}})\right] = \mathtt{N}^{\dagger a}\partial_{\mu}\sigma^{\mu}{}_{a\dot{c}}\mathtt{N}^{\dot{c}} + (\partial_{\mu}\mathtt{N}^{\dagger a})\sigma^{\mu}{}_{a\dot{c}}\mathtt{N}^{\dot{c}} \ ,$$

we arrive to

$$\mathtt{N}_{\dot{a}}\bar{\sigma}^{\mu\dot{a}c}\partial_{\mu}\mathtt{N}^{\dagger}_{c} = \mathtt{N}^{\dagger a}\sigma^{\mu}_{a\dot{c}}\partial_{\mu}\mathtt{N}^{\dot{c}} - \partial_{\mu}\left[\mathtt{N}^{\dagger a}(\sigma^{\mu}{}_{a\dot{c}}\mathtt{N}^{\dot{c}})\right] \ .$$

The last term is a total divergence, which, into the action, vanishes, therefore Eq. (B.23) becomes Eq. (B.24).

Finally, the mass terms for a Majorana field reads,

$$\begin{aligned}
\frac{1}{2}\psi_{M}^{T} C \psi_{M} &= \frac{1}{2}\begin{pmatrix}\mathtt{N}_a^{\dagger} & \mathtt{N}^{\dot{a}}\end{pmatrix}\begin{pmatrix}-\epsilon^{ac} & 0 \\ 0 & -\epsilon_{\dot{a}\dot{c}}\end{pmatrix}\begin{pmatrix}\mathtt{N}_c^{\dagger} \\ \mathtt{N}^{\dot{c}}\end{pmatrix} \\
&= \frac{1}{2}\mathtt{N}^{a\dagger}\mathtt{N}_a^{\dagger} + \frac{1}{2}\mathtt{N}_{\dot{a}}\mathtt{N}^{\dot{a}} \\
&= \frac{1}{2}\mathtt{N}_{\dot{a}}\mathtt{N}^{\dot{a}} + h.c. = \frac{1}{2}\overline{\psi}_{M}\psi_{M} \ .
\end{aligned} \tag{B.25}$$

B.4 A Short Introduction to the Standard Model of Particle Physics (SM)

The Standard Model of Particle Physics (SM) is the most sophisticated and successful description that we currently have to understand the physics of our Universe at the subatomic scale, to this end, the model specializes on the other three interactions we know exist aside from gravity, namely, electromagnetic, nuclear-strong and nuclear-weak interactions.

Since the SM is grounded on the quantum field theory, such interactions, which are also commonly called forces, together with the matter content are described using quantum fields.

The matter fields, are classified due to their interactions into quarks, which interact under all of the three forces, and lepton fields, which do not interact through the strong force. In turn, the lepton fields can be classified into charged-leptons and neutrinos, from which the former only interact by means of both, electromagnetic and weak interactions, whereas the latter do it by weak forces uniquely.

As for the spin, the matter fields are fermionic, i.e., their spin is semi-integer, particularly, both, the quark and lepton fields have spin 1/2. On the other hand, the interaction fields are bosonic, in particular, the three kinds of interactions are mediated by fields of spin 1 (vector fields).

The SM is a local gauge theory, i.e., it is built on the principle of local gauge invariance under symmetry transformations ruled by the group

$$SU(3) \otimes SU(2) \otimes U(1) \ ,$$

	matter (fermions)			H	(Higgs)
quarks	u	c	t	γ	forces (bosons)
	d	s	b	g	
leptons	e	μ	τ	Z	
	ν_e	ν_μ	ν_τ	W	

Table B.1: The fundamental fields of the SM. The three columns in which matter is organized are known as generations.

this is why the interaction fields are also called gauge fields.

The strong interactions are contained in the $SU(3)$ sector. Since there are eight generators for this group, the strong force is mediated by eight vector fields called gluons (g). Similarly, the electromagnetic and nuclear-weak interactions, or electroweak for short, are contained in the $SU(2) \otimes U(1)$ sector. The $SU(2)$ group has three generators, hence there are three vector fields carrying the weak force named W^+, W^- and Z^0. The vector field, which is responsible for the electromagnetic interaction is associated with the only one generator of $U(1)$, it is the photon (γ). The only SM field that remains to mention is the Higgs, which is a zero spin (scalar) field that couples with all other SM fields except the photon and the gluons.

The typical arrangement of the content of the SM is shown in table B.1. As shown there, there are six flavors of quarks, called: up (u), down (d), charm (c), strange (s), top (t) and bottom (b). For each flavor, there are three different colors , namely, red, blue and green. Since each quark has its antiparticle, there are in total, 36 different quarks. As for the electric charge, the quarks u, c and t have $+\frac{2}{3}$ of the elementary charge, whereas the quarks d, s and b have $-\frac{1}{3}$.

The electric charge of the electron (e), the muon (μ) and the tau (τ) equals the elementary charge (-1), on the other hand, the electric charge of the neutrinos equals zero. In the minimal extension of the SM it is assumed that neutrinos and anti-neutrinos are different, therefore, by adding particles and antiparticles, there are a total of 12 different leptons.

The electric charge of the Z, the photon and the gluons is zero. All of them are their own antiparticles, thereby, there are a total of 10 electrically neutral vector bosons. Unlike these, the electric charge of the W^- equals -1, and that of its antiparticle, the W^+ equals $+1$, thus, we have 2 electrically charged vector bosons.

Adding all the mentioned fields plus the Higgs, we obtain the famous 61 particles of the SM, understanding by particles the excitations around the vacuum state of the fields.

As a remark, a concept to take into account is that of generation (see the caption of B.1). The generation is also called family. Notice that one generation is formed by two quarks together with one lepton and its leptonic partner the neutrino.

B.4.1 The $SU(3)$ sector

This is a very short introduction to QCD theory, for deep developments and details see [102, 103, 104].

In its fundamental representation, the elements of the $SU(3)$ group have dimension three. They act on arrays of quarks written as

$$q = \begin{pmatrix} q_r \\ q_b \\ q_g \end{pmatrix}, \qquad \overline{q} = \left(\overline{q}_r \ \overline{q}_b \ \overline{q}_g\right),$$

where q_i is a Dirac spinor for a quark of each color, $i = \{r, b, g\}$ (red, blue or green), for a given flavor. This object rotates under $SU(3)$ in the three-dimensional color space, that is why the theory of strong interactions is commonly called Quantum Chromodynamics (QCD).

By assuming that the mass for the three colors of quarks at a given flavor is the same ($m = m_r = m_b = m_g$), we can write the $SU(3)$ invariant

Lagrangian, for that flavor, as

$$\mathcal{L}_{QCD} = i\bar{q}\gamma^{\mu} D_{\mu} q - m\bar{q}q - \frac{1}{16\pi}\mathbf{G}^{\mu\nu} \cdot \mathbf{G}_{\mu\nu} \ , \tag{B.26}$$

where, D_{μ} is the covariant derivative operator

$$D_{\mu} = \partial_{\mu} + i\mathrm{g}\boldsymbol{\lambda} \cdot \mathbf{G}_{\mu} \ , \tag{B.27}$$

wherein g is the coupling for the color charge, and where

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_8) \, , \tag{B.28}$$

is the array formed from the eight Gell-Mann matrices, which are the generators of $SU(3)$. Also there

$$\mathbf{G}_{\mu} = (G_{\mu 1}, G_{\mu 2}, \ldots, G_{\mu 8}) \, , \tag{B.29}$$

is the array formed from the eight gluon vector potentials, such that, the antisymmetric tensor field that appears in (B.26) is written as

$$\mathbf{G}_{\mu\nu} = \partial_{\mu}\mathbf{G}_{\nu} - \partial_{\nu}\mathbf{G}_{\mu} - 2\mathrm{g}\mathbf{G}_{\mu} \times \mathbf{G}_{\nu} \ , \tag{B.30}$$

where the last term is a shorthand notation for a product under $SU(3)$, like the cross-product, given by

$$[\mathbf{G}_{\mu} \times \mathbf{G}_{\nu}]_k \equiv G_{\mu i} G_{\nu j} f_{ijk} \ ,$$

with f_{ijk}, the structure constants of the $SU(3)$ algebra

$$[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k \ .$$

The gauge fields $G_{\mu i}$, are necessary to keep the invariance of the Lagrangian under the transformation

$$q \longrightarrow Sq \ , \tag{B.31}$$

where

$$S = e^{-i\mathrm{g}\boldsymbol{\lambda}\cdot\boldsymbol{\phi}(x)} \in SU(3), \tag{B.32}$$

with $\boldsymbol{\phi}(x) = (\phi_1(x), \phi_2(x), \ldots, \phi_8(x))$, the array of local gauge parameters.

Given the transformation (B.31), its covariant derivative becomes

$$D_{\mu} q \longrightarrow D_{\mu}(Sq) \ ,$$

which, by using the definition (B.27) yields to

$$D_\mu\,(Sq) = S\,(\partial_\mu q) + (\partial_\mu S)\,q + i\mathrm{g}\boldsymbol{\lambda}\cdot\mathbf{G}_\mu\,(Sq)\ \ . \tag{B.33}$$

From here, we see it is fulfilled that

$$D_\mu\,(Sq) = S\,(D_\mu q)\ \ , \tag{B.34}$$

provided that the term containing the gauge fields transforms as

$$\boldsymbol{\lambda}\cdot\mathbf{G}_\mu = S\,(\boldsymbol{\lambda}\cdot\mathbf{G'}_\mu)\,S^{-1} + \frac{i}{\mathrm{g}}\,(\partial_\mu S)\,S^{-1}\ . \tag{B.35}$$

By substitution of the previous condition into equation (B.33), it can be verified that

$$\begin{aligned} D_\mu\,(Sq) &= S\,[\partial_\mu + i\mathrm{g}\boldsymbol{\lambda}\cdot\mathbf{G'}_\mu]\,q \\ &= S\,(D_\mu q)\ \ . \end{aligned}$$

This means that, for the equation (B.34) to be fulfilled, which guarantees the invariance of the Lagrangian (B.26) under (B.31), it is necessary to involve the fields $G_{\mu i}$ which must transform ruled by the condition (B.35).

The explicit transformation of the fields $G_{\mu i}$ can be known from such a condition by using the infinitesimal transformations

$$S = e^{-i\mathrm{g}\boldsymbol{\lambda}\cdot\boldsymbol{\phi}(x)} \approx 1 - i\mathrm{g}\boldsymbol{\lambda}\cdot\boldsymbol{\phi}(x), \qquad S^{-1} = e^{i\mathrm{g}\boldsymbol{\lambda}\cdot\boldsymbol{\phi}(x)} \approx 1 + i\mathrm{g}\boldsymbol{\lambda}\cdot\boldsymbol{\phi}(x)\ .$$

By substitution of these in equation (B.35) we arrive to

$$\begin{aligned} \boldsymbol{\lambda}\cdot\mathbf{G}_\mu \approx\ & \{1 - i\mathrm{g}\boldsymbol{\lambda}\cdot\boldsymbol{\phi}(x)\}\,(\boldsymbol{\lambda}\cdot\mathbf{G'}_\mu)\,\{1 + i\mathrm{g}\boldsymbol{\lambda}\cdot\boldsymbol{\phi}(x)\} \\ & + \frac{i}{\mathrm{g}}\,\{\partial_\mu(1 - i\mathrm{g}\boldsymbol{\lambda}\cdot\boldsymbol{\phi}(x))\}\,\{1 + i\mathrm{g}\boldsymbol{\lambda}\cdot\boldsymbol{\phi}(x)\}, \\ & \approx \boldsymbol{\lambda}\cdot\mathbf{G'}_\mu + i\mathrm{g}\,[(\boldsymbol{\lambda}\cdot\mathbf{G'}_\mu),(\boldsymbol{\lambda}\cdot\boldsymbol{\phi(x)})] + \boldsymbol{\lambda}\cdot(\partial_\mu\boldsymbol{\phi}(x))\ \ . \end{aligned} \tag{B.36}$$

Next, by invoking the identity

$$(\boldsymbol{\lambda}\cdot\boldsymbol{a})(\boldsymbol{\lambda}\cdot\boldsymbol{b}) = \boldsymbol{a}\cdot\boldsymbol{b} + i\boldsymbol{\lambda}\cdot(\boldsymbol{a}\times\boldsymbol{b})\ , \tag{B.37}$$

we obtain

$$[(\boldsymbol{\lambda}\cdot\mathbf{G'}_\mu),(\boldsymbol{\lambda}\cdot\boldsymbol{\phi(x)})] = -2i\boldsymbol{\lambda}\cdot\boldsymbol{\phi}(x)\times\mathbf{G'}_\mu\ .$$

Finally, by replacing this in (B.36) we arrive at the transformation rule for the $G_{\mu i}$ fields, given by

$$\mathbf{G}_\mu \longrightarrow \mathbf{G'}_\mu + \partial_\mu \boldsymbol{\phi(x)} + 2\mathrm{g}\boldsymbol{\phi(x)} \times \mathbf{G'}_\mu \ . \tag{B.38}$$

From this last, together with (B.30), we arrive to the rule of transformation of the tensor field given by

$$\mathbf{G}_{\mu\nu} \longrightarrow \mathbf{G}'_{\mu\nu} + 2\mathrm{g}\left(\boldsymbol{\phi}(x) \times \mathbf{G}'_{\mu\nu}\right) \ . \tag{B.39}$$

There are not mass terms for the fields $G_{\mu i}$, because the Proca-like terms proportional to $\mathbf{G}^\mu \cdot \mathbf{G}_\mu$ are not invariant under (B.38), on the other hand, the kinetic term that appears at the end of (B.26) is invariant under (B.39).

The complete QCD Lagrangian requires six copies of (B.26), one for each flavor of quark, with their corresponding masses.

B.4.2 The $SU(2) \otimes U(1)$ sector

As said above, the electroweak interactions are contained in this sector. Unlike the QCD sector, in which the symmetry group is of dimension three, here the group has dimension two so that the objects in which the group acts have to be defined.

Let start by defining the SM spinors[1] in terms of the two-component notation we have used along this work, although this notation looks quite different respect to that commonly used, with due care it should not lead to confusion. Then, let us state the following conventions.

The Weyl fields of a generic lepton field are written in lowercase greek letters as

$$\psi = \begin{pmatrix} \chi^\dagger_a \\ \xi^{\dot{a}} \end{pmatrix} \ , \tag{B.40}$$

[1]These are only spinors, the two-dimensional objects in which the group acts will be defined from the components of these.

The Weyl fields of a generic Dirac neutrino[2], are written in typewriter capital letters as

$$\nu_D = \begin{pmatrix} \mathtt{U}^\dagger_a \\ \mathtt{V}^{\dot{a}} \end{pmatrix} \ . \tag{B.41}$$

The Weyl fields of an up-type quark are written in script typeface, as

$$u = \begin{pmatrix} \mathcal{P}^\dagger_a \\ \mathcal{Q}^{\dot{a}} \end{pmatrix} \ , \tag{B.42}$$

The Weyl fields of a down-type quark are written in typewriter lowercase characters, as follows

$$d = \begin{pmatrix} \mathtt{p}^\dagger_a \\ \mathtt{q}^{\dot{a}} \end{pmatrix} \ . \tag{B.43}$$

In all the previous definitions, if necessary, more than one generation can be denoted by a subscript, for instance

$$\nu_{iD} = \begin{pmatrix} \mathtt{U}^\dagger_{ia} \\ \mathtt{V}^{\dot{a}}_i \end{pmatrix}, \qquad d_j = \begin{pmatrix} \mathtt{p}^\dagger_{ja} \\ \mathtt{q}^{\dot{a}}_j \end{pmatrix}, \qquad u_k = \begin{pmatrix} \mathcal{P}^\dagger_{ka} \\ \mathcal{Q}^{\dot{a}}_k \end{pmatrix} \ ,$$

where, $i = \{e, \mu, \tau\}$ stands for the three flavors of leptons, where $j = \{d, s, b\}$ stands for the down, strange and bottom quarks, and where $k = \{u, c, t\}$ stands for the up, charm and top quarks.

Next, by using the components (Weyl fields) of the previous objects, we can define two kinds of objects that group $SU(2) \otimes U(1)$ acts on, which are, the doublets and the singlets, as follows.

By taking the left components of (B.40) and (B.41) it is defined the (left-handed) lepton doublet

$$\ell_a = \begin{pmatrix} \mathtt{U}^\dagger_a \\ \chi^\dagger_a \end{pmatrix} . \tag{B.44}$$

Similarly, by taking the left components of (B.42) and (B.43) it is defined the (left-handed) quark doublet

$$\mathrm{L}_a = \begin{pmatrix} \mathcal{P}^\dagger_a \\ \mathtt{p}^\dagger_a \end{pmatrix} \ . \tag{B.45}$$

[2]The SM only contains left-handed Weyl fields for the neutrinos, but here we consider the minimal extension of the SM, thus, we include the right-handed field to let Dirac mass terms for the neutrinos.

By taking the right components of (B.40), (B.41), (B.42) and (B.43), the (right-handed) lepton and quark singlets are defined respectively as

$$\xi^{\dot{a}}, \qquad \mathtt{V}^{\dot{a}}, \qquad \mathcal{Q}^{\dot{a}}, \qquad \mathtt{q}^{\dot{a}}. \tag{B.46}$$

Additionally, it is necessary to define the Higgs doublet and its $SU(2)$ conjugated, these are

$$\Phi = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}, \qquad \tilde{\Phi} = \begin{pmatrix} h^{0\dagger} \\ -h^- \end{pmatrix} . \tag{B.47}$$

From this separation into doublets and singlets, we can see that the symmetry group for electroweak interactions treats differentially the left-handed and the right-handed fields. Actually, it is only the $SU(2)$ group from which this arises, that is why it is commonly denoted with a subscript $_I$ standing for isospin, whereas the $U(1)$ group carries the subscript $_Y$ standing for hypercharge as follows

$$SU(2)_I \otimes U(1)_Y \ .$$

Let see how it works. The hypercharge is related to both, the third projection of the isospin and the electric charge Q by means of[3]

$$Y = 2(Q - I_3) \ . \tag{B.48}$$

All previously defined objects transform under the two-dimensional representation of the electro-weak group $SU(2)_L \otimes U(1)_Y$, for which, any element S_{EW} can be written as the exponential map of such a representation as

$$S_{EW}(\boldsymbol{\alpha}, \theta) = S_E \otimes S_W \ , \tag{B.49}$$

with

$$S_E = e^{-i\mathrm{a}\mathbf{Y}\theta(x)/2} \quad \text{and} \quad S_W = e^{-i\mathrm{f}\boldsymbol{\sigma}\cdot\boldsymbol{\alpha}(x)/2} \ , \tag{B.50}$$

where $\mathbf{Y}$ is the (unidimensional) hypercharge operator which is the generator of the $U_Y(1)$ group, $\theta(x)$ is a local gauge parameter and a is the coupling for the electric charge. Symilarly, $\boldsymbol{\sigma}/2 = (\sigma_1, \sigma_2, \sigma_3)/2$, with σ_i the Pauli matrices, is the operator of isospin, that generates the $SU(2)_L$ group, the array of local gauge parameters is $\boldsymbol{\alpha}(x) = (\alpha_1(x), \alpha_2(x), \alpha_3(x))$ and f is the coupling for the weak interaction.

[3]Some authors use another convention where the factor of 2 is omitted.

Υ	I	I_3	Q	Y
$\ell_a = \begin{pmatrix} \mathtt{U}_a^\dagger \\ \chi_a^\dagger \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	0 -1	-1
$\mathrm{L}_a = \begin{pmatrix} \mathcal{P}_a^\dagger \\ \mathtt{p}_a^\dagger \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$2/3$ $-1/3$	$1/3$
$\xi^{\dot{a}}$	0	0	-1	-2
$\mathtt{V}^{\dot{a}}$	0	0	0	0
$\mathcal{Q}^{\dot{a}}$	0	0	$2/3$	$4/3$
$\mathtt{q}^{\dot{a}}$	0	0	$-1/3$	$-2/3$
$\Phi = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	1 0	1
$\tilde{\Phi} = \begin{pmatrix} h^{0\dagger} \\ h^- \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	0 -1	-1

Table B.2: Quantum numbers for generic object Υ: isospin (I), its third projection (I_3), electric charge (Q), and hypercharge ($Y = 2(Q - I_3)$).

Let us write, for short, the transformation under S_{EW}, of the generic object Υ (defined in the table B.2) as

$$\Upsilon \xrightarrow{S_{EW}} e^{-i\{I+Y\}}\Upsilon \ . \tag{B.51}$$

The corresponding eigenvalues (I and Y) for the above defined doublets and singlets under the S_{EW} action are showed in the table B.2.

Unlike the QCD sector, it is not possible to write mass terms invariant under S_{EW}, instead, we can write the so called Yukawa terms. It can be verified by using (B.51) together with the values of the table B.2, that the invariant

Yukawa Lagrangian (one generation) we can write is

$$- \mathcal{L}_{yuk} = y^u \mathrm{L}^\dagger_{\dot{a}} \tilde{\varPhi} \mathcal{Q}^{\dot{a}} + y^d \mathrm{L}^\dagger_{\dot{a}} \varPhi \mathsf{q}^{\dot{a}} + y^\psi \ell^\dagger_{\dot{a}} \varPhi \xi^{\dot{a}} + y^\nu \ell^\dagger_{\dot{a}} \tilde{\varPhi} \mathsf{K}^{\dot{a}} + h.c. \; , \tag{B.52}$$

where y^u, y^d, y^ψ and y^ν are Yukawa couplings.

On the other hand, the fermionic kinetic Lagrangian,

$$\begin{aligned} \mathcal{L}_k = & \, i \mathrm{L}^\dagger_{\dot{a}} \bar{\sigma}^{\mu \dot{a} c} D_\mu \mathrm{L}_c + i \ell^\dagger_{\dot{a}} \bar{\sigma}^{\mu \dot{a} c} D_\mu \ell_c \\ & + i \mathcal{Q}^{\dagger a} \sigma^\mu_{a \dot{c}} D_\mu \mathcal{Q}^{\dot{c}} + i \mathsf{q}^{\dagger a} \sigma^\mu_{a \dot{c}} D_\mu \mathsf{q}^{\dot{c}} + i \mathsf{V}^{\dagger a} \sigma^\mu_{a \dot{c}} D_\mu \mathsf{V}^{\dot{c}} + i \xi^{\dagger a} \sigma^\mu_{a \dot{c}} D_\mu \xi^{\dot{c}} \; , \end{aligned} \tag{B.53}$$

is invariant under

$$\mathrm{L}_c \longrightarrow D_\mu \mathrm{L}_c \; , \quad \ell_c \longrightarrow D_\mu \ell_c \; , \quad \mathcal{Q}^{\dot{c}} \longrightarrow D_\mu \mathcal{Q}^{\dot{c}} \; , \text{ etc.,}$$

where D_μ stands for the covariant derivative operator which, for the generic object Υ, is given by

$$D_\mu \Upsilon = \left(\partial_\mu + \frac{i}{2} \mathrm{a} \mathbf{Y} A_\mu + \frac{i}{2} \mathrm{f} \boldsymbol{\sigma} \cdot \mathbf{W}_\mu \right) \Upsilon \; , \tag{B.54}$$

where, as before, there have been introduced four gauge fields in order to keep the invariance, one is the photon field

$$A_\mu \; , \tag{B.55}$$

and the other are the three vector fields of the weak interaction accomodated into the array

$$\mathbf{W}_\mu = (W_{\mu 1}, W_{\mu 2}, W_{\mu 3}) \, . \tag{B.56}$$

With this, we can check that when

$$\Upsilon \longrightarrow S_{EW} \Upsilon \; ,$$

it is fullfilled that

$$D_\mu \left(S_{EW} \Upsilon \right) = S_{EW} \left(D_\mu \Upsilon \right) , \tag{B.57}$$

so, the second term of the Lagrangian (B.53) is invariant under the local gauge. Let us check it. Taking into accoun (B.54), the left-hand side of the previous equation is

$$D_\mu \left(S_{EW} \Upsilon \right) = S_{EW} (\partial_\mu \Upsilon) + (\partial_\mu S_{EW}) \Upsilon + \frac{i}{2} \mathrm{a} \mathbf{Y} A_\mu S_{EW} \Upsilon + \frac{i}{2} \mathrm{f} \boldsymbol{\sigma} \cdot \mathbf{W}_\mu S_{EW} \Upsilon \; . \tag{B.58}$$

From here we can see that the condition (B.57) is accomplished provided that both terms, $\mathbf{Y}A_\mu/2$ and $\boldsymbol{\sigma}\cdot\mathbf{W}_\mu/2$ transform as

$$\frac{1}{2}\mathbf{Y}A_\mu = S_{\scriptscriptstyle E}\left(\frac{\mathbf{Y}}{2}A'_\mu\right)S_{\scriptscriptstyle E}^{-1} + \frac{i}{\mathrm{a}}\left(\partial_\mu S_{\scriptscriptstyle E}\right)S_{\scriptscriptstyle E}^{-1}, \tag{B.59}$$

$$\frac{1}{2}\boldsymbol{\sigma}\cdot\mathbf{W}_\mu = S_{\scriptscriptstyle W}\left(\frac{1}{2}\boldsymbol{\sigma}\cdot\mathbf{W}'_\mu\right)S_{\scriptscriptstyle W}^{-1} + \frac{i}{\mathrm{f}}\left(\partial_\mu S_{\scriptscriptstyle W}\right)S_{\scriptscriptstyle W}^{-1}\ . \tag{B.60}$$

By substituting (B.59) and (B.60) into (B.58) we arrive to

$$\begin{aligned}D_\mu\left(S_{\scriptscriptstyle EW}\Upsilon\right) = S_{\scriptscriptstyle EW}\left(\partial_\mu\Upsilon\right) + \left(\partial_\mu S_{\scriptscriptstyle EW}\right)\Upsilon \\ + i\mathrm{a}S_{\scriptscriptstyle E}\left(\frac{1}{2}\mathbf{Y}A'_\mu\right)S_{\scriptscriptstyle W}\Upsilon - \left(\partial_\mu S_{\scriptscriptstyle E}\right)S_{\scriptscriptstyle W}\Upsilon \\ + i\mathrm{f}S_{\scriptscriptstyle W}\left(\frac{1}{2}\boldsymbol{\sigma}\cdot\mathbf{W}'_\mu\right)S_{\scriptscriptstyle E}\Upsilon - \left(\partial_\mu S_{\scriptscriptstyle W}\right)S_{\scriptscriptstyle E}\Upsilon\ .\end{aligned}$$

By noticing that $S_{\scriptscriptstyle E}$ and $S_{\scriptscriptstyle W}$ commute, the term $\left(\partial_\mu S_{\scriptscriptstyle EW}\right)$ cancels to get

$$D_\mu\left(S_{\scriptscriptstyle EW}\Upsilon\right) = S_{\scriptscriptstyle EW}\left(\partial_\mu\Upsilon\right) + i\mathrm{a}S_{\scriptscriptstyle E}\left(\frac{1}{2}\mathbf{Y}A'_\mu\right)S_{\scriptscriptstyle W}\Upsilon + i\mathrm{f}S_{\scriptscriptstyle W}\left(\frac{1}{2}\boldsymbol{\sigma}\cdot\mathbf{W}'_\mu\right)S_{\scriptscriptstyle E}\Upsilon\ .$$

Next, by inserting $S_{\scriptscriptstyle W}S_{\scriptscriptstyle W}^{-1} = S_{\scriptscriptstyle E}S_{\scriptscriptstyle E}^{-1} = \mathbb{I}$ conveniently before each gauge field, we obtain

$$\begin{aligned}D_\mu\left(S_{\scriptscriptstyle EW}\Upsilon\right) &= S_{\scriptscriptstyle EW}\left(\partial_\mu + \frac{i}{2}\mathrm{a}\mathbf{Y}A'_\mu + \frac{i}{2}\mathrm{f}\boldsymbol{\sigma}\cdot\mathbf{W}'_\mu\right)\Upsilon \\ &= S_{\scriptscriptstyle EW}\left(D_\mu\Upsilon\right)\ ,\end{aligned}$$

so that, equation (B.57) is fullfilled.

It turns out that, because of the eigenvalues shown in table B.2, the covariant derivative operator (B.54) is slightly different for each object, as follows

$$D_\mu \ell_a = \left(\partial_\mu - \frac{i}{2} \mathrm{a} A_\mu + \frac{i}{2} \mathrm{f} \boldsymbol{\sigma} \cdot \mathbf{W}_\mu \right) \ell_a \ , \tag{B.61}$$

$$D_\mu \mathrm{L}_a = \left(\partial_\mu + \frac{i}{6} \mathrm{a} A_\mu + \frac{i}{2} \mathrm{f} \boldsymbol{\sigma} \cdot \mathbf{W}_\mu \right) \mathrm{L}_a \ , \tag{B.62}$$

$$D_\mu \mathcal{Q}^{\dot{a}} = \left(\partial_\mu + \frac{2i}{3} \mathrm{a} A_\mu \right) \mathcal{Q}^{\dot{a}} \ , \tag{B.63}$$

$$D_\mu \mathtt{q}^{\dot{a}} = \left(\partial_\mu - \frac{i}{3} \mathrm{a} A_\mu \right) \mathtt{q}^{\dot{a}} \ , \tag{B.64}$$

$$D_\mu \xi^{\dot{a}} = \left(\partial_\mu - i \mathrm{a} A_\mu \right) \xi^{\dot{a}} \ , \tag{B.65}$$

$$D_\mu \mathtt{V}^{\dot{a}} = \partial_\mu \mathtt{V}^{\dot{a}} \ . \tag{B.66}$$

As before, we have the antisymmetric tensor fields

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - 2\mathrm{f} \left(\mathbf{W}_\mu \times \mathbf{W}_\nu \right) \ , \tag{B.67}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ . \tag{B.68}$$

where the last term in (B.67) is a shorthand notation for the cross-product under $SU(2)$, given by

$$\left[\mathbf{W}_\mu \times \mathbf{W}_\nu \right]_k \equiv W_{\mu i} W_{\nu j} \epsilon_{ijk} \ ,$$

with ϵ_{ijk}, Levi-Civita symbol which corresponds with the structure constants of the $SU(2)$ algebra

$$[\sigma_i, \sigma_j] = 2i \epsilon_{ijk} \sigma_k \ ,$$

on the other hand, the tensor field (B.68) does not carry such a term because $U(1)$ is abelian.

As before, the rule under which the $W_{\mu i}$ fields transform can be obtained by writing the second equation appearing in (B.50) as

$$S_W \approx 1 - i\mathrm{f} \boldsymbol{\sigma} \cdot \boldsymbol{\alpha}(\boldsymbol{x})/2 \ , \qquad S_W^{-1} \approx 1 + i\mathrm{f} \boldsymbol{\sigma} \cdot \boldsymbol{\alpha}(\boldsymbol{x})/2 \ ,$$

and by substitution of these into (B.60) then by using the indentity (B.37), we arrive to

$$\mathbf{W}_\mu = \mathbf{W}'_\mu + \partial_\mu \boldsymbol{\alpha}(x) + \mathrm{f} \left(\boldsymbol{\alpha}(x) \times \mathbf{W}'_\mu \right) \ . \tag{B.69}$$

Similarly, the rule for transformation of A_μ can be obtained by writing the first equation apperaring in (B.50) as

$$S_{_E} \approx 1 - i\mathrm{a}\boldsymbol{Y}\theta(x)/2 \ , \qquad S_{_E}^{-1} \approx 1 + i\mathrm{a}\boldsymbol{Y}\theta(x)/2 \ ,$$

and by substitution of these into (B.59) it is obtained

$$A_\mu = A'_\mu + \partial_\mu \theta(x) \ . \tag{B.70}$$

From (B.69) and (B.70) it can be obtained the transformation rules for the tensor fields (B.67) and (B.68), as

$$\mathbf{W}_{\mu\nu} = \mathbf{W}'_{\mu\nu} + \mathrm{f}\left(\boldsymbol{\alpha}(x) \times \mathbf{W}'_{\mu\nu}\right) \ ,$$

and

$$F_{\mu\nu} = F'_{\mu\nu} \ .$$

Because of these, we must include the kinetic gauge fields terms

$$-\mathcal{L}_{\mathrm{g}} = \frac{1}{16\pi}\mathbf{W}^{\mu\nu} \cdot \mathbf{W}_{\mu\nu} + \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} \ . \tag{B.71}$$

The electroweak sector is completed by including the Higgs Lagrangian

$$\mathcal{L}_{\varPhi} = \left(D^\mu \varPhi\right)^\dagger D_\mu \varPhi - V(\varPhi) \ , \tag{B.72}$$

where the covariant derivative is given by

$$D_\mu \varPhi = \left(\partial_\mu + \frac{i}{2}\mathrm{a}A_\mu + \frac{i}{2}\mathrm{f}\boldsymbol{\sigma} \cdot \mathbf{W}_\mu\right)\varPhi \ ,$$

and the potential is

$$V(\varPhi) = \mu^2 \varPhi^\dagger \varPhi + \lambda\left(\varPhi^\dagger \varPhi\right)^2 \ .$$

In summary, the electroweak Lagrangian of the SM is given by

$$\mathcal{L}_{EW} = \mathcal{L}_{\text{yuk}} + \mathcal{L}_k + \mathcal{L}_g + \mathcal{L}_{\varPhi} \ ,$$

wherein the terms of the RHS are given respectively by (B.52), (B.53), (B.71) and (B.72).

B.5 Electroweak phase transition (Yukawa sector - one generation)

During the electro-weak phase transition, the Higgs field acquires a vacuum-expectation-value (v.e.v) different from zero, for instance, if

$$\Phi = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} h_1 + ih_2 \\ h_3 + ih_4 \end{pmatrix} \ ,$$

then,

$$\langle h_1 \rangle = \langle h_2 \rangle = \langle h_4 \rangle = 0, \qquad \langle h_3 \rangle = \langle H \rangle \geq 0 \ ,$$

where H is the physical Higgs. Thus, the v.e.v can be used to parametrize the Higgs doublet, in a convenient (unitary) gauge, such that

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \langle H \rangle + H \end{pmatrix}, \qquad \tilde{\Phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \langle H \rangle + H \\ 0 \end{pmatrix} \ . \tag{B.73}$$

By substituting these, together with (B.44) and (B.45), into the Yukawa Lagrangian (B.52) it is obtained

$$\begin{aligned} -\mathcal{L}_{yuk} = &\frac{y^u}{\sqrt{2}} \left(\mathcal{P}_{\dot{a}} \ \mathtt{p}_{\dot{a}} \right) \begin{pmatrix} \langle H \rangle + H \\ 0 \end{pmatrix} \mathcal{Q}^{\dot{a}} + \frac{y^d}{\sqrt{2}} \left(\mathcal{P}_{\dot{a}} \ \mathtt{p}_{\dot{a}} \right) \begin{pmatrix} 0 \\ \langle H \rangle + H \end{pmatrix} \mathtt{q}^{\dot{a}} \\ &+ \frac{y^\psi}{\sqrt{2}} \left(\mathtt{U}_{\dot{a}} \ \chi_{\dot{a}} \right) \begin{pmatrix} 0 \\ \langle H \rangle + H \end{pmatrix} \xi^{\dot{a}} + \frac{y^\nu}{\sqrt{2}} \left(\mathtt{U}_{\dot{a}} \ \chi_{\dot{a}} \right) \begin{pmatrix} \langle H \rangle + H \\ 0 \end{pmatrix} \mathtt{K}^{\dot{a}} + h.c. \end{aligned}$$

Therefore, the Higgs doublets, as given in (B.73), break the $SU(2)_L \times U(1)_Y$ symmetry of the Yukawa Lagrangian (B.52), yielding to

$$\begin{aligned} -\mathcal{L}_{yuk} = &\frac{y^u}{\sqrt{2}} (\langle H \rangle + H) \mathcal{P}_{\dot{a}} \mathcal{Q}^{\dot{a}} + \frac{y^d}{\sqrt{2}} (\langle H \rangle + H) \mathtt{p}_{\dot{a}} \mathtt{q}^{\dot{a}} \\ &+ \frac{y^\psi}{\sqrt{2}} (\langle H \rangle + H) \chi_{\dot{a}} \xi^{\dot{a}} + \frac{y^\nu}{\sqrt{2}} (\langle H \rangle + H) \mathtt{U}_{\dot{a}} \mathtt{K}^{\dot{a}} + h.c. \end{aligned}$$

which is the same as

$$-\mathcal{L}_{yuk} = \left[1 + \frac{H}{\langle H \rangle} \right] \left(m^u_D \mathcal{P}_{\dot{a}} \mathcal{Q}^{\dot{a}} + m^d_D \mathtt{p}_{\dot{a}} \mathtt{q}^{\dot{a}} + m^\psi_D \chi_{\dot{a}} \xi^{\dot{a}} + m^\nu_D \mathtt{U}_{\dot{a}} \mathtt{K}^{\dot{a}} \right) + h.c. \tag{B.74}$$

with the quark masses given by

$$m_D^u = \frac{y^u \langle H \rangle}{\sqrt{2}}, \qquad m_D^d = \frac{y^d \langle H \rangle}{\sqrt{2}} \ . \tag{B.75}$$

and similarly, the lepton masses given by

$$m_D^\psi = \frac{y^\psi \langle H \rangle}{\sqrt{2}}, \qquad m_D^\nu = \frac{y^\nu \langle H \rangle}{\sqrt{2}} \ . \tag{B.76}$$

In conclusion, the Higgs mechanism generates Dirac mass terms for the quarks, the lepton and the neutrino, as well as interaction terms between them and the physical Higgs field.

The Higgs mechanism also generates masses for the vector fields $W_{\mu i}$, although here, we will not go further in that sector. Details can be found in the references [102, 103, 104].

B.6 The type I See Saw Mechanism, simple example

In this section we illustrate the type I see-saw mechanism for an one-generation system, i.e, there are only, one left-handed neutrino Weyl field

$$\mathtt{X}_a^\dagger \ ,$$

and one right-handed neutrino Weyl field

$$\mathtt{K}^{\dot{a}} \ .$$

Both of them can be switched into the opposite chirality by charge conjugation accordingly to the rule (B.16).

$$\left(\mathtt{X}_a^\dagger\right)^{\mathcal{C}} = \mathtt{X}^{\dot{a}}, \qquad \text{and} \qquad \left(\mathtt{K}^{\dot{a}}\right)^{\mathcal{C}} = \mathtt{K}_a^\dagger \ .$$

With both of the previous fields, the following neutrinos are composed:
one Majorana neutrino (sterile)

$$\nu = \begin{pmatrix} \mathtt{K}_a^\dagger \\ \mathtt{K}^{\dot{a}} \end{pmatrix} \ , \tag{B.77}$$

one Majorana neutrino (active)

$$\psi_{M} = \begin{pmatrix} \mathtt{X}^{\dagger}_{a} \\ \mathtt{X}^{\dot{a}} \end{pmatrix} \ , \tag{B.78}$$

and one Dirac neutrino

$$\nu_{D} = \begin{pmatrix} \mathtt{X}^{\dagger}_{a} \\ \mathtt{K}^{\dot{a}} \end{pmatrix} \ . \tag{B.79}$$

The Majorana mass term for the active neutrino is,

$$-\mathcal{L}^{M}_{m_L} = \frac{1}{2} m_{L} \mathtt{X}_{\dot{a}} \mathtt{X}^{\dot{a}} + h.c. \tag{B.80}$$

The Majorana mass term for the sterile neutrino is,

$$-\mathcal{L}^{M}_{m_R} = \frac{1}{2} m_{R} \mathtt{K}_{\dot{a}} \mathtt{K}^{\dot{a}} + h.c. \tag{B.81}$$

The Dirac mass term for the Dirac neutrino is,

$$-\mathcal{L}^{D}_{m_L} = m^{\nu}_{D} \mathtt{X}_{\dot{a}} \mathtt{K}^{\dot{a}} + h.c. \tag{B.82}$$

In general, there exists the mixing into the Dirac-Majorana neutrino mass term

$$-\mathcal{L}^{D+M} = \frac{1}{2} m_{L} \mathtt{X}_{\dot{a}} \mathtt{X}^{\dot{a}} + \frac{1}{2} m_{R} \mathtt{K}_{\dot{a}} \mathtt{K}^{\dot{a}} + m^{\nu}_{D} \mathtt{X}_{\dot{a}} \mathtt{K}^{\dot{a}} + h.c. \tag{B.83}$$

Definition of the column matrix of right handed fields:

$$\mathbb{N}^{\dot{a}} = \begin{pmatrix} \mathtt{X}^{\dot{a}} \\ \mathtt{K}^{\dot{a}} \end{pmatrix} \ , \tag{B.84}$$

definition of the matrix:

$$\mathrm{A} = \begin{pmatrix} m_{L} & m^{\nu}_{D} \\ m^{\nu}_{D} & m_{R} \end{pmatrix} \ , \tag{B.85}$$

then, in therms of these, the Lagrangian (B.83) is written as

$$-\mathcal{L}^{D+M} = -\frac{1}{2} \mathbb{N}^{\dot{a}T} \epsilon_{\dot{a}\dot{c}} \mathrm{A} \mathbb{N}^{\dot{c}} + h.c. \tag{B.86}$$

The minus sign at the righ-hand side is neccesary in order to recover the equation (B.83) correctly, remember that the contraction of the symbol $\epsilon_{\dot{a}\dot{c}}$ with its first indice involves a minus sign $(-)$, also remember that $\mathtt{K}_{\dot{c}}\mathtt{X}^{\dot{c}} = -\mathtt{X}^{\dot{c}}\mathtt{K}_{\dot{c}}$

and finally remember that another extra minus sign comes from the change ${}^{\dot{c}}{}_{\dot{c}} \longrightarrow {}_{\dot{c}}{}^{\dot{c}}$.

Due to the mixing, the above Majorana and Dirac neutrinos (B.77)-(B.79) are not the observed ones, in order to get them it is neccesary to diagonalize the Lagrangian (B.86). The diagonalization is carried out throught the diagonalization matrix,

$$\mathrm{D} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \ , \tag{B.87}$$

wherein

$$\cos\theta = \frac{2m_{_D}^{\nu}}{\sqrt{2h(h+\Delta)}} \ , \qquad \sin\theta = \frac{2m_{_D}^{\nu}}{\sqrt{2h(h-\Delta)}} \ , \tag{B.88}$$

with

$$\Delta = m_{_L} - m_{_R}, \qquad h^2 = \Delta^2 + 4(m_{_D}^{\nu})^2 \ . \tag{B.89}$$

Whit this, it is obtained the diagonal matrix M'

$$\mathrm{M}' = \mathrm{D}\mathrm{A}\mathrm{D}^{T} \ , \tag{B.90}$$

which becomes,

$$\mathrm{M}' = \begin{pmatrix} m_1' & 0 \\ 0 & m_2' \end{pmatrix} \ , \tag{B.91}$$

with the eigenvalues given by

$$m_1' = (m_{_L} + m_{_R} - h)/2 \ , \tag{B.92}$$
$$m_2' = (m_{_L} + m_{_R} + h)/2 \ . \tag{B.93}$$

By diagonalization, the Lagrangian (B.86) becomes,

$$-\mathcal{L}^{D+M} = -\frac{1}{2}\mathcal{n}'^{\dot{a}T}\epsilon_{\dot{a}\dot{c}}\mathrm{M}'\mathcal{n}'^{\dot{c}} + h.c. \tag{B.94}$$

where

$$\mathcal{n}'^{\dot{a}} = \mathrm{D}\mathbb{N}^{\dot{a}} = \begin{pmatrix} \mathtt{N}_1'^{\dot{a}} \\ \mathtt{N}_2'^{\dot{a}} \end{pmatrix} \ . \tag{B.95}$$

In general, the Majorana mass term (B.80) could exist, but in the context of the minimally extended SM it is not included because it should come from a Yukawa term like (see equation (B.52))

$$\ell_a^{\dagger}\tilde{\Phi}\mathtt{X}^{\dot{a}} \ \longrightarrow \sim \mathtt{X}_{\dot{a}}\mathtt{X}^{\dot{a}} \ ,$$

which is forbiden by the SM symmetry, because it requieres that the $\mathtt{X}^{\dot{a}}$ field has $I_3 = 0$, which is in contradiction with the value showed in the table B.2, therefore we choose

$$m_L = 0 \ . \tag{B.96}$$

On the other side, the Majorana mass term (B.81) does is allowed because it is compossed by fiels transforming as singlets under SM symmetry, so they can be included in the minimally extension of the SM as done previously. Furthermore, it is assumed that

$$m_R \gg m_D^\nu \ , \tag{B.97}$$

With the previous considerations, the equations (B.88) can be approximated as

$$\cos\theta = \frac{m_R}{\sqrt{(m_D^\nu)^2 + m_R^2}} \ , \qquad \sin\theta = \frac{m_D^\nu}{\sqrt{(m_D^\nu)^2 + m_R^2}} \ , \tag{B.98}$$

and the eigenvalues (B.92) become

$$m_1' \approx -\frac{(m_D^\nu)^2}{m_R}, \tag{B.99}$$

$$m_2' \approx m_R + \frac{(m_D^\nu)^2}{m_R} \approx m_R \ . \tag{B.100}$$

Notice that m_1' is negative. This can be corrected by redefinition of the Weyl field $\mathtt{N}_1'^{\dot{a}}$ throught the operator

$$\mathcal{P} = \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mathcal{P}^\dagger\mathcal{P} = \mathcal{P}\mathcal{P}^\dagger = \mathbb{I} \ .$$

By inserting this into (B.94) we get

$$-\mathcal{L}^{D+M} = -\frac{1}{2}\mathcal{N}'^{\dot{a}T}\epsilon_{\dot{a}\dot{c}}\mathcal{P}^\dagger\mathcal{P}\mathrm{M}'\mathcal{P}\mathcal{P}^\dagger\mathcal{N}'^{\dot{c}} + h.c.,$$

from which, it is obtained

$$-\mathcal{L}^{D+M} = -\frac{1}{2}\mathcal{N}^{\dot{a}T}\epsilon_{\dot{a}\dot{c}}\mathrm{M}\mathcal{N}^{\dot{c}} + h.c. \tag{B.101}$$

wherein

$$\mathrm{M} = \mathcal{P}\mathrm{M}'\mathcal{P} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \qquad m_1 = -m_1', \quad m_2 = m_2' \ , \tag{B.102}$$

and where

$$\mathcal{n}^{\dot{a}} = \mathcal{P}^{\dagger}\mathcal{n}'^{\dot{a}} = \begin{pmatrix} \mathtt{N}_1^{\dot{a}} \\ \mathtt{N}_2^{\dot{a}} \end{pmatrix}, \qquad \mathtt{N}_1^{\dot{a}} = -i\mathtt{N}_1'^{\dot{a}}, \quad \mathtt{N}_2^{\dot{a}} = \mathtt{N}_2'^{\dot{a}} \;, \tag{B.103}$$

which, in virtue of the equation (B.95) together with (B.84), can be written explicitly as

$$\mathtt{N}_1^{\dot{a}} = -i\mathtt{X}^{\dot{a}} + i\frac{m_D^{\nu}}{m_R}\mathtt{K}^{\dot{a}}, \qquad \mathtt{N}_2^{\dot{a}} = \frac{m_D^{\nu}}{m_R}\mathtt{X}^{\dot{a}} + \mathtt{K}^{\dot{a}} \;. \tag{B.104}$$

By substituting (B.99) into (B.102) together with (B.103), the Lagrangian (B.101) becomes

$$-\mathcal{L}^{D+M} = \frac{1}{2}\frac{(m_D^{\nu})^2}{m_R}\mathtt{N}_{1\dot{c}}\mathtt{N}_1^{\dot{c}} + \frac{1}{2}m_R\mathtt{N}_{2\dot{c}}\mathtt{N}_2^{\dot{c}} + h.c. \tag{B.105}$$

which is a result that provides an explanation for the smallness of the mass of the observed neutrino $\mathtt{N}_1^{\dot{a}}$, whose mass is the quotient of the mass generated by the Higgs mechanisms (m_D^{ν}) and m_R, which could come from a beyond-SM sector. Thus, in the seesaw mechanism it can be assumed an initial value for m_D^{ν} in Eq. (B.76), on the same order of magnitude that of its lepton partner,

$$m_D^{\nu} \sim m_D^{\psi} \;. \tag{B.106}$$

Finally, the Higgs interaction term that appears in (B.74) must be rewritten in terms of $\mathtt{N}_{i=1,2}^{\dot{a}}$. It is achieved by inversion of the equations (B.104), yielding to

$$\begin{aligned} -\mathcal{L}_{yuk}^{I} &= \frac{m_D^{\nu}}{\langle H\rangle}H\mathtt{X}_{\dot{a}}\mathtt{K}^{\dot{a}} + h.c., & \text{(B.107)} \\ &= \frac{m_D^{\nu}}{\langle H\rangle}H\left[\frac{m_D^{\nu}}{m_R}\left(\mathtt{N}_{1\dot{a}}\mathtt{N}_1^{\dot{a}} + \mathtt{N}_{2\dot{a}}\mathtt{N}_2^{\dot{a}}\right) + i\mathtt{N}_{1\dot{a}}\mathtt{N}_2^{\dot{a}}\right] + h.c. & \text{(B.108)} \end{aligned}$$

which, in virtue of equation (B.76), is the same as

$$-\mathcal{L}_{yuk}^{I} = \frac{1}{2}(y^{\nu})^2\frac{\langle H\rangle}{m_R}H\left(\mathtt{N}_{1\dot{a}}\mathtt{N}_1^{\dot{a}} + \mathtt{N}_{2\dot{a}}\mathtt{N}_2^{\dot{a}}\right) + \frac{i}{\sqrt{2}}y^{\nu}H\mathtt{N}_{1\dot{a}}\mathtt{N}_2^{\dot{a}} + h.c. \tag{B.109}$$

B.7 Mandelstam Variables

This section is based on Ref. [105] [like there, we use the signature $(-+++)$].

B.7.1 Notation

The four-momentum and its squared are denoted as

$$k_i = k_i^\mu \ , \qquad k_i^2 = (k_i)^\mu (k_i)_\mu \ , \tag{B.110}$$

wherein

$$k_i^\mu = (E_i, \mathbf{k}_i) \ ,$$

with particles on shell

$$k_i^2 = -m_i^2 \ , \tag{B.111}$$

the dispersion relation is obtained,

$$E_i^2 = |\mathbf{k}_i|^2 + m_i^2 \ , \tag{B.112}$$

where $\mathbf{k}_i$ is the three-momentum.

If there are two particles incoming and scattering, the notation is:

$$\begin{aligned} k_1,\ k_2 &: \text{ four-momentum of incoming particles,} \\ k_1',\ k_2' &: \text{ four-momentum of outgoing particles.} \end{aligned}$$

Four-momentum conservation:

$$k_1 + k_2 = k_1' + k_2' \ .$$

At the center of mass (CM):

$$\mathbf{k}_1 + \mathbf{k}_2 = 0 \ , \tag{B.113}$$

and because of momentum conservation, it is also hold

$$\mathbf{k}_1' + \mathbf{k}_2' = 0 \ . \tag{B.114}$$

B.7.2 The variables s, t and u

The three Mandelstam variables, which are Lorentz scalars, are

$$s = -(k_1 + k_2)^2 = -(k_1' + k_2')^2 \ , \tag{B.115}$$
$$t = -(k_1 - k_1')^2 = -(k_2 - k_2')^2 \ , \tag{B.116}$$
$$u = -(k_1 - k_2')^2 = -(k_2 - k_1')^2 \ . \tag{B.117}$$

From (B.115) and (B.111) the Mandelstam variable s is

$$\begin{aligned} s &= -(k_1+k_2)^2 \ , \\ &= -\big(k_1^2+k_2^2+2k_1k_2\big) \ , \\ &= +\big(m_1^2+m_2^2+2E_1E_2-2\mathbf{k}_1\cdot\mathbf{k}_2\big) \ . \end{aligned} \tag{B.118}$$

Similarly, the Mandelstam variable t is

$$\begin{aligned} t &= -(k_1-k_1')^2 \ , \\ &= -\big(k_1^2+k_1'^2-2k_1k_1'\big) \ , \\ &= +\big(m_1^2+m_1'^2-2E_1E_1'+2\mathbf{k}_1\cdot\mathbf{k}_1'\big) \ . \end{aligned} \tag{B.119}$$

The Mandelstam variables fulfill the relations

$$k_1k_1' = \frac{1}{2}(t-m_1^2-m_1'^2) = k_2k_2' \ , \tag{B.120a}$$

$$k_1k_2' = \frac{1}{2}(u-m_1^2-m_2'^2) = k_2k_1' \ , \tag{B.120b}$$

$$k_1k_2 = \frac{1}{2}(m_1^2+m_2^2-s) \ , \tag{B.120c}$$

$$k_1'k_2' = \frac{1}{2}(m_1'^2+m_2'^2-s) \ . \tag{B.120d}$$

Also, the three variables are related to each other through

$$t+u+s = (m_1^2+m_2^2+m_1'^2+m_2'^2) \ . \tag{B.121}$$

B.7.3 The Mandelstam variable s in the center of mass frame (CM).

Because of equation (B.113), the scalar product in (B.118) becomes

$$\mathbf{k}_1\cdot\mathbf{k}_2 = -|\mathbf{k}_1|^2_{CM} \ ,$$

with this into (B.118) it is obtained

$$s = \big(m_1^2+m_2^2+2E_1E_2+2|\mathbf{k}_1|^2_{CM}\big) \ . \tag{B.122}$$

By using again the Eq. (B.113) into (B.112) it is possible to write

$$E_1^2 = |\mathbf{k}_1|^2_{CM}+m_1^2, \quad \text{and} \quad E_2^2 = |\mathbf{k}_1|^2_{CM}+m_2^2 \ , \tag{B.123}$$

therefore

$$m_1^2 + m_2^2 = E_1^2 + E_2^2 - 2|\mathbf{k}_1|^2_{CM} \ ,$$

consequently, the equation (B.122) becomes

$$s = (E_1 + E_2)^2 \ , \tag{B.124}$$

which, provided both, equation (B.114) and (B.115) (or from energy conservation), is also true for the outgoing particles,

$$s = (E_1' + E_2')^2 \ . \tag{B.125}$$

The previous result means that in the CM frame the Mandelstam variable s becomes the total square energy of the pair of particles colliding and scattering. By using (B.123) together with (B.124) it is possible to solve for $|\mathbf{k}_1|^2_{CM}$ in terms of s and the masses m_1 and m_2, to obtain

$$|\mathbf{k}_1|^2_{CM} = \frac{1}{4s}\Big[s^2 + (m_1^2 - m_2^2)^2 - 2s(m_1^2 + m_2^2)\Big] \ , \tag{B.126}$$

which also holds for the primed version.

B.7.4 Mandelstam Triangular Function (MTF)

The Mandelstam triangular function is defined as

$$\begin{aligned} \lambda(s, m_1^2, m_2^2) &= [s - (m_1 + m_2)^2][s - (m_1 - m_2)^2] \\ &= s^2 + (m_1^2 - m_2^2)^2 - 2s(m_1^2 + m_2^2) \ . \end{aligned} \tag{B.127}$$

B.7.5 Relative Velocity

The relativistic relative velocity between particles 1 and 2 in one dimension is ($c = 1$):

$$v_{rel} = \frac{v_1 - v_2}{1 - v_1 v_2} \ ,$$

clearly it is not the same way as Galilean velocities add. In three dimensions it generalizes to

$$v_{rel} = \frac{\sqrt{(\mathbf{v}_1 - \mathbf{v}_2)^2 - (\mathbf{v}_1 \times \mathbf{v}_2)^2}}{1 - \mathbf{v}_1 \cdot \mathbf{v}_2} \ .$$

With the γ factor ($c = 1$)

$$\gamma = \frac{1}{\sqrt{1 - v_{rel}^2}}$$

and with the 4−velocity $\eta^\mu = \gamma(1, \mathbf{v})$ together with the definition of the 4−momentum $k^\mu = m\eta^\mu$, it is possible to write the relative velocity as

$$v_{rel} = -\frac{\sqrt{(k_1 k_2)^2 - m_1^2 m_2^2}}{k_1 k_2} \ .$$

It turns out that, with the MTF (B.127) and the definition of the Mandelstam variable s (B.115), the previous result can be put as

$$v_{rel} = \frac{\sqrt{\lambda(s, m_1^2, m_2^2)}}{s - (m_1^2 + m_2^2)} \ . \tag{B.128}$$

Clearly, the relative velocity depends only on the Mandelstam variable s and the masses of the colliding particles.

B.8 Cross Section Theory

This section is based mainly on Ref. [105], see also Ref. [106] and the appendices of Ref. [107].

The Lorentz invariant normalization of plane waves is given by

$$\langle k|k'\rangle = 2k^0(2\pi)^3\delta^3(\mathbf{k} - \mathbf{k}') \ . \tag{B.129}$$

The scattering matrix element, which conects the initial (i_{in}) and the final (f_{out}) state is

$$\langle f_{out}|i_{in}\rangle = (2\pi)^4\delta^4(k_{in} - k_{out})i\mathcal{M} \ , \tag{B.130}$$

where $\mathcal{M}$ is the transition amplitud. In order to get the probability of transition between such a states, the matrix element has to be squared and normalized by dividing by the norms of the initial and final states, then

$$P = \frac{|\langle f_{out}|i_{in}\rangle|^2}{\langle f_{out}|f_{out}\rangle \langle i_{in}|i_{in}\rangle} \ , \tag{B.131}$$

where according to (B.130)

$$|\langle f_{out}|i_{in}\rangle|^2 = [(2\pi)^4\delta^4(k_{in} - k_{out})]^2|\widetilde{\mathcal{M}}|^2 \ . \tag{B.132}$$

To face the problem of squaring the δ function it is usual to write it in the following way

$$[(2\pi)^4\delta^4(k_{in}-k_{out})]^2 = (2\pi)^4\delta^4(k_{in}-k_{out}) \times (2\pi)^4\delta^4(0) \ . \tag{B.133}$$

By definition

$$(2\pi)^4\delta^4(p) = \int d^4xe^{-ipx} \ ,$$

then, by assuming that the whole experiment is taking place in a big space of volume V and lasting for a large amount of time T, and by using the definition of the δ function it is gotten

$$(2\pi)^4\delta^4(0) = \int d^4x = VT \ .$$

With the last equation together with (B.133) into (B.132), it becomes

$$|\langle f_{out}|i_{in}\rangle|^2 = (2\pi)^4\delta^4(k_{in}-k_{out})|\widetilde{\mathcal{M}}|^2VT \ . \tag{B.134}$$

Also, under the same assumptions in the three dimensional case and by the definition, it is fulfilled

$$(2\pi)^3\delta^3(0) = \int d^3x = V \ .$$

Therefore, from (B.129) the norm of a single particle state (with $k^0 = E$) is

$$\langle k|k\rangle = 2E(2\pi)^3\delta^3(0) = 2EV \ ,$$

and since the initial state is formed from two particles incoming with energy E_1 and E_2, its normalization takes the form

$$\langle i_{in}|i_{in}\rangle = 4E_1E_2V^2 \ . \tag{B.135}$$

In the case of n' outgoing particles the normalization becomes

$$\langle f_{out}|f_{out}\rangle = \prod_{j=1}^{n'}\{2E_j'V\} \ . \tag{B.136}$$

With (B.134), (B.135) and (B.136), the equation (B.131) becomes

$$\frac{P}{T} = \frac{(2\pi)^4\delta^4(k_{in}-k_{out})|\widetilde{\mathcal{M}}|^2V}{(4E_1E_2V^2)\prod_{j=1}^{n'}\{2E_j'V\}} = \dot{P} \ . \tag{B.137}$$

In the last equallity, the partition of the time T into infinitesimal intervales has been considered , this in order to get the probability per unit of time of two incomming particles of momenta k_1 and k_2 to scatter into many outgoing ones of momenta k'_j. To get the differential cross section $d\sigma$ from $\dot{P}$, it is neccesary to divide it by the incident flux (F_{in}), and multiply it by the differential factor $d^3e'_j$ for each outgoing particle, so that

$$d\sigma = \frac{\dot{P}}{F_{in}} \times \prod_{j=1}^{n'} d^3 e'_j \ . \tag{B.138}$$

The factor $d^3e'_j$ apperaring in this equation accounts for the contributions of all the possible values wich an individual vector $\mathbf{k}'_j$ can take. Remember that the experiment is considered to occur into a big volume, say of size L such that $V = L^3$, therefore it must be taking into account the quantized values of $\mathbf{k}'_j$, labeled by the integer numbers e'_j into such a volume, (for example, in the cartesian case):

$$(k_j)_\alpha = \frac{(2\pi)}{L}(e'_j)_\alpha, \quad \alpha = x, y, z, \quad (e_j)_\alpha \in \mathbb{Z} \ .$$

So that

$$\begin{aligned} d^3e'_j &= d(e'_j)_x d(e'_j)_y d(e'_j)_z \\ &= \left(\frac{L}{2\pi}\right)^3 d(k'_j)_x d(k'_j)_y d(k'_j)_z \\ &= \frac{V}{(2\pi)^3} d^3k'_j \ , \end{aligned}$$

With the previous result into (B.138) it is obtained

$$d\sigma = \frac{\dot{P}}{F_{in}} \times \prod_{j=1}^{n'} \frac{V}{(2\pi)^3} d^3k'_j \ . \tag{B.139}$$

Thereby, the differential cross section is defined as the probability of an incoming state of two particles, scatter into an state of many outgoing ones considering all possible values of the quantized momenta, per unit of time and respect to the incident flux.

By substitution of equation (B.137) into equation (B.139) it becomes

$$d\sigma = \frac{1}{F_{in}} \frac{(2\pi)^4 \delta^4(k_{in} - k_{out}) |\widetilde{\mathcal{M}}|^2}{(4E_1 E_2 V)} \prod_{j=1}^{n'} \widetilde{d^3 k'_j} \ , \tag{B.140}$$

where the argument of the productory is the Lorentz-invariant differential phase-space, given by

$$\widetilde{d^3 k'_j} \equiv \frac{d^3 k'_j}{(2\pi)^3 2E'_j} \ . \tag{B.141}$$

The incident flux F_{in}, is defined as the number density of particles approaching to the target, times the relative speed.

$$F_{in} = \frac{N_{part}}{V} \times v_{rel} \ .$$

Since there are only two particles colliding, the target is one of them and the insident flux is simply

$$F_{in} = \frac{v_{rel}}{V} \ .$$

Substituting the last result into equation (B.140), the differential cross section becomes

$$d\sigma = \frac{(2\pi)^4 \delta^4(k_{in} - k_{out}) |\widetilde{\mathcal{M}}|^2}{4E_1 E_2 v_{rel}} \prod_{j=1}^{n'} \widetilde{d^3 k'_j} \ . \tag{B.142}$$

It is customary to define the n'-body Lorentz-invariant phase-space differential as

$$\mathrm{dLIPS}_{n'}(k_{in}) = (2\pi)^4 \delta^4(k_{in} - k_{out}) \prod_{j=1}^{n'} \widetilde{d^3 k'_j} \ . \tag{B.143}$$

Finally, by using the previous definition into (B.142) the differential cross section becomes

$$d\sigma = \frac{|\widetilde{\mathcal{M}}|^2}{4E_1 E_2 v_{rel}} \mathrm{dLIPS}_{n'}(k_{in}) \ . \tag{B.144}$$

This defines the differential cross section for the scattering of two incoming particles into a set of n' outgoing ones.

B.8.1 Two outgoing particles case

If only two particles are scatered, the equation (B.143) becomes

$$\mathrm{dLIPS}_2(k_{in}) = (2\pi)^4\delta^4(k_{in} - (k_1' + k_2'))\,\widetilde{d^3k_1'}\,\widetilde{d^3k_2'}\ , \tag{B.145}$$

where

$$k_{in} = k_1 + k_2\ .$$

The δ function can be writen in the CM frame by using its symmetry properties and both equations (B.113) and (B.124) to get

$$\begin{aligned}\delta^4(k_1 + k_2 - (k_1' + k_2')) &= \delta(E_1 + E_2 - (E_1' + E_2'))\delta^3(\mathbf{k}_1 + \mathbf{k}_2 - (\mathbf{k}_1' + \mathbf{k}_2')) \\ &= \delta((E_1' + E_2') - \sqrt{s})\delta^3(\mathbf{k}_1' + \mathbf{k}_2')\ ,\end{aligned}$$

with both, the previous equation and (B.141), the equation (B.145) becomes

$$\mathrm{dLIPS}_2(k_{in}) = \delta((E_1' + E_2') - \sqrt{s})\delta^3(\mathbf{k}_1' + \mathbf{k}_2')\frac{1}{4(2\pi)^2}\frac{d^3k_1'}{E_1'}\frac{d^3k_2'}{E_2'}\ ,$$

where the dependece of E_i' in $\mathbf{k}_i'$ is given by (B.112). This expresion can be simplified by performing the integration over k_2', using the properties of the δ function. (Note that $d^3k_i' \equiv d\mathbf{k}_i'$). This leads to

$$\mathrm{dLIPS}_2(k_{in}) = \frac{\delta\Big(E_1'(|\mathbf{k}_1'|) + E_2'(|\mathbf{k}_1'|) - \sqrt{s}\Big)}{E_1'(|\mathbf{k}_1'|)\,E_2'(|\mathbf{k}_1'|)}\,\frac{d^3k_1'}{4(2\pi)^2}\ . \tag{B.146}$$

Let it define

$$f(|\mathbf{k}_1'|) \equiv E_1'(|\mathbf{k}_1'|) + E_2'(|\mathbf{k}_1'|)\ .$$

With this definition and the equation (B.125) it is clear that $\sqrt{s} = f(|\mathbf{k}_1'|_{CM})$. At this point it is important to note that the δ function becomes null when $|\mathbf{k}_1'| = |\mathbf{k}_1'|_{CM}$, therefore it can be invoked the property of the δ, such that

$$\delta\big(f(x) - f(x_0)\big) = \frac{\delta(x - x_0)}{\left|\dfrac{\partial f(x)}{\partial x}\right|_{x_0}}\ ,$$

with

$$x = |\mathbf{k}_1'| \quad \text{and} \quad x_0 = |\mathbf{k}_1'|_{CM}\ .$$

Therefore, by performing the derivative it leads to

$$\frac{\partial f(|\mathbf{k}_1'|)}{\partial|\mathbf{k}_1'|}=\frac{|\mathbf{k}_1'|\sqrt{s}}{E_1'(|\mathbf{k}_1'|)E_2'(|\mathbf{k}_1'|)}\ .$$

By evaluation in $|\mathbf{k}_1'|_{CM}$ and by substitution into (B.146), it leads to

$$\mathrm{dLIPS}_2(k_{in})=\frac{E_1'(|\mathbf{k}_1'|_{CM})E_2'(|\mathbf{k}_1'|_{CM})}{E_1'(|\mathbf{k}_1'|)E_2'(|\mathbf{k}_1'|)}\frac{\delta\big(|\mathbf{k}_1'|-|\mathbf{k}_1'|_{CM}\big)}{|\mathbf{k}_1'|_{CM}\sqrt{s}}\frac{d^3k_1'}{4(2\pi)^2}\ .$$

By writing the differential element d^3k_1' as

$$d^3k_1'=|\mathbf{k}_1'|^2d|\mathbf{k}_1'|d\Omega_{CM},\quad d\Omega_{CM}=\sin\theta d\theta d\phi\ ,$$

where θ is measured between $\mathbf{k}_1$ and $\mathbf{k}_1'$ in the CM frame, and by performing the integration over $d|\mathbf{k}_1'|$, with evaluation in $|\mathbf{k}_1'|_{CM}$ given by the δ function, it is obtained,

$$\mathrm{dLIPS}_2(\mathrm{k_{in}})=\frac{1}{16\pi^2}\frac{|\mathbf{k}_1'|_{CM}}{\sqrt{s}}\,d\Omega_{CM}\ . \tag{B.147}$$

The previous expresion can be put in an explicitly Lorentz invariant form, first of all, by writing the $d\Omega_{CM}$ in terms of t by using the equation (B.119) in the CM frame, at fixed s

$$\begin{aligned}dt&=2|\mathbf{k}_1|_{CM}|\mathbf{k}_1'|_{CM}d\cos\theta\\&=2|\mathbf{k}_1|_{CM}|\mathbf{k}_1'|_{CM}\frac{d\Omega_{CM}}{2\pi}\ ,\end{aligned}$$

by using this result into (B.147) it becomes

$$\mathrm{dLIPS}_2(\mathrm{k_{in}})=\frac{1}{16\pi\sqrt{s}}\frac{dt}{|\mathbf{k}_1|_{CM}}\ .$$

Finally, by writing the equation (B.126) in terms of the MTF given by equation (B.127), and substituting into the previous result, it is obtained

$$\mathrm{dLIPS}_2(\mathrm{k_{in}})=\frac{1}{8\pi}\frac{dt}{\sqrt{\lambda(s,m_1^2,m_2^2)}}\ . \tag{B.148}$$

By subtituting this result into (B.144) the differential cross section becomes

$$d\sigma=\frac{|\widetilde{\mathcal{M}}|^2}{32\pi E_1E_2v_{rel}}\frac{dt}{\sqrt{\lambda(s,m_1^2,m_2^2)}}\ . \tag{B.149}$$

B.8.2 Total cross section

The total cross section is obtained by integration over $d\sigma$ divided by the symmetry factor Σ.

$$\sigma = \frac{1}{\Sigma} \int d\sigma \ . \tag{B.150}$$

The symmetry factor is defined as

$$\Sigma = \prod_i n_i'! \ , \tag{B.151}$$

This factor is neccesary to avoid overcounting of particles, only if them are identical, this is so because, in general, for n' outgoing particles, the integration over the $\mathrm{dLIPS}_{\mathrm{n}'}$, considers the ordering of n' distinguisable particles. But in the case of identical particles the final state cannot be determined by an ordered list, there could be an overcounting, for example the state $a_1^\dagger a_2^\dagger |0\rangle$ is identical to the state $a_2^\dagger a_1^\dagger |0\rangle$. The symmetry factor corrects this.

Appendix C

Details on the $SO(1,1)$ Model Building

C.1 Diagonalization of the Lagrangian

In this appendix, we present in detail the diagonalization analysis of our model Lagrangian whose results are used along the discussion in the main text. First, we consider the scalar sector, whose Lagrangian (3.14) in terms of the doublet complex field components becomes

$$\mathcal{L}_\Phi = \partial^\mu \phi^* \partial_\mu \phi + \partial^\mu \varphi^* \partial_\mu \varphi - V(\phi, \varphi),$$

with the potential

$$V(\phi, \varphi) \;=\; \alpha_0\big(|\phi|^2 + |\varphi|^2\big) + \alpha_1\big(\phi^* \varphi + \varphi^* \phi\big) + \alpha_3\big(\phi^2 - \varphi^2\big) + c.c. \quad \text{(C.1)}$$

Next, we rewrite the Lagrangian in terms of the hermitian base

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \qquad \varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2),$$

where $\phi_i,\ \varphi_i,\ i = 1, 2$ are real scalar fields. This let us put the potential in a matrix form which we will diagonalize in order to identify physical fields having separated dynamics. The potential (C.1) becomes

$$V = \frac{1}{2}\Phi_R^T A \Phi_R,$$

with Φ_R being the vector formed from above real scalar fields components of ϕ and φ, given by $\Phi_R^T = (\phi_1, \phi_2, \varphi_1, \varphi_2)$, and A is the 4×4 mass coupling matrix

$$A = \begin{pmatrix} m_1^2 & \lambda^2 & \mu_1^2 & 0 \\ \lambda^2 & m_2^2 & 0 & \mu_1^2 \\ \mu_1^2 & 0 & m_2^2 & -\lambda^2 \\ 0 & \mu_1^2 & -\lambda^2 & m_1^2 \end{pmatrix},$$

where we have defined

$$m_1^2 = \mu_0^2 + \mu_3^2, \quad m_2^2 = \mu_0^2 - \mu_3^2, \quad \lambda^2 = 2Re(i\alpha_3),$$

and

$$\mu_0^2 \equiv 2Re(\alpha_0), \quad \mu_1^2 \equiv 2Re(\alpha_1), \quad \mu_3^2 = 2Re(\alpha_3)\ .$$

Notice that by definition all the involved mass terms, m_1^2, m_2^2, λ^2, μ_0^2, μ_3^2 and μ_1^2 are real and by construction, we have chosen them to be positive.

Since the A matrix is real and symmetric, by means of the proper orthogonal rotation of the field base, $\mathbb{S}$, through which we redefine

$$\Phi_D = \mathbb{S}\Phi_R, \qquad A_D = \mathbb{S}A\mathbb{S}^T,$$

we should get a diagonal mass sector. It is not difficult to check that such a matrix can be expressed as

$$\mathbb{S} = \big(\mathbb{I}_{2\times 2} \otimes \mathbb{B} - i\sigma_2 \otimes \mathbb{H}\big)\cos(\omega),$$

where

$$\mathbb{B} = \begin{pmatrix} \cos(\rho) & 0 \\ 0 & \cos(\rho) \end{pmatrix}, \qquad \mathbb{H} = \begin{pmatrix} \tan(\omega) & \sin(\rho) \\ \sin(\rho) & -\tan(\omega) \end{pmatrix}\ .$$

In the above, we have made use of the shorthand notation where

$$\cos(\rho) = \frac{\mu_1^2}{\sqrt{\mu_1^4 + \lambda^4}}, \qquad \sin(\rho) = \frac{\lambda^2}{\sqrt{\mu_1^4 + \lambda^4}}\ ,$$

$$\cos(\omega) = \frac{\alpha^2}{\sqrt{2h^2(h^2 + \Delta^2)}}, \qquad \sin(\omega) = \frac{\alpha^2}{\sqrt{2h^2(h^2 - \Delta^2)}},$$

and

$$\alpha^4 = 4(\mu_1^4 + \lambda^4), \qquad \Delta^2 = m_1^2 - m_2^2, \qquad h^4 = \Delta^4 + \alpha^4.$$

After performing the $\mathbb{S}$ rotation, the potential becomes

$$V = \frac{1}{2}\Phi_D^T A_D \Phi_D,$$

with $\Phi_D^T = (\mathrm{Q}_1, \xi_1, \xi_2, \mathrm{Q}_2)^T$ and

$$A_D = diag\left(m^2, \quad M^2, \quad M^2, \quad m^2\right),$$

where the eigenvalues m^2 and M^2 are given by

$$m^2 = \mu_0^2 - \mu^2 \quad \text{and} \qquad M^2 = \mu_0^2 + \mu^2 \ , \tag{C.2}$$

where

$$\mu^2 = \sqrt{\mu_3^4 + \mu_1^4 + \lambda^4}.$$

In terms of the α couplings, we get

$$\mu_0^2 = 2Re\ \alpha_0 \quad \text{and} \quad \mu^2 = 2\sqrt{(Re\ \alpha_1)^2 + |\alpha_3|^2}\ .$$

The requirement that $M^2, m^2 > 0$, which guarantees that the potential is bounded from below, is fulfilled if $\mu_0^2 > \mu^2 > 0$. If both parameters were of the same order, $\mu_0^2 \approx \mu^2 > 0$, we would naturally get $M^2 \gg m^2 \approx 0$. In such a scenario it becomes natural to identify ξ with the inflaton and Q with the DE field, provided M is as large as the inflation scale.

Notice that the mass eigenstates in Φ_D can be rearranged in a more natural ordering by the permutation matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \ ,$$

such that $(\mathrm{Q}_1, \mathrm{Q}_2, \xi_1, \xi_2)^T = \mathbb{S}'\Phi_R$ with $\mathbb{S}' = P\mathbb{S}$.

In terms of the diagonal base and given that there are two degenerate scalar degrees of freedom for each mass, the potential finally can be expressed as

$$V = m^2|\mathrm{Q}|^2 + M^2|\xi|^2, \tag{C.3}$$

where we have introduced the new complex scalar fields

$$\mathrm{Q} = \frac{1}{\sqrt{2}}(\mathrm{Q}_1 + i\mathrm{Q}_2), \qquad \text{and} \qquad \xi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2). \tag{C.4}$$

Analogously, the scalar kinetic term can be easily put in terms of the new fields after the $\mathbb{S}'$ rotation on Φ_R, to get the also diagonal terms $\partial^\mu \mathrm{Q}^* \partial_\mu \mathrm{Q} + \partial^\mu \xi^* \partial_\mu \xi$.

Finally, by introducing the doublet

$$\boldsymbol{\varphi} = \begin{pmatrix} \mathrm{Q} \\ \xi \end{pmatrix}, \tag{C.5}$$

the whole Lagrangian of the scalar sector becomes

$$\mathcal{L}_{\boldsymbol{\varphi}} = \partial^\mu \boldsymbol{\varphi}^\dagger \partial_\mu \boldsymbol{\varphi} - \boldsymbol{\varphi}^\dagger \mathbb{M} \boldsymbol{\varphi}, \tag{C.6}$$

where $\mathbb{M}$ is the diagonal mass matrix

$$\mathbb{M} = \begin{pmatrix} m^2 & 0 \\ 0 & M^2 \end{pmatrix}. \tag{C.7}$$

We should emphasize that this new doublet notation is not a faithful representation of $SO(1,1)$, since the $SO(4)$ rotation, $\mathbb{S}'$, and the $SO(1,1)$ transformations do not commute. Therefore, the diagonal Lagrangian (C.7), which provides the decoupled field system which evolves explaining inflation and the late accelerated expansion of the Universe, is not explicitly invariant under $SO(1,1)$, even though the original model is so.

Let us now move into analyzing the fermion sector of the theory, for which the corresponding kinetic terms, as given in Eq. (3.23), are

$$\mathcal{L}_{N_i} = \sum_{i=0}^{2} N_i^{\dagger a} i \sigma^\mu_{a\dot{c}} \partial_\mu N_i^{\dot{c}}\ , \tag{C.8}$$

and the interaction terms (3.24) which take the form

$$\begin{aligned} -\mathcal{L}_I = N_{0\dot{a}} \big\{ & a_0(\phi^* N_1^{\dot{a}} + \varphi^* N_2^{\dot{a}}) + a_1(\phi^* N_2^{\dot{a}} + \varphi^* N_1^{\dot{a}}) \\ & + a_2(\phi N_2^{\dot{a}} - \varphi N_1^{\dot{a}}) + a_3(\phi N_1^{\dot{a}} - \varphi N_2^{\dot{a}}) \big\} + h.c. \end{aligned} \tag{C.9}$$

Last, written in terms of the real field components in Φ_R, leads to

$$-\mathcal{L}_I = \frac{1}{\sqrt{2}} N_{0\dot{a}} \Phi_R^T \{ \mathbb{V} N_1^{\dot{a}} + \mathbb{\Gamma} \mathbb{V} N_2^{\dot{a}} \} + h.c., \tag{C.10}$$

where $\mathbb{V}$ is the vector formed from the complex couplings a_i, given by

$$\mathbb{V} = \begin{pmatrix} a_3 + a_0 \\ i(a_3 - a_0) \\ a_1 - a_2 \\ -i(a_1 + a_2) \end{pmatrix},$$

and $\mathbb{\Gamma}$ is a 4×4 matrix given by $\mathbb{\Gamma} = -\sigma_1 \otimes \sigma_2$. After the $\mathbb{S}$ rotation in the scalar sector is set in, and noticing that $\mathbb{\Gamma}$ is actually an invariant matrix, since $\mathbb{\Gamma} = \mathbb{S}\mathbb{\Gamma}\mathbb{S}^T$, the interaction Lagrangian becomes

$$-\mathcal{L}_I = \frac{1}{\sqrt{2}} N_{0\dot{a}} \Phi_D^T \left\{ \mathbb{V}' N_1^{\dot{a}} + \mathbb{\Gamma} \mathbb{V}' N_2^{\dot{a}} \right\} + h.c., \tag{C.11}$$

where $\mathbb{V}' = \mathbb{S}\mathbb{V}$.

It is important to note that $\mathbb{V}'$ just corresponds to a redefinition of the Yukawa couplings, for which one can always assume a convenient parameterization, implicitly defined in terms of the initial $a_{i=0,\ldots,3}$ couplings. Hence, using this freedom we choose the following combinations to define the couplings in the rotated scalar base:

$$\mathbb{V}' = \frac{1}{\sqrt{2}} \begin{pmatrix} g_1 + g_2 \\ h_1 - h_2 \\ -i(h_1 + h_2) \\ i(g_1 - g_2) \end{pmatrix}, \tag{C.12}$$

where $g_{i=1,2}$ and $h_{i=1,2}$ are complex numbers. Substituting the last expression and the redefinition of the scalar fields given in Eq. (C.4) into Eq. (C.11), after some simple algebra, we finally rewrite the interaction terms as

$$-\mathcal{L}_I = N_{0\dot{a}} \{ g_1 \mathrm{Q} F_1^{\dot{a}} + g_2 \mathrm{Q}^* F_2^{\dot{a}} + h_1 \xi^* F_1^{\dot{a}} - h_2 \xi F_2^{\dot{a}} \} + h.c., \tag{C.13}$$

where the new Weyl fields $F_{i=1,2}^{\dot{a}}$ are the components of the doublet

$$\mathbf{F} = \begin{pmatrix} F_1^{\dot{a}} \\ F_2^{\dot{a}} \end{pmatrix}, \tag{C.14}$$

which in turn comes from the transformation

$$e^{-i\sigma_2\pi/4}\Psi = \mathbf{F}, \tag{C.15}$$

i.e., the diagonalization of the scalar potential through $\mathbb{S}$, induces an $SO(2)$ rotation over the doublet Eq. (3.16), by an angle of $\pi/4$. Note that we can still define the $U(1)$ global transformation used in (3.27) with the same charge for the new Weyl fields as $\mathbf{F} \longrightarrow e^{iq}\mathbf{F}$, and so this convenient transformation does not alter the argument used to remove the mass of N_0 in the main text. Nevertheless, as for the scalar sector, the transformations used to rewrite the interactions hide the $SO(1,1)$ symmetry of the theory, but on the other hand, allows to write down Eq. (C.13) in a simple and compact way, as

$$-\mathcal{L}_I = N_{0\dot{a}}\{\boldsymbol{\varphi}^\dagger \mathbb{G}_1 \mathbf{F} + \boldsymbol{\varphi}^T \mathbb{G}_2 \mathbf{F}\} + h.c., \tag{C.16}$$

where we have defined the coupling matrices as

$$\mathbb{G}_1 = \begin{pmatrix} 0 & g_2 \\ h_1 & 0 \end{pmatrix} \qquad \mathbb{G}_2 = \begin{pmatrix} g_1 & 0 \\ 0 & -h_2 \end{pmatrix}. \tag{C.17}$$

Notice that the transformation given in Eq. (C.15) keeps the diagonal form of fermion kinetic terms, as expected, which can now be expressed as

$$\mathcal{L}_F = N_0^{\dagger a} i\sigma^\mu_{a\dot{c}}\partial_\mu N_0^{\dot{c}} + \mathbf{F}^\dagger i\sigma^\mu \partial_\mu \mathbf{F} \ . \tag{C.18}$$

Finally, the complete Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_{\boldsymbol{\varphi}} + \mathcal{L}_F + \mathcal{L}_I, \tag{C.19}$$

where the three sectors are respectively given by (C.6), (C.18) and (C.16).

C.2 Including phase fields on the model

Here we explore some of the possible effects that considering dynamical phase fields for the cosmological scalars may have in the model outcomes discussed in the main text, as well as other interesting aspects that we believe might be of further interest for field dynamics. For this, we assume that after reheating, the Q field remains dynamically trapped in a homogeneous and isotropic false vacuum configuration, which sources DE and breaks the $U(1)$ global symmetry in the neutrino sector, whereas the inflaton field ξ has already settled on its null value, and thus, quantum perturbation for our cosmological scalar fields can be conveniently introduced in a polar base as

$$\mathrm{Q} = \frac{(\langle \mathrm{Q} \rangle + \mathcal{X})}{\sqrt{2}} e^{i\vartheta/\langle \mathrm{Q} \rangle}, \qquad \xi = \frac{1}{\sqrt{2}} |\xi| e^{i\theta/\langle \mathrm{Q} \rangle}, \tag{C.20}$$

where the degrees of freedom of the complex scalar field Q are now given by the real scalar field $\mathcal{X}$, and the dynamical phase ϑ. Similarly, for ξ, its degrees of freedom are given by its modulus and its own dynamical phase θ.

Next, we proceed to rewrite the Lagrangian of our model in terms of the above parameterization, for this we first notice that the doublet (C.5) can be written as

$$\boldsymbol{\varphi} = \mathbb{P} \boldsymbol{\varphi}_{R}, \tag{C.21}$$

where we have defined the radial field part as

$$\boldsymbol{\varphi}_{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} \langle \mathrm{Q} \rangle + \mathcal{X} \\ |\xi| \end{pmatrix}, \tag{C.22}$$

and the field phase matrix given by

$$\mathbb{P} = \begin{pmatrix} e^{i\vartheta/\langle \mathrm{Q} \rangle} & 0 \\ 0 & e^{i\theta/\langle \mathrm{Q} \rangle} \end{pmatrix}. \tag{C.23}$$

C.2.1 Scalar sector

By substitution of equations (C.21) into the kinetic part that appears in (C.6), it becomes

$$\begin{aligned} \partial^\mu \boldsymbol{\varphi}^\dagger \partial_\mu \boldsymbol{\varphi} = \partial^\mu \boldsymbol{\varphi}_{R}^{T} \partial_\mu \boldsymbol{\varphi}_{R} + \boldsymbol{\varphi}_{R}^{T} \left(\partial^\mu \mathbb{P}^\dagger \right) \left(\partial_\mu \mathbb{P} \right) \boldsymbol{\varphi}_{R} \\ \boldsymbol{\varphi}_{R}^{T} \left(\partial^\mu \mathbb{P}^\dagger \right) \mathbb{P} \left(\partial_\mu \boldsymbol{\varphi}_{R} \right) + \left(\partial^\mu \boldsymbol{\varphi}_{R}^{T} \right) \mathbb{P}^\dagger \left(\partial_\mu \mathbb{P} \right) \boldsymbol{\varphi}_{R}. \end{aligned} \tag{C.24}$$

Note that

$$\left(\partial^\mu \mathbb{P}^\dagger \right) \mathbb{P} = -\frac{i}{\langle \mathrm{Q} \rangle} \partial^\mu \begin{pmatrix} \vartheta & 0 \\ 0 & \theta \end{pmatrix}, \qquad \mathbb{P}^\dagger \left(\partial_\mu \mathbb{P} \right) = \frac{i}{\langle \mathrm{Q} \rangle} \partial_\mu \begin{pmatrix} \vartheta & 0 \\ 0 & \theta \end{pmatrix}.$$

By using the previous, it can be shown that the two last terms in the RHS of (C.24) cancel each other, yielding to

$$\partial^\mu \boldsymbol{\varphi}^\dagger \partial_\mu \boldsymbol{\varphi} = \partial^\mu \boldsymbol{\varphi}_{R}^{T} \partial_\mu \boldsymbol{\varphi}_{R} + \mathcal{T}(\boldsymbol{\varphi}_{R}, \mathbb{P}), \tag{C.25}$$

where we have defined the term

$$\mathcal{T}(\boldsymbol{\varphi}_{R}, \mathbb{P}) = \boldsymbol{\varphi}_{R}^{T} \left(\partial^{\mu}\mathbb{P}^{\dagger}\right) \left(\partial_{\mu}\mathbb{P}\right) \boldsymbol{\varphi}_{R}, \tag{C.26}$$

which is a dimension six and highly suppressed operator. Thus we do not expect it to be relevant for the later dynamics of DE.

In terms of (C.22), the potential appearing in the Lagrangian (C.6) takes the simple form

$$\boldsymbol{\varphi}^{\dagger}\mathbb{M}\boldsymbol{\varphi} = \boldsymbol{\varphi}_{R}^{T}\mathbb{M}\boldsymbol{\varphi}_{R}, \tag{C.27}$$

so that, the Lagrangian for the scalar sector becomes

$$\mathcal{L}_{\boldsymbol{\varphi}} = \partial^{\mu}\boldsymbol{\varphi}_{R}^{T}\partial_{\mu}\boldsymbol{\varphi}_{R} + \boldsymbol{\varphi}_{R}^{T}\mathbb{M}\boldsymbol{\varphi}_{R} + \mathcal{T}(\boldsymbol{\varphi}_{R}, \mathbb{P}). \tag{C.28}$$

C.2.2 Interaction sector

As for the interaction with fermions, by substitution of (C.21) into the Lagrangian (C.16), it can be written as

$$-\mathcal{L}_{I} = N_{0\dot{a}}\boldsymbol{\varphi}_{R}^{T}\left\{\mathbb{P}^{\dagger}\mathbb{G}_{1} + \mathbb{P}^{T}\mathbb{G}_{2}\right\}\mathbf{F} + h.c., \tag{C.29}$$

which in turn can be rewritten as

$$-\mathcal{L}_{I} = N_{0\dot{a}}\boldsymbol{\varphi}_{R}^{T}\mathbb{G}\mathbf{F}' + h.c., \tag{C.30}$$

where the new coupling matrix is given by

$$\mathbb{G} = \begin{pmatrix} g_{1} & g_{2} \\ h_{1}e^{-i(\theta+\vartheta)/\langle \mathrm{Q}\rangle} & -h_{2}e^{i(\theta+\vartheta)/\langle \mathrm{Q}\rangle} \end{pmatrix}, \tag{C.31}$$

and where we have defined the object

$$\mathbf{F}' = \begin{pmatrix} F_{1}^{\prime\dot{a}} \\ F_{2}^{\prime\dot{a}} \end{pmatrix}, \tag{C.32}$$

with

$$F_{1}^{\prime\dot{a}} = e^{i\vartheta/\langle \mathrm{Q}\rangle}F_{1}^{\dot{a}}, \qquad F_{2}^{\prime\dot{a}} = e^{-i\vartheta/\langle \mathrm{Q}\rangle}F_{2}^{\dot{a}}. \tag{C.33}$$

This redefinition of the fermion fields removes the dynamical phases on the $\mathcal{X}$-sector, as can be seen from (C.31), nonetheless, they will reappear as currents coming from the transformation of the kinetic terms (C.18), as we will see next.

C.2.3 Fermionic sector

As stated above, definitions (C.33) produce additional terms apart from purely kinetic terms when we substitute them in (C.18), then we have

$$\mathcal{L}_F = N_0^{\dagger a} i\sigma^{\mu}_{a\dot{c}}\partial_\mu N_0^{\dot{c}} + \mathbf{F}'^{\dagger} i\sigma^{\mu}\partial_\mu \mathbf{F}' + \frac{\partial_\mu \vartheta}{\langle \mathrm{Q} \rangle}\mathbf{F}'^{\dagger}\sigma^{\mu}\sigma_3\mathbf{F}', \tag{C.34}$$

where in the last term the effect of σ_3 is to switch the sign of the lower entry of the doublet. Notice that once again the phase field enters in a suppressed way. Apart from these new terms where the phase fields are explicit, the part of the Lagrangian that matters for the model remains the same.

C.2.4 Revisiting massive neutrino base

Let us now execute a new transformation with the aim to remove the constant phases of the couplings g_1 and g_2 appearing in (C.31), by means of a $SU(2)$ rotation on the doublet fermion sector

$$\boldsymbol{\eta} = \mathbb{R}\mathbf{F}' = \begin{pmatrix} \eta_1^{\dot{a}} \\ \eta_2^{\dot{a}} \end{pmatrix}, \tag{C.35}$$

with

$$\mathbb{R} = \frac{1}{a_c}\begin{pmatrix} g_1 & g_2 \\ -g_2* & g_1* \end{pmatrix}, \tag{C.36}$$

where the coupling a_c is defined as

$$a_c = \sqrt{|g_1|^2 + |g_2|^2}. \tag{C.37}$$

After this rotation, the interaction term (C.30) becomes

$$-\mathcal{L}_I = N_{0\dot{a}}\boldsymbol{\varphi}_R^T \mathbb{G}'\boldsymbol{\eta} + h.c., \tag{C.38}$$

where now, the coupling matrix is

$$\mathbb{G}' = \mathbb{G}\mathbb{R}^{\dagger} = \begin{pmatrix} a_c & 0 \\ C_1(\theta,\vartheta) & C_2(\theta,\vartheta) \end{pmatrix}. \tag{C.39}$$

In above we have used for a shorthand notation

$$\begin{aligned} C_1(\theta,\vartheta) &= \quad (g_{11}e^{-i(\theta+\vartheta)/\langle \mathrm{Q}\rangle} - g_{22}e^{i(\theta+\vartheta)/\langle \mathrm{Q}\rangle})/a_c, \\ C_2(\theta,\vartheta) &= -(g_{12}e^{i(\theta+\vartheta)/\langle \mathrm{Q}\rangle} + g_{21}e^{-i(\theta+\vartheta)/\langle \mathrm{Q}\rangle})/a_c, \end{aligned} \tag{C.40}$$

where $g_{11} = g_1^* h_1$, $g_{22} = g_2^* h_2$, $g_{12} = g_1 h_2$, and $g_{21} = g_2 h_1$. On the other hand, by substitution of equation (C.35) into the equation (C.34), leads to

$$\mathcal{L}_F = N_0^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu N_0^{\dot{c}} + \boldsymbol{\eta}^\dagger i\sigma^\mu \partial_\mu \boldsymbol{\eta} + \frac{\partial_\mu \vartheta}{\langle \mathrm{Q} \rangle} \boldsymbol{\eta}^\dagger \sigma^\mu \mathbb{Y} \boldsymbol{\eta}, \tag{C.41}$$

where $\mathbb{Y}$ is a couplings matrix, that comes from the transformation of σ_3 under (C.36), given by

$$\mathbb{Y} = \begin{pmatrix} y_1 & -y_2 \\ -y_2^* & -y_1 \end{pmatrix},$$

where $y_1 = \left(|g_1|^2 - |g_2|^2\right)/a_c^2$, and $y_2 = 2g_1 g_2 / a_c^2$, i.e., $y_1 \in \mathbb{R}$ and $y_2 \in \mathbb{C}$. (Notice that $y_1^2 + |y_2|^2 = 1$.)

Next, we will proceed to separate the interaction Lagrangian (C.38) into two parts, one corresponding to the interaction between the inflaton and the neutrinos, and the other corresponding to the interactions with the DE field. So that by substituting (C.22), (C.35) and (C.39) into (C.38) we arrive to

$$\mathcal{L}_I = \mathcal{L}_g + \mathcal{L}_{\nu \mathcal{X}}, \tag{C.42}$$

where the two parts above mentioned are given by

$$-\mathcal{L}_g = N_{0\dot{a}} \left\{ C_1(\theta, \vartheta) \eta_1^{\dot{a}} + C_2(\theta, \vartheta) \eta_2^{\dot{a}} \right\} \frac{|\xi|}{2} + h.c., \tag{C.43}$$

and

$$-\mathcal{L}_{\nu \mathcal{X}} = \frac{a_c}{\sqrt{2}} (\langle \mathrm{Q} \rangle + \mathcal{X}) \left\{ N_{0\dot{a}} \eta_1^{\dot{a}} + h.c. \right\}. \tag{C.44}$$

Let us now concentrate our analysis towards the interaction among neutrinos and the DE field. The part between braces on the previous equation, can be expressed also as

$$\begin{aligned} N_{0\dot{a}} \eta_1^{\dot{a}} + h.c. &= N_{0\dot{a}} \eta_1^{\dot{a}} + \eta_1^{\dagger a} N_{0a}^\dagger \\ &= \frac{1}{2} \{ N_{0\dot{a}} \eta_1^{\dot{a}} + N_{0\dot{a}} \eta_1^{\dot{a}} + \eta_1^{\dagger a} N_{0a}^\dagger + \eta_1^{\dagger a} N_{0a}^\dagger \} \\ &= \frac{1}{2} \{ N_{0\dot{a}} \eta_1^{\dot{a}} + \eta_{1\dot{a}} N_0^{\dot{a}} + \eta_1^{\dagger a} N_{0a}^\dagger + N_0^{\dagger a} \eta_{1a}^\dagger \}, \end{aligned} \tag{C.45}$$

wherein both, the second and the fourth terms in the last line, we have used the anti-commutation properties plus an extra minus sign coming from the change from ${}^{\dot{a}}{}_{\dot{a}}$ to ${}_{\dot{a}}{}^{\dot{a}}$ (and similarly for the undotted indices). Now, we define two four-component Dirac neutrinos as

$$u_1 = \begin{pmatrix} N^{\dagger}_{0a} \\ \eta^{\dot{a}}_1 \end{pmatrix}, \qquad u_2 = \begin{pmatrix} \eta^{\dagger}_{1a} \\ N^{\dot{a}}_0 \end{pmatrix}, \tag{C.46}$$

in terms of which the last line in Eq. (C.45) can be written as

$$N_{0\dot{a}}\eta^{\dot{a}}_1 + h.c. = \frac{1}{2}\{\bar{u}_1 u_1 + \bar{u}_2 u_2\}. \tag{C.47}$$

As it can be seen from (C.46), the neutrinos u_1 and u_2 are charge conjugates of each other, this let us put them in terms of two Majorana neutrinos ν_1 and ν_2, through of another rotation, which is given by

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}. \tag{C.48}$$

Therefore, Eq. (C.47) directly becomes

$$N_{0\dot{a}}\eta^{\dot{a}}_1 + h.c. = \frac{1}{2}\{\bar{\nu}_1 \nu_1 + \bar{\nu}_2 \nu_2\}, \tag{C.49}$$

which explicitly provide the neutrino mass eigenstates, with a mass given by

$$m_k = \frac{a_c \langle \mathrm{Q} \rangle}{\sqrt{2}} \ . \tag{C.50}$$

Notice that this same rearrangement of the neutrinos provide the interaction Lagrangian with $\mathcal{X}$ fields,

$$-\mathcal{L}_{I\mathcal{X}} = \frac{a_c}{2\sqrt{2}} \mathcal{X} \left(\bar{\nu}_1 \nu_1 + \bar{\nu}_2 \nu_2\right), \tag{C.51}$$

that we use on our discussions along the paper. We stress that these results are independent of the phase fields and link the origin of the heavy right handed neutrino masses to DE, as already argued in the main text.

As a final note on this regard, notice that the Majorana neutrinos, in four-component notation, can be expressed as

$$\nu_i = \begin{pmatrix} \mathtt{K}^{\dagger}_{ia} \\ \mathtt{K}^{\dot{a}}_i \end{pmatrix}, \quad i = 1, 2. \tag{C.52}$$

In the last equation, we have introduced the new right-handed Weyl field in two-component notation: $\mathtt{K}^{\dot{a}}_{i=1,2}$. Note that the transformation (C.48) together with (C.46) are equivalent to the transformations

$$\begin{pmatrix} \mathtt{K}^{\dot{a}}_1 \\ \mathtt{K}^{\dot{a}}_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \begin{pmatrix} N^{\dot{a}}_0 \\ \eta^{\dot{a}}_1 \end{pmatrix}, \tag{C.53}$$

and

$$\begin{pmatrix} \mathtt{K}^{\dagger}_{1a} \\ \mathtt{K}^{\dagger}_{2a} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \begin{pmatrix} \eta^{\dagger}_{1a} \\ N^{\dagger}_{0a} \end{pmatrix}. \tag{C.54}$$

It is important to remark that these transformations do not respect the $U(1)$ invariance of the fermionic sector since it mixes fields with different global charges.

Summarizing, we can either, substitute (C.52) into (C.49) or directly operate over (C.45) through of (C.53) and (C.54) to get

$$N_{0\dot{a}}\eta^{\dot{a}}_1 + h.c. = \frac{1}{2}\left\{\mathtt{K}_{1\dot{a}}\mathtt{K}^{\dot{a}}_1 + \mathtt{K}_{2\dot{a}}\mathtt{K}^{\dot{a}}_2\right\} + h.c. \tag{C.55}$$

By substituting equation (C.55) into equation (C.44) it is obtained

$$\mathcal{L}_{\nu\mathcal{X}} = \mathcal{L}_m + \mathcal{L}_{I\mathcal{X}}, \tag{C.56}$$

where the Lagrangian corresponding to the mass terms is

$$-\mathcal{L}_m = \frac{1}{2}m_k\left(\mathtt{K}_{1\dot{a}}\mathtt{K}^{\dot{a}}_1 + \mathtt{K}_{2\dot{a}}\mathtt{K}^{\dot{a}}_2\right) + h.c., \tag{C.57}$$

with the mass given as before and the interaction term

$$-\mathcal{L}_{I\mathcal{X}} = \frac{a_c}{2\sqrt{2}}\mathcal{X}\left(\mathtt{K}_{1\dot{a}}\mathtt{K}^{\dot{a}}_1 + \mathtt{K}_{2\dot{a}}\mathtt{K}^{\dot{a}}_2\right) + h.c. \tag{C.58}$$

In the same footing, and for future use, we also write the inflaton to neutrino interactions, as derived from Eq. (C.43), for which we also rename $\mathtt{K}^{\dot{a}}_3 \equiv \eta^{\dot{a}}_2$, to write

$$\begin{aligned} -\mathcal{L}_g = \frac{1}{4}C_1(\theta,\vartheta)|\xi|\left(\mathtt{K}_{1\dot{a}}\mathtt{K}^{\dot{a}}_1 + \mathtt{K}_{2\dot{a}}\mathtt{K}^{\dot{a}}_2\right) \\ + \frac{1}{2\sqrt{2}}C_2(\theta,\vartheta)|\xi|\left(\mathtt{K}_{1\dot{a}} - i\mathtt{K}_{2\dot{a}}\right)\mathtt{K}^{\dot{a}}_3 + h.c. \end{aligned} \tag{C.59}$$

C.2.5 The whole Lagrangian in terms of $\mathtt{K}^{\dot{a}}_{i=1,2,3}$ and the scalar fields

As a summary, we write the complete Lagrangian in terms of the Weyl fields $\mathtt{K}^{\dot{a}}_{i=1,2,3}$ and the scalar fields $|\xi|$, $\mathcal{X}$ and the phases ϑ and θ. The Lagrangian (C.19) is

$$\mathcal{L} = \mathcal{L}_{\varphi} + \mathcal{L}_I + \mathcal{L}_F \ .$$

C.2.5.1 Scalar sector

The scalar sector (C.28) becomes.

$$\mathcal{L}_{\varphi} = \frac{1}{2}\partial^\mu|\xi|\partial_\mu|\xi| + \frac{1}{2}\partial^\mu\mathcal{X}\partial_\mu\mathcal{X} \\ + \frac{1}{2}m^2(\langle \mathrm{Q}\rangle + \mathcal{X})^2 + \frac{1}{2}M^2|\xi|^2 + \mathcal{T}_{(\xi,\mathcal{X},\vartheta,\theta)}, \quad \text{(C.60)}$$

where the last term on the RHS, which corresponds with (C.26) is given by

$$\mathcal{T}_{(\xi,\mathcal{X},\vartheta,\theta)} = \frac{|\xi|^2}{2\langle \mathrm{Q}\rangle^2}\partial^\mu\theta\partial_\mu\theta + \frac{1}{2}\left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2\partial^\mu\vartheta\partial_\mu\vartheta. \quad \text{(C.61)}$$

C.2.5.2 Interaction sector

By using (C.56) the interaction sector (C.42) is

$$\mathcal{L}_I = \mathcal{L}_g + \mathcal{L}_m + \mathcal{L}_{I\mathcal{X}}, \quad \text{(C.62)}$$

when the RHS terms are correspondingly given by (C.59), (C.57) and (C.58).

C.2.5.3 Fermionic sector

Finally, by expanding (C.41) and by the transformation (C.53), (remember that $\eta^{\dot{c}}_2 = \mathtt{K}^{\dot{c}}_3$) the fermionic sector becomes

$$\mathcal{L}_F = \mathcal{L}_K + \mathcal{L}_c \quad \text{(C.63)}$$

where the first term on the RHS corresponds to the kinetic terms for $\mathtt{K}^{\dot{a}}_{i=1,2,3}$,

$$\mathcal{L}_K = \sum_{i=1}^{3} \mathtt{K}^{\dagger a}_i i\sigma^\mu_{a\dot{c}}\partial_\mu \mathtt{K}^{\dot{c}}_i, \quad \text{(C.64)}$$

and the other account of the currents between the scalar $\partial_\mu\vartheta$ and the neutrinos,

$$\mathcal{L}_c = \mathcal{L}_{c_1} + \mathcal{L}_{c_2}, \tag{C.65}$$

where

$$\begin{aligned}\mathcal{L}_{c_1} = y_1 \frac{\partial_\mu\vartheta}{\langle \mathrm{Q}\rangle} \Bigg\{ & \frac{1}{2}\left(\mathtt{K}_1^{\dagger a}\sigma^\mu_{a\dot{c}}\mathtt{K}_1^{\dot{c}} + \mathtt{K}_2^{\dagger a}\sigma^\mu_{a\dot{c}}\mathtt{K}_2^{\dot{c}}\right) \\ & + \frac{i}{2}\left(\mathtt{K}_1^{\dagger a}\sigma^\mu_{a\dot{c}}\mathtt{K}_2^{\dot{c}} - \mathtt{K}_2^{\dagger a}\sigma^\mu_{a\dot{c}}\mathtt{K}_1^{\dot{c}}\right) - \mathtt{K}_3^{\dagger a}\sigma^\mu_{a\dot{c}}\mathtt{K}_3^{\dot{c}}\Bigg\},\end{aligned} \tag{C.66}$$

and

$$\mathcal{L}_{c_2} = -\frac{\partial_\mu\vartheta}{\langle \mathrm{Q}\rangle}\left\{\frac{y_2}{\sqrt{2}}(\mathtt{K}_1^{\dagger a} - i\mathtt{K}_2^{\dagger a})\sigma^\mu_{a\dot{c}}\mathtt{K}_3^{\dot{c}} + h.c.\right\}. \tag{C.67}$$

C.2.6 Energy density and equations of motion for the DE sector

We close this appendix by presenting the results of the calculation of the equation-of-state (parameter ω) for DE, the slow-roll condition and the dynamic system of the homogeneous background in the present model. For this purpose, we made explicit use of the model Lagrangian, as defined in Eq. (C.60), where the DE part is written as

$$\mathcal{L}_{\mathfrak{X},\vartheta} = \frac{1}{2}\partial^\mu\mathfrak{X}\partial_\mu\mathfrak{X} + \frac{1}{2}\left(1 + \frac{\mathfrak{X}}{\langle \mathrm{Q}\rangle}\right)^2 \partial^\mu\vartheta\partial_\mu\vartheta + V(\mathfrak{X}), \tag{C.68}$$

where the potential is defined as

$$V(\mathfrak{X}) = \frac{1}{2}m^2(\langle \mathrm{Q}\rangle + \mathfrak{X})^2. \tag{C.69}$$

By writing the energy momentum tensor

$$T_{\mu\nu} = +2\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} - g_{\mu\nu}\mathcal{L}\ ,$$

then, by subtitution of (C.68), it is obtained

$$T_{\mu\nu} = \partial_\mu\mathfrak{X}\partial_\nu\mathfrak{X} + \left(1 + \frac{\mathfrak{X}}{\langle \mathrm{Q}\rangle}\right)^2 \partial_\mu\vartheta\partial_\nu\vartheta - g_{\mu\nu}\mathcal{L}_{\mathfrak{X},\vartheta},$$

from here we know both, the energy density and the pressure in terms of $\mathcal{X}$ and the phase ϑ, which are given by

$$\rho_{DE} = \frac{1}{2}\dot{\mathcal{X}}^2 + \frac{1}{2}\left(1+\frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2\dot{\vartheta}^2 + \frac{1}{2a^2}(\nabla\mathcal{X})^2 + V(\mathcal{X}) + \frac{1}{2a^2}\left(1+\frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2(\nabla\vartheta)^2 \ , \quad \text{(C.70)}$$

and

$$P_{DE} = \frac{1}{2}\dot{\mathcal{X}}^2 + \frac{1}{2}\left(1+\frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2\dot{\vartheta}^2 - \frac{1}{6a^2}(\nabla\mathcal{X})^2 - V(\mathcal{X}) - \frac{1}{6a^2}\left(1+\frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2(\nabla\vartheta)^2 \ . \quad \text{(C.71)}$$

where in general $\mathcal{X} = \mathcal{X}(t,\mathbf{x})$ and $\vartheta = \vartheta(t,\mathbf{x})$, however, in the FLRW background universe $\mathcal{X} = \mathcal{X}(t)$ and $\vartheta = \vartheta(t)$, therefore $\nabla\mathcal{X}, \nabla\rho \to 0$, such that, the energy density and the pressure reduces to

$$\rho_{DE} = \frac{1}{2}\dot{\mathcal{X}}^2 + \frac{1}{2}\left(1+\frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2\dot{\vartheta}^2 + V(\mathcal{X}), \quad \text{(C.72)}$$

and

$$P_{DE} = \frac{1}{2}\dot{\mathcal{X}}^2 + \frac{1}{2}\left(1+\frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2\dot{\vartheta}^2 - V(\mathcal{X}). \quad \text{(C.73)}$$

In order to realize the accelerated expansion, the DE field has to accomplish an equation of state such that

$$\omega \equiv \frac{P_{DE}}{\rho_{DE}} \approx -1,$$

which means, according to (C.72) and (C.73), that the first slow-roll condition is of the form

$$\frac{1}{2}\dot{\mathcal{X}}^2 + \frac{1}{2}\left(1+\frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2\dot{\vartheta}^2 \ll \frac{1}{2}m^2(\langle \mathrm{Q}\rangle + \mathcal{X})^2. \quad \text{(C.74)}$$

Equations (C.72) and (C.74) impliy that the DE density is given by

$$\rho_{DE} \approx V(\mathcal{X}), \quad \text{(C.75)}$$

which means that the phase ϑ does not play any role regarding the DE field composition, as is should be expected for a potential of the form

$$V(\mathrm{Q}) = m^2 \mathrm{Q}\mathrm{Q}^*.$$

Although the phase does not contribute effectively to the DE density, it still could have an indirect effect, because it appears explicitly in the condition (C.74) and the fulfillment of it could depend on the initial values of the phase and its velocity.

In what follows, we write the complete dynamic system involving both of the scalar fields and impose the fulfillment of the FSRC, this allows us to check under what initial conditions the system evolves consistently with our requirements. To this end, let us turn back to the Lagrangian (C.68), which in the homogeneous limit reduces to

$$\mathcal{L}_{\mathcal{X}\vartheta} = \frac{1}{2}\dot{\mathcal{X}}^2 + \frac{1}{2}\left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 \dot{\vartheta}^2 - V(\mathcal{X}),$$

from here, the Euler-Lagrange equations are:

$$\frac{d}{dt}\Big[\frac{\partial}{\partial \phi_i}(\sqrt{-g}\mathcal{L}_{\mathcal{X}\vartheta})\Big] - \frac{\partial}{\partial \phi_i}(\sqrt{-g}\mathcal{L}_{\mathcal{X}\vartheta}) = 0,$$

where $i = 1, 2$, with $\phi_1 = \vartheta$, and $\phi_2 = \mathcal{X}$ (remember that the variation is done over the action in a flat FLRW Universe, there is where the volume element $\sqrt{-g} = a^3$ comes from).

Then, the dynamical equations for the scalars $\mathcal{X}$ and ϑ are

$$\ddot{\vartheta} + \left(\frac{2\dot{\mathcal{X}}}{\langle \mathrm{Q} \rangle + \mathcal{X}} + 3\mathsf{H}\right)\dot{\vartheta} = 0, \tag{C.76}$$

and

$$\ddot{\mathcal{X}} + 3\mathsf{H}\dot{\mathcal{X}} + W_{,\mathcal{X}} = 0, \tag{C.77}$$

where

$$W = \frac{1}{2}m^2(\langle \mathrm{Q} \rangle + \mathcal{X})^2 - \frac{1}{2}\left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 \dot{\vartheta}^2. \tag{C.78}$$

As said above, we impose on this system the fulfillment of the first slow roll condition (C.74), which must hold separately for both of the left-hand side members, thus we have

$$\frac{1}{2}\dot{\mathcal{X}}^2 \ll V(\mathcal{X}), \tag{C.79}$$

and,

$$\frac{1}{2}\left(1+\frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2\dot{\vartheta}^2 \ll V(\mathcal{X}). \tag{C.80}$$

Next, by writing (C.79) as

$$\frac{1}{\sqrt{2}}\frac{|\dot{\mathcal{X}}|}{\sqrt{V}} \ll 1, \tag{C.81}$$

and by using (A.100) to rewrite it as

$$\frac{|\dot{\mathcal{X}}|m}{\sqrt{2V}} \ll m \ll \sqrt{3}\mathsf{H},$$

we arrive to

$$\frac{|\dot{\mathcal{X}}|}{\langle \mathrm{Q}\rangle + \mathcal{X}} \ll 3\mathsf{H}.$$

Therefore, in the SR regimen, the equation (C.76) reduces to

$$\ddot{\vartheta} + 3\mathsf{H}\dot{\vartheta} = 0. \tag{C.82}$$

In the same way, because of the condition (C.80), the equation (C.77) becomes the usual Klein-Gordon equation for a real scalar field in an expanding universe, namely

$$\ddot{\mathcal{X}} + 3\mathsf{H}\dot{\mathcal{X}} + V_{,\mathcal{X}} = 0. \tag{C.83}$$

As it is shown in the Appendix A.11.1, the dynamics of the DE system can be analized by means of the parameter β which is defined in equation (A.93), analogously to it, we define here

$$\beta \equiv \frac{\ddot{\mathcal{X}}}{3\mathsf{H}\dot{\mathcal{X}}},$$

with this into (C.83) and by solving for $\dot{\mathcal{X}}$ we get

$$\dot{\mathcal{X}} = -\frac{V_{,\mathcal{X}}}{3\mathsf{H}(\beta+1)}, \tag{C.84}$$

by substituting the previous quantities into (C.83) it is gotten

$$\beta^2 + \beta\left(1 - \frac{3}{2}\Pi\right) = \frac{3}{2}\Pi - \frac{1}{3}\eta. \tag{C.85}$$

The quantity Π is defined as

$$\Pi = -\frac{2}{3}\frac{\dot{\mathsf{H}}}{\mathsf{H}^2}, \tag{C.86}$$

similarly, the quantity η is given by

$$\eta = \frac{V_{,xx}}{3\mathsf{H}^2}. \tag{C.87}$$

The previous definition clearly corresponds with the second parameter of slow roll for quintessence given in the equation (A.107).

As shown in the appendix A.11.2.2, there are two possible cases for the value of η, namely,

$$\eta \to \frac{9}{2}\Pi \qquad \text{when} \qquad \beta \to 0,$$

which defines the evolution in the *Freezing Quintessence regime*, and

$$\eta \ll 1 \qquad \text{when} \qquad \beta \approx \mathcal{O}(1),$$

which corresponds to the *Thawing Quintessence regime* [49].

Notice that, with the potential as given in (C.69) the slow roll parameter defined in equation (A.99), coincides with (C.87), namely

$$\eta = \epsilon \ll 1.$$

Since the First Slow Roll Parameter ϵ por quintessence [see Eq. (A.99)] is always very much smaller than unity, the system with this potential evolves in the *thawing quintessence* regime, therefore $\Pi \approx \mathcal{O}(1)$, except in the pure scalar field domination era, as it is show in the figure C.2.6.

By using the fact that $\eta \ll 1$ into the equation (C.85), it becomes

$$\beta^2 + \beta\left(1 - \frac{3}{2}\Pi\right) - \frac{3}{2}\Pi = 0, \tag{C.88}$$

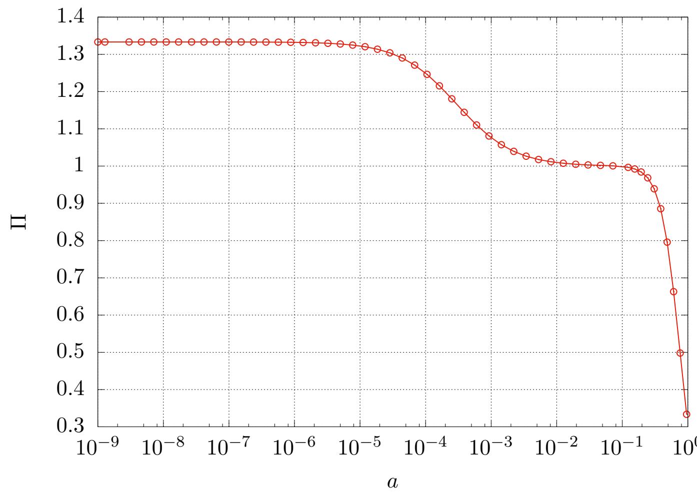

Figure C.1: The parameter Π as defined in (C.86). Note that its value remains close order unity for times prior to DE domination, then, as stated in the text, the system evolves in the thawing regime.

which is a quadratic algebraic equation, which one can easily solve to get

$$\beta^- = -1 \qquad \text{and} \qquad \beta^+ = \frac{3}{2}\Pi.$$

The solution β^- contradicts the first slow-roll condition since $\dot{\mathfrak{X}} \to \infty$ [see Eq. (C.84)], therefore the only acceptable solution is β^+.

As for the equation (C.82), it can be written as a first order system by means of the definitions as follows

$$y_0 = \vartheta \quad \text{and} \quad y_1 = \dot{\vartheta}. \tag{C.89}$$

Finally, by taking into account all of the previous quantities, the whole complex coupled dynamic system can be raised. By completness we also consider the densities of Dark Matter (ρ_{DM}), baryons (b), light active neutrinos (n), and photons (γ), as components of the background. By including the Friedmann equations, the whole system is

$$
\begin{aligned}
&\mathsf{H}^2 = \frac{1}{3M_{pl}^2} V(\mathcal{X}), \\
&\ddot{\mathcal{X}} + 3\mathsf{H}\dot{\mathcal{X}} + V(\mathcal{X})_{,\mathcal{X}} = 0, \\
&\ddot{\vartheta} + 3\mathsf{H}\dot{\vartheta} = 0, \\
&\dot{\mathsf{H}} = \frac{-1}{2M_{pl}^2}\left(\rho_{DM} + \rho_b + \frac{4}{3}\rho_\gamma + \frac{4}{3}\rho_n\right), \\
&\dot{\rho}_{DM,b} + 3\mathsf{H}\rho_{DM,b} = 0, \\
&\dot{\rho}_{\gamma,n} + 4\mathsf{H}\rho_{\gamma,n} = 0.
\end{aligned}
\tag{C.90}
$$

Appendix D

Phenomenology Complementary Calculations

D.1 Cosmological quantities used

D.1.1 Temperatures

CMB temperature today [6]

$$\begin{aligned} T_{\gamma,0} &= 2.7255 \text{ K} \\ &= 2.34865337 \times 10^{-4} \text{ eV}. \end{aligned} \tag{D.1}$$

The reheating temperature is approximately

$$T_r \approx 10^{14} \text{ GeV} - \ 10^{15} \text{ GeV}.$$

D.1.2 Fundamental constants

Plank mass ($\hbar = c = 1$)

$$m_{pl} = \sqrt{\frac{1}{G}} = 1.220890 \times 10^{19} \text{ GeV}. \tag{D.2}$$

Reduced Plank mass ($\hbar = c = 1$)

$$M_{\text{pl}} = \sqrt{\frac{1}{8\pi G}} = 2.4353232036 \times 10^{18} \text{ GeV}. \tag{D.3}$$

Note from the above that the reduced Planck mass is

$$M_{\rm pl} = \frac{m_{pl}}{\sqrt{8\pi}} \ . \tag{D.4}$$

There is a useful conversion factor

$$1({\rm GeV})^4 = 2.845 \times 10^{-74} \ M_{\rm pl}^4 \ , \tag{D.5}$$

D.1.3 Hubble and critical density

Hubble parameter today [6]

$$\mathsf{H}_0 = 1.4382431715 \times 10^{-33} \ {\rm eV}. \tag{D.6}$$

Critical density of the Universe

$$\begin{aligned} \rho_{crit} &= 3M_{\rm pl}^2 \mathsf{H}_0^2 \\ &= 3.6894315078 \times 10^{-47} \ {\rm GeV}^4 \ . \end{aligned} \tag{D.7}$$

D.1.4 Density parameters

Observed Dark Energy (DE) density parameter [6]

$$\Omega_\Lambda = 0.685 \tag{D.8}$$

Radiation density parameter (CMB photons) observed today [6]

$$\Omega_{\gamma,0} = 5.38 \times 10^{-5}. \tag{D.9}$$

Neutrino density parameter today [6]

$$0.0012 \leq \Omega_{\nu,0} < 0.003 \ . \tag{D.10}$$

DM density parameter today [6]

$$\Omega_{DM} = 0.265 \ . \tag{D.11}$$

Baryon density parameter today [6]

$$\Omega_b = 0.0493 \ . \tag{D.12}$$

The matter density parameter is defined as

$$\Omega_{M,0} = \Omega_{DM} + \Omega_b,$$

which, with the values given in (D.11) and (D.12) becomes

$$\Omega_{M,0} = 0.3143 \ . \tag{D.13}$$

Spatial curvature density parameter today [6]

$$\Omega_\kappa = 0.0007 \ . \tag{D.14}$$

In a FLRW universe, we have

$$1 + \Omega_\kappa = \Omega_{DM} + \Omega_b + \Omega_\gamma + \Omega_\nu + \Omega_\Lambda,$$

then we can know the radiation density parameter $\Omega_R = \Omega_\gamma + \Omega_\nu$, as

$$\Omega_R = 1 + \Omega_\kappa - \Omega_{DM} - \Omega_b - \Omega_\Lambda \ ,$$

then with (D.8), (D.11), (D.12) and (D.14) we get

$$\Omega_{R,0} = 1.4 \times 10^{-3} \ . \tag{D.15}$$

D.1.5 Energy densities

With the definition of the density parameter

$$\Omega := \frac{\rho}{\rho_{crit}} \ , \tag{D.16}$$

and with Eqs. (D.8) and (D.7) we have that the observed DE energy density is

$$\rho_\Lambda^{(obs)} = 2.5272605829 \times 10^{-47}\mathrm{GeV}^4. \tag{D.17}$$

With (D.9), (D.7) and (D.16) we have that the radiation energy density today is

$$\rho_{r,0} = 2.93 \times 10^{-51} \ \mathrm{GeV}^4. \tag{D.18}$$

With (D.10), (D.7) and (D.16) we have that the energy density of neutrinos today is

$$\rho_{\nu,0} \lesssim 0.87 \times 10^{-48} \ \mathrm{GeV}^4. \tag{D.19}$$

D.1.6 Cosmological constant

With

$$\rho_\Lambda^{(obs)} = M_{\rm pl}^2\Lambda\ , \tag{D.20}$$

and by using (D.17) y (D.3) we arrive to

$$\Lambda = 4.261248 \times 10^{-84}\ \mathrm{GeV}^2. \tag{D.21}$$

D.2 Numerical density of relativistic particles in equilibrium

The numerical density of particles in thermal equilibrium with the photons at the relativistic limit (radiation) is

$$n = \frac{\zeta(3)}{\pi^2} g_{*n}(T)T^3, \tag{D.22}$$

with

$$g_{*n}(T) = g_b(T) + \frac{3}{4}g_f(T),$$

and $\zeta(3)$ the Riemann zeta function evaluated in 3. (Appery constant).

$$\zeta(3) \approx 1.20205 \tag{D.23}$$

D.3 Table of number of degrees of freedom in energy density, entropy density and numerical density

Temperatures in GeV.

$[T_a,$	$T_b)$	g_*	g_{*s}	g_{*n}
10^{16}	1.67×10^{14}	111.125	111.125	99.25
1.67×10^{14}	30	107.625	107.625	96.25
30	10	97.125	97.125	87.25
10	0.7	87.125	87.125	77.25
0.7	0.3	76.625	76.625	68.25
0.3	0.17	73.25	73.25	65.25
0.17	0.15	62.625	62.625	56.25
0.15	0.023	18.125	18.125	16.25
0.023	0.018	15.125	15.125	13.25
0.018	85.2×10^{-6}	11.625	11.625	10.25
85.2×10^{-6}	today	3.61	4.25	3.93

D.4 Calculations related to the process $\bar{\nu}\nu \to \mathcal{X}\mathcal{X}$

In this section, we perform the calculations needed for writing the thermally averaged cross-section for the process

$$\bar{\nu}\nu \to \mathcal{X}\mathcal{X} \, .$$

We start by calculating the corresponding annihilation amplitude.

D.4.1 Annihilation amplitude for the process $\bar{\nu}\nu \to \mathcal{X}\mathcal{X}$

We start by calculating the total cross-section for the part of the Lagrangian given in Eq. (4.5) and showed in Fig. 4.1. Since the outgoing particles are indistinguishable, there are two annihilation channels allowed by (4.5), which are characterized by the Mandelstam variables t and u.

In order to calcutate the annihilation amplitude

$$|\widetilde{\mathcal{M}}|_{\bar{\nu}\nu\to\chi\chi},$$

we apply the Feynmann rules, which in general for Majorana fields, must be used in their compact form [108, 109], however, since the previous diagrams involve only one fermionic line, the Feynman rules can be applied in their ordinary form, then it is obtained

$$\begin{aligned} i\mathcal{M} &= \bar{v}_{s_2}(\mathbf{p}_2) i\frac{a_c}{2\sqrt{2}}(-i)\frac{-\not{p}_1 + \not{p}'_1 + m_k}{-t + m_k^2} i\frac{a_c}{2\sqrt{2}} u_{s_1}(\mathbf{p}_1) \\ &+ \bar{v}_{s_2}(\mathbf{p}_2) i\frac{a_c}{2\sqrt{2}}(-i)\frac{-\not{p}_1 + \not{p}'_2 + m_k}{-u + m_k^2} i\frac{a_c}{2\sqrt{2}} u_{s_1}(\mathbf{p}_1), \end{aligned}$$

which is the same as

$$\mathcal{M} = \frac{a_c^2}{8}\bar{v}_{s_2}(\mathbf{p}_2) A u_{s_1}(\mathbf{p}_1),$$

with

$$A = \frac{\not{p}'_1 + 2m_k}{-t + m_k^2} + \frac{\not{p}'_2 + 2m_k}{-u + m_k^2},$$

where we have used the relation

$$-\not{p}u(\mathbf{p}) = m_k u(\mathbf{p}).$$

Notice that, since A is linearly proportional to a Dirac operator, it is accomplished that $\bar{A} = A$, thereby, the squared modulus of $\mathcal{M}$ becomes

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{a_c^4}{8}\bar{v}_{s_2}(\mathbf{p}_2) A u_{s_1}(\mathbf{p}_1)\bar{u}_{s_1}(\mathbf{p}_1) A v_{s_2}(\mathbf{p}_2) \\ &= \frac{a^4}{8} Tr\left[v_{s_2}(\mathbf{p}_2)\bar{v}_{s_2}(\mathbf{p}_2) A u_{s_1}(\mathbf{p}_1)\bar{u}_{s_1}(\mathbf{p}_1) A\right]. \end{aligned}$$

After averaging over the spin values, it leads to

$$|\widetilde{\mathcal{M}}|^2 = \frac{1}{\Sigma_s}\frac{a_c^4}{8} Tr\left[(-\not{p}_2 - m_k) A (-\not{p}_1 + m_k) A\right],$$

where, for two incoming particles of spin $\frac{1}{2}$

$$\Sigma_s = (2s_1 + 1)(2s_2 + 2) = 4.$$

Next, by expanding the argument of Tr, the amplitude squared modulus spin-averaged becomes

$$
\begin{aligned}
|\widetilde{\mathcal{M}}|^2 = \frac{a_c^4}{32} Tr \Bigg[& \frac{(-\not{p}_2 - m_k)(\not{p}'_1 + 2m_k)(\not{p}_1 + m_k)(\not{p}'_1 + 2m_k)}{(-t + m_k^2)^2} \\
& + \frac{(-\not{p}_2 - m_k)(\not{p}'_2 + 2m_k)(\not{p}_1 + m_k)(\not{p}'_2 + 2m_k)}{(-u + m_k^2)^2} \\
& + \frac{(-\not{p}_2 - m_k)(\not{p}'_1 + 2m_k)(\not{p}_1 + m_k)(\not{p}'_2 + 2m_k)}{(-t + m_k^2)(-u + m_k^2)} \\
& + \frac{(-\not{p}_2 - m_k)(\not{p}'_2 + 2m_k)(\not{p}_1 + m_k)(\not{p}'_1 + 2m_k)}{(-t + m_k^2)(-u + m_k^2)} \Bigg].
\end{aligned}
$$

After expanding and making use of the properties of the gamma matrices products, and by taking into account the rules for the Mandelstam variables given in equations (B.120) and (B.121), we arrive to

$$
|\widetilde{\mathcal{M}}|^2_{\bar{\nu}\nu \to \chi\chi} = \frac{a_c^4}{128} P(t, s), \tag{D.24}
$$

where $P(t, s)$ is a polynomial, given by

$$
P(t, s) = \frac{P_{tt}}{(m_k^2 - t)^2} + \frac{P_{uu}}{(m_k^2 - u)^2} + \frac{P_{ut}}{(m_k^2 - u)(m_k^2 - t)}, \tag{D.25}
$$

wherein

$$
\begin{aligned}
P_{tt} &= -\frac{1}{2} t^2 + \left(m_\chi^2 - 3m_k^2 - \frac{1}{2} s \right) t + \frac{1}{2} m_k^2 s - \frac{1}{2} (m_k^2 + m_\chi^2)^2 + 4m_k^2 (m_\chi^2 - m_k^2), \\
P_{uu} &= -\frac{1}{2} t^2 + \left(m_\chi^2 + 5m_k^2 - \frac{1}{2} s \right) t + \frac{9}{2} m_k^2 s - \frac{1}{2} (m_k^2 + m_\chi^2)^2 - 4m_k^2 (m_\chi^2 + 3m_k^2), \\
P_{ut} &= + \; t^2 + \left[s - 2(m_k^2 + m_\chi^2) \right] t + 3m_k^2 s + (m_k^2 + m_\chi^2)^2 - 16 m_k^4.
\end{aligned}
$$

The previous polynomials, as well as the polynomial P [see equation (B.121)], are functions that only depend on the Mandelstam variables t and s.

Next, we will proceed to calculate the total cross section.

D.4.2 Cross section for the process $\bar{\nu}\nu \to \mathcal{X}\mathcal{X}$

As explained in appendix B.8, the differential cross-section for two identical outgoing particles [see Eq. (B.149)], calculated in the center of mass frame (CM) is given by

$$d\sigma_{\bar{\nu}\nu\to\mathcal{X}\mathcal{X}} = \frac{|\widetilde{\mathcal{M}}|^2_{\bar{\nu}\nu\to\mathcal{X}\mathcal{X}}}{8\pi\, s\, v_{rel}(s)} \frac{dt}{\sqrt{\lambda(s, m_k^2)}}, \tag{D.26}$$

where s is a Mandelstam variable, which in the CM becomes

$$s = 4E^2, \quad E = E_1 = E_2, \tag{D.27}$$

wherein E_i is the energy of each incoming particle, $\lambda(s, m_k^2)$ is the Mandelstam triangular function given in (B.127), which, evaluated in $m_k = m_1 = m_2$, becomes,

$$\lambda(s, m_k^2) = s(s - 4m_k^2), \tag{D.28}$$

and v_{rel} is the relative velocity between the particles [see appendix B.7.5].

The total cross-section, accordingly to equation (B.150), is given by

$$\sigma = \frac{1}{\Sigma} \int d\sigma,$$

where Σ is the symmetry factor [see Eq. (B.151)], which for two identical outgoing particles yields, $\Sigma = 2$.

By inserting (D.24) into (D.26) and by integrating over the polynomial (D.25), the total cross-section becomes

$$\sigma_{\bar{\nu}\nu\to\mathcal{X}\mathcal{X}} = \frac{1}{8\pi\, s\, v_r(s)} \frac{1}{\sqrt{\lambda(s, m_k^2)}} \frac{a_c^4}{256} \int_{t_{in}}^{t_{fin}} P(t, s)\; dt. \tag{D.29}$$

The integration limits of the previous are calculated by means of equation (B.119), in order to write them we insert (D.27) into (B.126) to get

$$|\mathbf{p}_1|_{CM} = \sqrt{E^2 - m_k^2}. \tag{D.30}$$

Notice that, because the outgoing particles have equal masses, the equation (D.27) is also fulfilled for its primed version, such that

$$s = 4E'^2, \quad \to \quad E' = E,$$

with this into the primed version of (B.126) it is obtained

$$|\mathbf{p}'_1|_{CM} = \sqrt{E^2 - m_{\mathcal{X}}^2}. \tag{D.31}$$

By substitution of (D.30) and (D.31) into equation (B.119), we arrive to

$$t = m_k^2 + m_{\mathcal{X}}^2 - 2E^2 + 2\sqrt{(E^2 - m_k^2)(E^2 - m_{\mathcal{X}}^2)}\cos\theta,$$

where θ is the angle formed by $\mathbf{p}_1$ and $\mathbf{p}'_1$. From here, the integration limits appearing in (D.29) are easily written as

$$\begin{aligned} t_{min} &= m_k^2 + m_{\mathcal{X}}^2 - 2E^2 - 2\sqrt{(E^2 - m_k^2)(E^2 - m_{\mathcal{X}}^2)} \\ t_{max} &= m_k^2 + m_{\mathcal{X}}^2 - 2E^2 + 2\sqrt{(E^2 - m_k^2)(E^2 - m_{\mathcal{X}}^2)}. \end{aligned} \tag{D.32}$$

After integration, the total cross-section (D.29) in the CM yields

$$\sigma_{\bar{\nu}\nu\to\mathcal{X}\mathcal{X}} = \frac{1}{2048\pi}\frac{a_c^4}{s\, v_r(s)\sqrt{\lambda(s, m_k^2)}}T(s),$$

where

$$\begin{aligned} T(s) = &\left\{\frac{16m_k^2(s/4 - 4m_k^2 + m_{\mathcal{X}}^2)}{4m_k^2(s/4 - m_{\mathcal{X}}^2) + m_{\mathcal{X}}^4} - 12\right\}\sqrt{(s/4 - m_k^2)(s/4 - m_{\mathcal{X}}^2)} \\ &+ \left\{2(s/2 + 4m_k^2 - m_{\mathcal{X}}^2) + \frac{16m_k^2(s/4 - m_k^2) + m_{\mathcal{X}}^4}{s/2 - m_{\mathcal{X}}^2}\right\} \\ &\times \log\frac{s/2 - m_{\mathcal{X}}^2 + 2\sqrt{(s/4 - m_k^2)(s/4 - m_{\mathcal{X}}^2)}}{s/2 - m_{\mathcal{X}}^2 - 2\sqrt{(s/4 - m_k^2)(s/4 - m_{\mathcal{X}}^2)}}. \end{aligned}$$

Because of the $\mathcal{X}$-particles are ultra-relativistic, their masses can be neglected respect to s and m_k, leading to

$$T(s) \to F(s),$$

with

$$\begin{aligned} F(s) = &\left[s + 16m_k^2\left(1 - \frac{2m_k^2}{s}\right)\right]\log\left[\frac{s + \sqrt{\lambda(s, m_k^2)}}{s - \sqrt{\lambda(s, m_k^2)}}\right] \\ &- 2\left(1 + \frac{8m_k^2}{s}\right)\sqrt{\lambda(s, m_k^2)}. \end{aligned} \tag{D.33}$$

Finally, the total cross-section is given by

$$\sigma_{\mathcal{X}} \equiv \sigma_{\bar{\nu}\nu\to\mathcal{X}\mathcal{X}} = \frac{1}{2048\pi}\frac{a_c^4}{s\, v_r(s)\sqrt{\lambda(s, m_k^2)}}F(s). \tag{D.34}$$

D.4.3 Thermally averaged cross-section for the process $\bar{\nu}\nu \to \mathcal{X}\mathcal{X}$

In this section we calculate the TACS for the process $\bar{\nu}\nu \to \mathcal{X}\mathcal{X}$ by means of the methods explained in appendix E.2, which in turn is based on [110, 111].

Firstly, by using the equation (D.27) into (D.34), we define the function

$$W_{\mathcal{X}}(s) \equiv E_1 E_2 v_r \sigma_{\mathcal{X}} = \frac{1}{8192\pi} \frac{a_c^4}{\sqrt{\lambda(s, m_k^2)}} F(s), \tag{D.35}$$

notice that, this coincides with the definition (E.19), so that, we can substitute it into the definition (E.21), to get

$$\langle \sigma_{\mathcal{X}} v_r \rangle = \frac{a_c^4}{4 \times 8192\pi m_k^4 T K_2^2(m_k/T)} \int_{4m_k^2}^{\infty} ds \frac{F(s)}{\sqrt{s}} K_1(\sqrt{s}/T),$$

where we have used the equation (D.28). After the change of integration variable

$$s \to 4m_k^2/x, \qquad F(s) \to 4m_k^2 g_{\mathcal{X}}(x), \tag{D.36}$$

where

$$g_{\mathcal{X}}(x) = \left(\frac{1}{x} + 4 - 2x\right) \log \frac{1 + \sqrt{1-x}}{1 - \sqrt{1-x}} - 2\left(\frac{1}{x} + 2\right)\sqrt{1-x},$$

whe arrive to

$$\langle \sigma_{\mathcal{X}} v_r \rangle = \frac{a_c^4}{4096\, \pi m_k T\, K_2^2(m_k/T)} \mathcal{I}_{\mathcal{X}}(m_k; T), \tag{D.37}$$

where K_2 is the modified Bessel function of the second kind of order 2, and where we have defined the integral

$$\mathcal{I}_{\mathcal{X}}(m_k; T) \equiv \int_0^1 dx\, \frac{g_{\mathcal{X}}(x)}{x\sqrt{x}} K_1\left(\frac{2m_k}{T\sqrt{x}}\right), \tag{D.38}$$

with K_1 the modified Bessel function of the second kind of order 1.

D.5 Calculations related to the process $\bar{\nu}\nu \to h^0 h^{0\dagger}$

In this section, we perform the calculations needed for write the thermally averaged cross-section for the process

$$\bar{\nu}_i + \nu_i \longrightarrow h^0 + h^{0\dagger}, \tag{D.39}$$

We start by calculating the corresponding annihilation amplitude.

D.5.1 Annihilation amplitude for the process $\bar{\nu}\nu \to h^0 h^{0\dagger}$

In this section we calculate the annihilation amplitude for the process given in Eq. (D.39), which is allowed by the Lagrangian (4.6). Since the outgoing particles are distinguishable, there is only one annihilation channel allowed which is characterized by the Mandelstam variable t, as shown in Fig. 4.2.

In order to calcutate the annihilation amplitude

$$|\widetilde{\mathcal{N}}|_{\bar{\nu}\nu\to h^0 h^{0\dagger}},$$

we consider all of the Yukawa couplings to be about the same order, namely $y^{ni} \sim y$, they are accompanied by the chiral projectors, such that after applying the Feynman rules, it is obtained

$$i\mathcal{N} = \bar{v}_{s_2}(\mathbf{p}_2)(iyP_L))(-i)\frac{-\not{p}_1 + \not{k}'_1 + m_\psi}{-t + m_\psi}(iyP_R)u_{s_1}(\mathbf{p}_1),$$

which is the same as

$$i\mathcal{N} = y^2 \bar{v}_{s_2}(\mathbf{p}_2) A u_{s_1}(\mathbf{p}_1),$$

with

$$A = P_L \frac{-\not{p}_1 + \not{k}'_1}{-t} P_R = P_L \frac{-\not{p}_1 + \not{k}'_1}{-t},$$

wherein, we have neglected the neutrino mass m_ψ because this process happens on the interaction basis, and where we have use the properties of the chiral projectors. As before, notice that $\bar{A} = A$, then we can write

$$|\mathcal{N}|^2 = y^4 \bar{v}_{s_2}(\mathbf{p}_2) A u_{s_1}(\mathbf{p}_1) \bar{u}_{s_1}(\mathbf{p}_1) A v_{s_2}(\mathbf{p}_2),$$

then,

$$|\mathcal{N}|^2 = y^4 Tr\left\{v_{s_2}(\mathbf{p}_2)\bar{v}_{s_2}(\mathbf{p}_2)A u_{s_1}(\mathbf{p}_1)\bar{u}_{s_1}(\mathbf{p}_1)A\right\},$$

by averaging over the spins we get

$$|\widetilde{\mathcal{N}}|^2 = \frac{y^4}{\Sigma} Tr\left\{(-\not{p}_2 - m_k)A(-\not{p}_1 + m_k)A\right\},$$

where $\Sigma = (2s_2+1)(2s_1+1) = 4$. Remember that $\not{p}u_s(\mathbf{p}) = -m u_s(\mathbf{p})$, such that, we can write

$$A = P_L \frac{m_k + \not{k}_1'}{-t},$$

then we have

$$|\widetilde{\mathcal{N}}|^2 = \frac{y^4}{4t^2} Tr\left\{\left[P_R(-\not{p}_2 m_k - \not{p}_2\not{k}_1') + P_L(-m_k^2 - m_k\not{k}_1')\right] \times \left[P_R(-\not{p}_1 m_k - \not{p}_1\not{k}_1') + P_L(m_k^2 + m_k\not{k}_1')\right]\right\},$$

after expanding and making use of the properties of the gamma matrices products, it yields

$$|\widetilde{\mathcal{N}}|^2 = \frac{y^4}{4t^2} Tr\left\{-P_R(2m_k^2\not{p}_2\not{k}_1') + P_L(m_k^2\not{k}_1'\not{p}_1 - m_k^4)\right\},$$

after taking the trace, by taking into account the rules for the Mandelstam variables given in equations (B.120) and (B.121), and by considering $m_k \gg m_H$, we arrive to

$$|\widetilde{\mathcal{N}}|^2 = \frac{y^4}{4}\frac{m_k^2}{t^2}(m_k^2 - 3t - 2s). \tag{D.40}$$

With this, we can proceed to calculate the total cross section for this process.

D.5.2 Cross section for the process $\bar{\nu}\nu \to h^0 h^{0\dagger}$

Analogously as what it was done in section D.4.2, the total cross section for this annihilation channel is calculated by means of

$$\sigma_H = \frac{3}{8\pi\, s\, v_r\sqrt{\lambda(s, m_k^2)}}\int_{t_1}^{t_+} |\widetilde{\mathcal{N}}|^2 dt,$$

where the factor of 3 in the numerator is introduced because there are three ways to accommodate the vertex given in (4.6) to achieve the annihilation process showed in (4.8). As before, $v_r(s)$ the relative velocity between the incomming particles depending on the Mandelstan variable s.

By substituting of equation (D.40) into the previous, and after integration, it is obtained

$$\sigma_{H} = \frac{3}{32\pi}\frac{1}{s v_r(s)}\frac{y^4}{\sqrt{\lambda(s, m_k^2)}} T_{H}(s), \tag{D.41}$$

where

$$T_{H}(s) = \frac{m_k^2 - 2s}{m_k^2}\sqrt{\lambda(s, m_k^2)} + 3m_k^2 \log\frac{t_-}{t_+}, \tag{D.42}$$

with

$$\frac{t_-}{t_+} = \frac{(s - 2m_k^2) + \sqrt{\lambda(s, m_k^2)}}{(s - 2m_k^2) - \sqrt{\lambda(s, m_k^2)}}.$$

D.5.3 Thermally averaged cross-section for the process $\bar{\nu}\nu \to h^0 h^{0\dagger}$

By substituting the equation (D.27) into (D.41), it is easy defining

$$W_{H}(s) \equiv E_1 E_2 v_r \sigma_{H} = \frac{3}{128\pi}\frac{y^4}{\sqrt{\lambda(s, m_k^2)}} T_{H}(s),$$

after replacing this into (E.21) it is obtained

$$\langle \sigma_{H} v_r \rangle = \frac{3y^4}{4 \times 128\pi m_k^4 T K_2^2\left(m_k/T\right)} \int_{4m_k^2}^{\infty} \frac{ds}{\sqrt{s}} T_{H}(s) K_1\left(\sqrt{s}/T\right).$$

As before, by switching the integration variable

$$s \to 4m_k^2/x, \qquad T_{H}(s) \to 4m_k^2 g_{H}(x),$$

we arrive to

$$\langle \sigma_{H} v_r \rangle = \frac{3}{64\pi}\frac{y^4}{m_k T K_2^2\left(m_k/T\right)} \mathcal{I}_{H}(m_k; T), \tag{D.43}$$

where

$$I_{H}(m_k; T) = \int_0^1 dx \frac{g_{H}(x)}{x\sqrt{x}} K_1\left(\frac{2m_k}{T\sqrt{x}}\right), \tag{D.44}$$

with

$$g_{H}(x) = \frac{2 - x}{x}\sqrt{1 - x} + \frac{3}{4}\log\frac{1 - \frac{x}{2} + \sqrt{1 - x}}{1 - \frac{x}{2} - \sqrt{1 - x}}. \tag{D.45}$$

D.6 Process of disintegration of heavy neutrinos into Higgs and leptons

Finally, we calculate the decay width for the decay process allowed by Eq. (4.6) as given in Eq. (4.7). It is the simplest, as before, after applying the Feyman rules we get

$$i\mathcal{T} = \bar{u}_{s'}(\mathbf{p}')(iy)u_s(\mathbf{p}) \quad \longrightarrow \quad |\mathcal{T}|^2 = \bar{y}^2 u_{s'}(\mathbf{p}')u_s(\mathbf{p})\bar{u}_s(\mathbf{p})\bar{u}_{s'}(\mathbf{p}'),$$

which leads to

$$\begin{aligned} |\mathcal{T}|^2 &= y^2 Tr\left[u_{s'}(\mathbf{p}')\bar{u}_{s'}(\mathbf{p}')u_s(\mathbf{p})\bar{u}_s(\mathbf{p})\right] = \frac{y^2}{\Sigma} Tr\left[(-\not{p}' + m_n)(-\not{p} + m_k)\right], \\ &= \frac{y^2}{2} Tr\left[\not{p}'\not{p} + m_n m_k\right] = 2y^2(-p'^{\mu}p_{\mu} + m_n m_k), \end{aligned}$$

next, by using $p'^{\mu}p_{\mu} = p'^{\mu}k'_{\mu} - m_n^2$, where $p'^{\mu}k'_{\mu} = \frac{1}{2}(m_n^2 + m_H^2 - s)$, we arrive to

$$|\mathcal{T}|^2 = y^2(2m_n m_K + m_n^2 - m_H^2 + s),$$

provided that $m_k \gg m_H \gg m_n$, we finally get

$$|\mathcal{T}|^2 = y^2 m_k^2.$$

With this amplitude, we perform the integration,

$$\Gamma_d = \frac{1}{64\pi^2 m_k^2} \int |\mathbf{k}'|_{CM} |\mathcal{T}|^2 d\Omega,$$

with

$$|\mathbf{k}'|_{CM}^2 = \frac{1}{4s}\left[s^2 + (m_H^2 - m_n^2)^2 - 2s(m_H^2 + m_n^2)\right] \approx \frac{1}{4} m_k^2,$$

where we have used $s = m_k^2$.

Finally, by taking into acccount the six similar processes of disintegration of heavy neutrino, the total decay width becomes

$$\Gamma_d = \frac{3}{32\pi} y^2 m_k \ . \tag{D.46}$$

D.7 The background dynamics

D.7.1 Numerical evolution of the backgroun dynamics

In order to evolve numerically the background system given in Eqs. (4.98), we use the well known change of variables as follows,

$$q = \frac{\sqrt{V(\mathcal{X})}}{\sqrt{3}M_{pl}\mathsf{H}}, \tag{D.47}$$

$$f = \frac{\sqrt{\rho_{DM}}}{\sqrt{3}M_{pl}\mathsf{H}}, \tag{D.48}$$

$$b = \frac{\sqrt{\rho_b}}{\sqrt{3}M_{pl}\mathsf{H}}, \tag{D.49}$$

$$z = \frac{\sqrt{\rho_\gamma}}{\sqrt{3}M_{pl}\mathsf{H}}, \tag{D.50}$$

$$v = \frac{\sqrt{\rho_\nu}}{\sqrt{3}M_{pl}\mathsf{H}}, \tag{D.51}$$

$$x = \frac{y_1}{\sqrt{3}M_{pl}\mathsf{H}}, \tag{D.52}$$

$$r = \frac{y_0}{\sqrt{3}M_{pl}}, \tag{D.53}$$

$$\Pi = -\frac{2}{3}\frac{\dot{\mathsf{H}}}{\mathsf{H}^2}, \tag{D.54}$$

$$P = \frac{m}{\mathsf{H}}. \tag{D.55}$$

The dynamic system takes the following form (the evolution is done in the e-folding N),

$$F = f^2 + b^2 + z^2 + v^2 + q^2 = 1, \tag{D.56}$$

$$\Pi = f^2 + b^2 + \frac{4}{3}z^2 + \frac{4}{3}v^2, \tag{D.57}$$

$$\frac{dq}{dN} = \frac{3}{2}\Pi q - \frac{2}{3}\frac{p^2 q}{(3\Pi + 2)}, \tag{D.58}$$

$$\frac{df}{dN} = \frac{3}{2}(\Pi - 1)f, \tag{D.59}$$

$$\frac{db}{dN} = \frac{3}{2}(\Pi - 1)b, \tag{D.60}$$

$$\frac{dz}{dN} = \frac{3}{2}(\Pi - \frac{4}{3})z, \tag{D.61}$$

$$\frac{dv}{dN} = \frac{3}{2}(\Pi - \frac{4}{3})v, \tag{D.62}$$

$$\frac{dx}{dN} = \frac{3}{2}(\Pi - 2)x, \tag{D.63}$$

$$\frac{dr}{dN} = x, \tag{D.64}$$

$$\frac{dP}{dN} = \frac{3}{2}\Pi P. \tag{D.65}$$

By using the equation (C.84) together with (D.47), the kinetic energy density of the field $\mathfrak{X}$ can be written as

$$\frac{1}{2}\dot{\mathfrak{X}} = \frac{1}{3}A_v \frac{q^2}{(\beta + 1)^2}, \tag{D.66}$$

where

$$A_v = 3M_{pl}^2 m^2. \tag{D.67}$$

Similarly, by using (C.89), (D.52) and (D.55), for the phase field ϑ, we can write

$$\frac{1}{2}\dot{\vartheta} = \frac{1}{2}A_v \frac{x^2}{P^2}. \tag{D.68}$$

As for the potential, it can be writen by means of equations (D.47) and (D.55) as

$$V = A_v \frac{q^2}{P^2}. \tag{D.69}$$

Next, we use the potential (C.69) to write the term

$$\frac{1}{2}\left(1+\frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2 = \frac{V}{m^2\langle \mathrm{Q}\rangle^2},$$

then, by assuming $\langle \mathrm{Q}\rangle \sim M_{pl}$ and by virtue of equations (D.67) and (D.47), it becomes

$$\frac{1}{2}\left(1+\frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2 = 9\frac{M_{pl}^2\mathsf{H}^2}{A_v}q^2,$$

this last together with equations (D.66) and (D.68) allows writing the equation (4.97) as

$$\frac{1}{9M_{pl}^2\mathsf{H}^2}\frac{A_v}{(\beta+1)^2} + 3\frac{x^2}{P^2} \ll 1,$$

and by substitution of (D.55) and (D.67) into the previous, the first slow roll condition (4.97) becomes

$$\frac{P^2}{(\beta+1)^2} + 9\frac{x^2}{P^2} \ll 1. \tag{D.70}$$

Appendix E

The Boltzmann Equation and the Termally Averaged Cross Section

E.1 The Boltzmann equation

The main references for this section are [13, 95, 96].

The distribution of momenta etc, in the phase space of a set of particles is described by the distribution function

$$f = f(\mathbf{x}, \mathbf{p}, t) \ ,$$

whose evolutin is driven by the *Boltzmann equation*, which is definded as

$$\hat{L}[f] = C[f] \ , \tag{E.1}$$

where

$$\hat{L} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\alpha p^\gamma \frac{\partial}{\partial p^\alpha} \ ,$$

is the generalized, covariant, relativistic Liouville operator, and $C[f]$ is the *collision term.*

In an FLRW Universe, the only non-zero Christoeffel symbols are

$$\Gamma^0_{ii} = -\frac{1}{2} g^{00} g_{ii,0}, \qquad \Gamma^i_{0i} = \frac{1}{2} g^{ii} g_{ii,0}, \qquad \Gamma^i_{kj} = \frac{1}{2} g^{ii} (g_{ki,j} + g_{ji,k} - g_{kj,i}) \ ,$$

then, the Liouvile operator becomes

$$\hat{L} = p^0 \frac{\partial}{\partial x^0} + p^i \frac{\partial}{\partial x^i} - \Gamma^0_{ii} p^i p^i \frac{\partial}{\partial p^0} - \Gamma^i_{0i} p^0 p^i \frac{\partial}{\partial p^i} - \Gamma^i_{kj} p^k p^j \frac{\partial}{\partial p^j} \ ,$$

but in turn, in an FLRW Universe, because of the homogeneity, the distribution function does not depend on $\mathbf{x}$, and because of the isotropy neither depends on $\mathbf{p}$ but only on its magnitude $|\mathbf{p}|$, which can be swithched by the energy $E = p^0$, such that

$$f = f(E, t) \ ,$$

thus, the left hand side of the Boltzmann equation becomes simply

$$\hat{L}[f] = p^0 \frac{\partial f}{\partial x^0} - \Gamma^0_{ii} p^i p^i \frac{\partial f}{\partial p^0} \ .$$

Notice that the spatial part of the FLRW metric can be written as

$$g_{ij} = a^2(t)\gamma_{ij} \ ,$$

such that

$$g_{ii,0} = 2a\dot{a}\gamma_{ii} = 2\frac{\dot{a}}{a} g_{ii} \quad \longrightarrow \quad \Gamma^0_{ii} = \frac{\dot{a}}{a} g_{ii} \ ,$$

then the Boltzman equation (E.1) becomes

$$\frac{\partial f}{\partial t} - \frac{\dot{a}}{a} \frac{|\mathbf{p}|^2}{E} \frac{\partial f}{\partial E} = \frac{C[f]}{E} \ .$$

On the other hand, we know that the particle number density in the real space is given by

$$n(t) = \frac{g}{(2\pi)^3} \int f d^3p \ , \tag{E.2}$$

thus we can write

$$\frac{g}{(2\pi)^3} \frac{d}{dt} \int f d^3p - \frac{\dot{a}}{a} \frac{g}{(2\pi)^3} \int \frac{|\mathbf{p}|^2}{E} \frac{\partial f}{\partial E} d^3p = \frac{g}{(2\pi)^3} \int d^3p \frac{C[f]}{E} \ . \tag{E.3}$$

Notice that, the factor appearing in the second term in the LHS of the previous equation yields

$$\frac{g}{(2\pi)^3} \int \frac{|\mathbf{p}|^2}{E} \frac{\partial f}{\partial E} d^3p = -3n(t) \ , \tag{E.4}$$

for illustrative purposes let us show it simplified by considering $\frac{\partial f}{\partial E} = \frac{df}{dE}$ (for more rigorous demonstrations see [13, 95, 96]), then by using $EdE = |\mathbf{p}|d|\mathbf{p}|$ we have

$$\frac{g}{(2\pi)^3}\int \frac{|\mathbf{p}|^2}{E}\frac{\partial f}{\partial E}d^3p = 4\pi\frac{g}{(2\pi)^3}\int \frac{|\mathbf{p}|^4}{|\mathbf{p}|}\frac{df}{d|\mathbf{p}|}d|\mathbf{p}| = 4\pi\frac{g}{(2\pi)^3}\int |\mathbf{p}|^3 df \ ,$$

after integrating by parts we are left with

$$4\pi\frac{g}{(2\pi)^3}\int |\mathbf{p}|^3 df = 4\pi\frac{g}{(2\pi)^3}|\mathbf{p}|^3 f - 3\frac{g}{(2\pi)^3}\int f d^3p \ ,$$

the first term on the RHS member becomes like a boundary term, with the distribution function asymptotically null, this yields to the equation (E.4).

Then, by using the previous result together with (E.2) into (E.3) we left with

$$\frac{dn}{dt} + 3\frac{\dot{a}}{a}n = \frac{g}{(2\pi)^3}\int d^3p\frac{C[f]}{E} \ . \tag{E.5}$$

If we are focus on the study of the specie ψ, we most write the Boltzmann equation as

$$\frac{dn_\psi}{dt} + 3\frac{\dot{a}}{a}n_\psi = \mathrm{CT}_\psi \ , \tag{E.6}$$

where the collision term (CT_ψ) is given by

$$\mathrm{CT}_\psi \equiv \frac{g}{(2\pi)^3}\int C[f]\frac{d^3p_\psi}{E_\psi} \ .$$

By considering the process in which a set of particles involving the specie ψ, namely, $\psi, a, b, \ldots$ transform into a set $i, j, \ldots$,

$$\psi + a + b + \ldots \longleftrightarrow i + j + \ldots$$

then, the most general collision term is writen as

$$\begin{aligned}\mathrm{CT}_\psi = -\int & d\Pi_\psi d\Pi_a d\Pi_b \ldots d\Pi_i d\Pi_j \ldots \\ & \times (2\pi)^4\delta^4(p_\psi + p_a + p_b \ldots - p_i - p_j \ldots) \\ & \times \big[f_a f_b \ldots f_\psi (1 \pm f_i)(1 \pm f_j)|M|^2_{\psi+a+b+\ldots\to i+j\ldots} \\ & \quad - f_i f_j (1 \pm f_a)(1 \pm f_b)\ldots(1 \pm f_\psi)|M|^2_{i+j+\ldots\to\psi+a+b+\ldots}\big] \ ,\end{aligned}$$

where,

$$d\Pi_i = \frac{g_i}{(2\pi)^3}\frac{d^3p_i}{2E_i} \ , \tag{E.7}$$

and where the 4-dimensional delta Dirac function guarantees the energy-momentum conservation, the $|M|^2$ are the corresponding squared transition aplitudes, and f_i the phase space densities of the i-specie. The $1 \pm f$ terms, with $+$ for bosons and $-$ for fermions, accounts for the effect of Bose enhancement and Pauli blocking.

The CT can be greatly simplified by neglecting the quantum statistical effects, namely, by assuming that the distribution functions are Maxwell-Boltzman instead of Fermi-Dirac of Bose-Einstein, this approximation can be done by considerig that all three distributions are similar for values of the momentum near the maximun of the distributions, thus this simplification does not alter significative changes, so the $1 \pm f$ terms can be ignored [13].

Let us now focus on annihilation proccesses in which two incomming particles interact to produce two new particles,

$$1 + 2 \longleftrightarrow 3 + 4 \ ,$$

then, the CT becomes

$$\begin{aligned}\mathrm{CT}_1 = -\int & d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 \times (2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4) \\ & \times \left[f_1 f_2 |M|^2_{1+2\rightarrow 3+4} - f_3 f_4 |M|^2_{3+4\rightarrow 1+2}\right] \ ,\end{aligned}$$

which is the same that

$$\begin{aligned}\mathrm{CT}_1 = &-\int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 \times (2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4) f_1 f_2 |M|^2_{1+2\rightarrow 3+4} \\ &+ \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 \times (2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4) f_3 f_4 |M|^2_{3+4\rightarrow 1+2} \ .\end{aligned} \tag{E.8}$$

The distribution functions in the Maxwell-Boltzmann statistic are

$$f_i = e^{\mu_i/T} e^{-E_i/T} \ ,$$

where μ_i is the chemical potential for the specie i. The relation between the chemical potential and the number density is given by [96]

$$n_i = e^{\mu_i/T} g_i \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T} = e^{\mu_i/T} n_i^{\rm eq} \ ,$$

where $n_i^{\rm eq}$ is the number density of the specie i in thermal equilibrium

$$n_i^{\rm eq} = g_i \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T} \ . \tag{E.9}$$

Thus, we can write

$$f_1 f_2 = \frac{n_1 n_2}{n_1^{\rm eq} n_2^{\rm eq}} e^{-(E_1+E_2)/T}, \qquad \text{and} \qquad f_3 f_4 = \frac{n_3 n_4}{n_3^{\rm eq} n_4^{\rm eq}} e^{-(E_1+E_2)/T} \ , \tag{E.10}$$

where we have used the fact that energy is conserved in the collision, $E_1 + E_2 = E_3 + E_4$. By substitution of (E.10) into (E.8) we arrive to

$$\mathrm{CT}_1 = -\langle \sigma_{12\to 23} v_r \rangle n_1 n_2 + \langle \sigma_{34\to 12} v_r \rangle n_3 n_4 \ , \tag{E.11}$$

where it has been defined the termally averaged cross section (TACS) for the process $1 + 2 \longleftrightarrow 3 + 4$ as

$$\langle \sigma_{12\to 34} v_r \rangle = \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 \\ \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |M|^2_{1+2\to 3+4} \frac{e^{-(E_1+E_2)/T}}{n_1^{\rm eq} n_2^{\rm eq}} \ ,$$

and similarly for the proccess $3 + 4 \longleftrightarrow 1 + 2$

$$\langle \sigma_{34\to 12} v_r \rangle = \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 \\ \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |M|^2_{3+4\to 1+2} \frac{e^{-(E_1+E_2)/T}}{n_3^{\rm eq} n_4^{\rm eq}} \ .$$

In the previous equations σ is the total cross section (see section B.8), and v_r is the relative velocity between the incomming particles 1 and 2, further explained in appendix B.7.5.

Finally, by substitution of (E.11) into (E.6), the standard, simple form, of the Boltzmann equation for the study of the specie 1 can be written as

$$\frac{dn_1}{dt} + 3\frac{\dot{a}}{a}n_1 = -\alpha n_1 n_2 + \beta n_3 n_4 \ , \tag{E.12}$$

where

$$\alpha = \langle \sigma_{12\to 34} v_r \rangle \quad \text{and} \quad \beta = \langle \sigma_{34\to 12} v_r \rangle \ , \tag{E.13}$$

therefore, the α-term accounts for the decrease on the number density of the specie 1, due to its annihilation and the β-term accounts for it increase due to the inverse proccess.

E.2 Termally Averaged Cross Section

Let us rewrite the termally averaged cross section (TACS), for the proccess $1 + 2 \longrightarrow 3 + 4$, as it was defined in the previous section, it is

$$\begin{aligned}\langle \sigma_{12\to 34} v_r \rangle = &\int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 \\ &\times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)|M|^2_{1+2\to 3+4} \frac{e^{-(E_1+E_2)/T}}{n_1^{\rm eq} n_2^{\rm eq}} \ ,\end{aligned} \tag{E.14}$$

by using the equation (E.7) together with the first of the equations (E.10), it becomes

$$\begin{aligned}\langle \sigma_{12\to 34} v_r \rangle = &\int \frac{g_1}{(2\pi^3)}\frac{d^3p_1}{2E_1}\frac{g_2}{(2\pi^3)}\frac{d^3p_2}{2E_2}\frac{g_3}{(2\pi^3)}\frac{d^3p_3}{2E_3}\frac{g_4}{(2\pi^3)}\frac{d^3p_4}{2E_4} \\ &\times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)|M|^2_{1+2\to 3+4} \frac{f_1 f_2}{n_1 n_2} \ ,\end{aligned} \tag{E.15}$$

which is the same as

$$\begin{aligned}\langle \sigma_{12\to 34} v_r \rangle = &\frac{g_1 g_2}{(2\pi)^3 (2\pi)^3} \int d^3p_1 \int d^3p_2 \times \frac{f_1 f_2}{n_1 n_2} \\ &\times g_3 g_4 \int \frac{d^3p_3}{(2\pi)^3} \int \frac{d^3p_4}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)|M|^2_{1+2\to 3+4}}{2E_1 2E_2 2E_3 2E_4} \ ,\end{aligned}$$

next, in virtue of equation (B.142) the second line in the last equation equals the cross section ($\sigma \equiv \sigma_{12\to 34}$) times the relative velocity between the incoming particles, therefore, with this and the equation (E.2) the previous equation becomes

$$\langle \sigma v_r \rangle = \frac{\int d^3p_1 d^3p_2 \; f_1(E_1) f_2(E_2) \; \sigma v_r}{\int d^3p_1 d^3p_2 \; f_1(E_1) f_2(E_2)} \; , \tag{E.16}$$

which is the most common expression used for the integration the TACS.

In the next section, we proceeded to perform this integral.

E.2.1 Integrating the TACS

The main reference for this section is Ref. [110, 111].

As explainded above, it is possible to make use of distribucions de Maxwell-Boltzmann, in this approximation

$$f(E) \approx e^{-(E-\mu)/T} \; .$$

Furthermore, when the particle 2 is the anti particle of the particle 1 it turns out that $\mu_2 = -\mu_1$, (or when the particle is its own anti particle $\mu = 0$), then:

$$f(E_1)f(E_2) = e^{-(E_1+E_2)/T} \; .$$

Denominator

We will now procced to integrate the denominator of (E.16), for short we will call it DEN, it is:

$$\begin{aligned} \mathrm{DEN} &= \int d^3p_1 d^3p_2 f(E_1) f(E_2) = \int d^3p_1 e^{-E_1/T} \int d^3p_2 e^{-E_2/T} \\ &= \left(\int d^3p \, e^{-E/T} \right)^2 \; . \end{aligned}$$

The integral:

$$\int d^3p \, e^{-E/T} = 4\pi \int_0^\infty d|\mathbf{p}||\mathbf{p}|^2 \, e^{-E/T} \; ,$$

with

$$E^2 = m^2 + |\mathbf{p}|^2 \qquad \to \qquad EdE = |\mathbf{p}|d|\mathbf{p}|, \quad \text{when} \qquad |\mathbf{p}| = 0 \to E = m \; ,$$

$$\int d^3p\, e^{-E/T} = 4\pi \int_m^\infty dE\, E\sqrt{E^2 - m^2}\; e^{-E/T} \; .$$

By shwitching the variable

$$y = \frac{E}{m} \qquad dy = \frac{dE}{m} \qquad E = m \to y = 1 \; ,$$

it yields

$$\int d^3p\, e^{-E/T} = 4\pi m^3 \int_1^\infty dy\, y\sqrt{y^2 - 1} e^{-my/T} \; ,$$

after integration by parts with $u = e^{-my/T}$ y $dv = y\sqrt{y^2 - 1}dy$ (the part of uv nulls), thererfore

$$\begin{aligned}\int d^3p\, e^{-E/T} &= \frac{4\pi}{3}\frac{m^4}{T}\int_1^\infty dy(y^2 - 1)^{3/2} e^{-my/T} \\ &= 4\pi m^2 T \left(\frac{m^2}{3T^2}\int_1^\infty dy(y^2 - 1)^{3/2} e^{-my/T}\right) \; .\end{aligned}$$

The part in parenthesis can be rewritten by means of the definition of the modificated Bessel function of order 2, whis is given by

$$K_2(z) = \frac{z^2}{3}\int_1^\infty dt\, (t^2 - 1)^{3/2} e^{-zt} \; ,$$

therefore

$$\int d^3p\, e^{-E/T} = 4\pi m^2 T K_2(m/T) \; .$$

Consequently, the denominator of (E.16) becomes:

$$\mathrm{DEN} = \left[4\pi m^2 T K_2(m/T)\right]^2 \; . \tag{E.17}$$

Notice that the quantity DEN is proportional to the square of the number density of particles in equilibrium [see equation (E.9)], namely

$$\frac{(2\pi)^6}{g_i^2}(n_i)^2_{eq} = \left[4\pi m^2 T K_2(m/T)\right]^2 \; . \tag{E.18}$$

Numerator

We now proceed to integrate the numerator that appears in equation (E.16), for short, from now on, the previous integral will be called NUM.

$$\text{NUM} = \int d^3p_1 d^3p_2 \; f_1(E_1) f_2(E_2) \; \sigma v_r = \int d^3p_1 d^3p_2 e^{-(E_1+E_2)/T} \, \sigma \, v \; .$$

In spherical coordinates

$$\begin{aligned} d^3p_1 d^3p_2 &= d|\mathbf{p}_1| d|\mathbf{p}_2| d\phi_1 d\phi_2 d\cos(\theta_1) d\cos(\theta_2) |\mathbf{p}_1|^2 |\mathbf{p}_2|^2 \\ &= 8\pi^2 |\mathbf{p}_1|^2 |\mathbf{p}_2|^2 d|\mathbf{p}_1| d|\mathbf{p}_2| d\cos(\theta) \; , \end{aligned}$$

whit θ being the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$.

By changing from θ to the Mandesltan variable s [see eq. (B.118)]

$$\begin{aligned} s &= -(p_1 + p_2)^2 \\ &= -(-m_1^2 - m_2^2 + 2[-E_1 E_2 + \mathbf{p}_1 \cdot \mathbf{p}_2]) \\ &= 2m^2 + 2E_1 E_2 - 2|\mathbf{p}_1||\mathbf{p}_2| \cos(\theta) \; , \end{aligned}$$

with $m = m_1 = m_2$. Therefore by taking the derivative (with E fixed, so $|\mathbf{p}|$), it leads to

$$ds = -2|\mathbf{p}_1||\mathbf{p}_2| d\cos(\theta) \; ,$$

then

$$\begin{aligned} d^3p_1 d^3p_2 &= 8\pi^2 d|\mathbf{p}_1| d|\mathbf{p}_2| |\mathbf{p}_1|^2 |\mathbf{p}_2|^2 \frac{ds}{-2|\mathbf{p}_1||\mathbf{p}_2|} \\ &= -4\pi^2 d|\mathbf{p}_1| d|\mathbf{p}_2| |\mathbf{p}_1||\mathbf{p}_2| \; ds \; , \end{aligned}$$

where the integration over both p_1 and p_2 are performed between 0 and ∞, and the limits of integration over s depend on E_1 y E_2. Since

$$E dE = |\mathbf{p}| d|\mathbf{p}| \; ,$$

then

$$d^3p_1 d^3p_2 = -4\pi^2 E_1 E_2 dE_1 dE_2 \; ds \; .$$

With the transformation

$$E_+ = E_1 + E_2 \qquad E_- = E_1 - E_2 \; ,$$

by calculating the jacobian of this transformation it is obtainded

$$dE_1 dE_2 = \frac{1}{2} dE_+ dE_- \ ,$$

and by making use of the definition of E_+

$$\int d^3p_1 d^3p_2 e^{-(E_1+E_2)/T} \sigma v = -2\pi^2 \int dE_+ \int dE_- \int ds \ E_1 E_2 \sigma v \ e^{-E_+/T} \ .$$

The limits of integration are the following

$$-\sqrt{(1-4m^2/s)(E_+^2 - s)} \leqslant E_- \leqslant \sqrt{(1-4m^2/s)(E_+^2 - s)} \ ,$$

$$s > 4m^2 \qquad E_+ > \sqrt{s} \ ,$$

and by defining the function

$$W(s) = E_1 E_2 \sigma v_{rel} \ , \tag{E.19}$$

the numerator becomes

$$\mathrm{NUM} = -2\pi^2 \int dE_+ e^{-E_+/T} \int dE_- \int ds \, W(s) \ .$$

By switching the order of integration it leads to

$$\mathrm{NUM} = 2\pi^2 \int_{4m^2}^{\infty} ds \int_{\sqrt{s}}^{\infty} dE_+ \int_{-\sqrt{(1-4m^2/s)(E_+^2-s)}}^{\sqrt{(1-4m^2/s)(E_+^2-s)}} dE_- \, W(s) e^{-E_+/T} \ .$$

The integral over E_- can be performed to get

$$\mathrm{NUM} = 4\pi^2 \int_{4m^2}^{\infty} ds \, \sqrt{(1-4m^2/s)} \, W(s) \int_{\sqrt{s}}^{\infty} dE_+ \sqrt{(E_+^2 - s)} \, e^{-E_+/T} \ .$$

By invoking the definition of the modified Bessel function of order 1 with real argument,given by

$$K_1(x) = x \int_1^{\infty} dt \, e^{-xt} \sqrt{t^2 - 1} \ ,$$

namely

$$K_1(\sqrt{s}/T) = \frac{\sqrt{s}}{T} \int_1^{\infty} dt \, e^{-\frac{\sqrt{s}}{T} t} \sqrt{t^2 - 1} \ ,$$

next, by substitution of

$$E_+ = \sqrt{s}t \to dE_+ = \sqrt{s}dt \ ,$$

it can be written

$$\int_{\sqrt{s}}^{\infty} dE_+ \sqrt{(E_+^2 - s)}\, e^{-E_+/T} = T\sqrt{s}K_1(\sqrt{s}/T) \ .$$

Therefore, the numerator becomes

$$\mathrm{NUM} = 4\pi^2 T \int_{4m^2}^{\infty} ds\, \sqrt{(s-4m^2)} W(s) K_1(\sqrt{s}/T) \ . \tag{E.20}$$

TACS
Finally, with (E.17) and (E.20), the form that we will use in order to calculate the TACS along this work is given by

$$\langle \sigma v_r \rangle = \frac{4\pi^2 T}{\left[4\pi m^2 T K_2(m/T)\right]^2} \int_{4m^2}^{\infty} ds\, \sqrt{(s-4m^2)} W(s) K_1(\sqrt{s}/T) \ . \tag{E.21}$$

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