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Luis Felipe Zapata Figueroa

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Luis Felipe Zapata Figueroa

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Resumen

La búsqueda de soluciones de Sitter (dS) en compactificaciones de teoría de cuerdas con todos los módulos estabilizados, esto es, sin direcciones planas en el potencial han sido llevadas a cabo en el contexto del modelo KKLT dirigiendo el problema de la constante cosmológica sugerida por la fuerte evidencia de un universo en expansión acelerada. Además esto es importante debido a algunas conjeturas motivadas por teoremas de imposibilidad que imponen restricciones a la existencia de soluciones dS en supergravedad de teoría de cuerdas.

En esta tesis estudiamos una variante del paradigma principal para la construcción de vacío de Sitter metaestable de soluciones Anti de Sitter (AdS) supersimétricas obtenidas de teoría de supergravedad de bajas energías cuatro-dimensional con todos los módulos establizados basado en introducir anti-D3-branas proporcionando una cantidad de energía positiva a la energía potencial, este proceso es conocido como levantamiento del mínimo ya que eso levanta el vacío a un mínimo de vacío dS. En cambio consideramos la posibilidad de D-branas envolviendo ciclos torsionales de una variedad mitad plana y estudiamos sus consecuencias en el vacío AdS. Mostramos que para estas branas torsionales el vacío permanece en un mínimo AdS con todos los módulos estabilizados y también calculamos la contribución a la energía potencial viniendo de estas branas torsionales.

Abstract

The search for de Sitter (dS) solutions in string theory compactifications with all stabilized moduli i.e., no flat direction of the potential energy has been carried out in the context of KKLT model addressing the problem of the positive cosmological constant suggested by the strong evidence of an accelerated expanding universe. In addition this is important because of some conjectures motivated by no-go theorems that impose constraints to the existence of dS solutions in string theory supergravity.

In this thesis we work on a variant to the leading paradigm for the construction of metastable de Sitter vacuum of supersymmetric Anti de Sitter (AdS) solutions obtained by four-dimensional low-energy supergravity theory with all muduli stabilized which is based on introduce anti-D3-branes providing a positive energy to the potential energy and this process is known as uplifting since it uplifts the AdS vacua to a dS vacua. Instead of we consider the possibility of D-branes wrapped around of torsional cycles of a half-flat manifold and study its consequences in the AdS vacuum. We show that for these torsional D3-branes the vacua remains in an AdS minimum with all moduli stabilized and work out the contribution to the energy potential arising of this torsional branes as well.

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Dedication

To my Mon and Brother

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1. Introduction

One of the most surprising discoveries is the accelerated expansion of the universe on a large scale, a fact supported by a plenty of experimental evidence collected by high-precision cosmology experiments. As a result of this, it is necessary to offer an explanation to this problem leading presumably because of a vacuum energy or cosmological constant, and then why it has an extremely small value but in fact non-zero value in the field equations of general relativity under quantum effects. One could try to address the problem from the perspective of Quantum Field Theory, that is, include quantum corrections, however the different contributions that are calculated in this approach contradict the evidence, the extent in which theory and experiment failure to thrive is around 60 to 120 orders of magnitude.

What makes this problem truly challenging is that it arises from two theories with the greatest experimental success. On the one hand General Relativity with more than a century of its discovery and on the other hand the Standard Model of Particle Physics. It seems clear in term of the evidence that an interplay between these two frameworks is indispensable to address this problem of the cosmological constant, for an introductory description to this problem see [1]. At this stage the emergence of a quantum theory of gravity is necessary and in the light of the developments the most prominent candidate we have today is String Theory. We have five perturbative theories precisely formulated in ten space-time dimensions known as superstrings theories and one more theory formulated in eleven space-time dimensions known as eleven-dimensional sugra. All of them are limits of the so called M-theory [2, 3].

To describe the universe in its early stage (this problem mostly driven by the inflationary paradigm) as well as the observation of late-time cosmic acceleration, the search for dS solutions was launched, i.e. vacuum with positive energy density. There are many no-go theorems [4] that complicate the construction of de Sitter vacuum in string theory as well as from a pure supergravity perspective, additionally to the string swampland/landscape conjectures (at this point the anthropic principle is expected play an important role) suppose another problem. The first steps in this search were taken by the KKLT model [5] (by the names of its discoverers S. Kachru, R. Kallosh, A. Linde and S. Trivedi) in the context of highly warped compactifications on a Calabi-Yau (CY) manifold of type IIB/F theory [6] . For this it was necessary turn on background fluxes for NSNS (Neveu-Schwarz-Neveu-Schwarz) and RR (Ramond-Ramond) forms fixing all of the complex structure moduli and the axion-dilaton of CY compactification. Focusing on models having only one Kähler modulus, they use nonperturbative effects to fix this remaining modulus thus this allowed them to carry out the moduli stabilization problem (which by it self is a difficult problem when we are looking for realistic string compactifications). This combined with a remnant superpotential characterized by the presence of fluxes, have shown that the minimum of the potential is a supersymmetric four-dimensional AdS vacuum. For an analysis of inflationary context to Cosmology see [7] and for the importance of the moduli stabilization problem see [8]. The addition of an anti-D3-brane has a significant impact providing an extra term for the potential which depends on Kähler modulus given rise to an extra source of positive potential energy uplifting the AdS minimum, finally resulting in a metastable de Sitter vacua with broken supersymmetry in four-dimension and of lifetime much greater than the cosmological timescale.

On the other hand a beautiful connection between the type II theories without fluxes is based on a duality known as mirror symmetry which under some precise requirements asserts that the low-energy four-dimensional effective theories derived from type IIA compactified on a CY manifold X_3 is equivalent to type IIB compactified on a CY manifold Y_3 mirror to X_3. In order to have these mirror pairs (X_3, Y_3) is necessary, among other things, the possibility of relate their cohomology groups, more precisely there exists a relation between even (odd) cohomology groups on X_3 and odd (even) cohomology groups on Y_3. However in this context of mirror symmetry the compactifications of type II theories in the presence of RR and NSNS fluxes it has been necessary to resort to more sophisticated manifolds than the commonly used CY. The main problem has arisen from the fact that in the most general compactifications, one finds that some of the fluxes appear to be related with (con)torsional elements of the Levi-Civita connection into the mirror symmetric manifold, in consequence most of the forms (as the holomorphic (3,0) form Ω and the Kähler form J) do not close under the standard differential operator d of the differential geometry, and the mirror symmetry is less clear and our knowledge of the standard cohomology is lost.

The mirror symmetry with fluxes is guaranteed in compactifications known as compactifications of Type II theories on generalized manifolds, e.g. half-flat manifolds [9, 10], in despite of not to have the cohomology structure than CY manifolds. And this leads to the existence of torsional elements in the (co)homology [11] which is important for us on search for implications in Cosmology. In addition we suppose, if mirror symmetry holds in the limit of small torsion, i.e. the limit of large complex structure thus the moduli space of the CY manifold, at least locally, should coincide with the moduli space of the half-flat manifold in particular its metric deformations and in both cases we used the same tool for describe the complex structure deformation [9, 10]. In this thesis by simplicity we do not consider any quantum correction to Kähler potential and therefore work out in the large volume limit. For an analysis of the nonperturbative correction to the Kähler potential, see [12].

This thesis is organized as follows. In Chapter 2 we briefly review the elements of type II theories, recall the necessary fact on CY compactifications and flux compactification and we focus on the complex structure moduli. We briefly describe the structure of the corresponding supergravity theory, superpotential, Kähler potential and the supersymmetry conditions. In Chapter 3 we introduce half-flat manifolds in the context of mirror symmetry and their associated torsional components that we then use in generic context to expand fluxes into both torsional and non-torsional elements, we rewrite the Kähler potential and the superpotential. Chapter 4 contains a brief review of the KKLT model and the strategy in the mechanism to obtain dS vacuum. We explore the effects of D3-branes wrapped in torsional cycles instead of the last step of the KKLT strategy. Finally some remarks are include in Chapter 5

2. Preliminaries of String Theory

2.1 String Theory Elements

String theory is a theory of quantum gravity, which can at the present be precisely formulated in high dimensions and several weakly coupled versions. There are two versions that are very important for us, these are the superstring theories in ten space-time dimensions, called Type IIA and Type IIB. These theories are described at low energies by effective ten dimensional supergravity theories. We will now briefly describe both type IIA and type IIB string theories focusing our discussion throughout the content of bosonic fields of the NSNS and RR sectors [2, 3, 8]. There are other three ten-dimensional versions, these are Type I, and the Heterotic $E_8 \times E_8$ and $SO(32)$ theories which can be found in e.g. [2] and there is one more limit eleven dimensional known as M-theory can be seen in [3]. On the other hand, there are other two sectors corresponding to NS-R and R-NS sectors where the content of fermionic fields is present but for this thesis they will not play an important role in our discussion, so we refer the reader for more details to [2, 3].

The NSNS sector of type II supergravities in ten dimensions contains the metric G_{MN}, the dilaton Φ, and the two-form B_2. The string-frame action for this field is (characterized by the exponential dilaton dependence)

$$S_{\text{NS}} = \frac{1}{2\kappa_{10}^2} \int d^{10}X \sqrt{-G} e^{-2\Phi} \left(R + 4(\partial\Phi)^2 - \frac{1}{2}|H_3|^2 \right) \ , \tag{2.1}$$

where $H_3 = dB_2$. The ten-dimensional gravitational coupling constant κ_{10}^2, corresponding to the Newton constant in ten-dimensions G_{10}, can be related to the string tension (it has been introduced the string length scale $l_{\text{s}} = \sqrt{2\alpha'}$) by comparing the worldsheet and supergravity actions; one finds [2, 3]

$$2\kappa_{10}^2 = (2\pi)^7(\alpha')^4 \ . \tag{2.2}$$

In addition, type IIA supergravity have R-R p-forms that is, an one-form C_1 and a three-form C_3 which appear in the complete action as form field strengths in the kinetic terms. The type IIA complete action then take the form

$$S_{\text{IIA}} = S_{\text{NS}} + S_{\text{R}}^{(\text{IIA})} + S_{\text{CS}}^{(\text{IIA})} \ , \tag{2.3}$$

where the RR and Chern-Simons (CS) actions are

$$S_{\text{R}}^{(\text{IIA})} = -\frac{1}{4\kappa_{10}^2}\int d^{10}X\sqrt{-G}\left(|F_2|^2 + |\tilde{F}_4|^2\right) \ , \tag{2.4}$$

$$S_{\text{CS}}^{(\text{IIA})} = -\frac{1}{4\kappa_{10}^2}\int B_2 \wedge F_4 \wedge F_4 \ , \tag{2.5}$$

with $F_p = dC_{p-1}$ and $\tilde{F}_4 = F_4 + C_1 \wedge H_3$. The field content in the type IIB supergravity are a zero-form C_0, a two-form C_2 and the four-form C_4. The type IIB complete action is

$$S_{\text{IIB}} = S_{\text{NS}} + S_{\text{R}}^{(\text{IIB})} + S_{\text{CS}}^{(\text{IIB})} \ , \tag{2.6}$$

where the RR and CS actions are

$$S_{\text{R}}^{(\text{IIB})} = -\frac{1}{4\kappa_{10}^2}\int d^{10}X\sqrt{-G}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right) \ , \tag{2.7}$$

$$S_{\text{CS}}^{(\text{IIB})} = -\frac{1}{4\kappa_{10}^2}\int C_4 \wedge H_3 \wedge \tilde{F}_3 \ , \tag{2.8}$$

with $F_p = dC_{p-1}$, $\tilde{F}_3 = F_3 - C_0 \wedge H_3$, and $\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$ with the observation that F_5 has to satisfy a self-duality constraint in type IIB theory.

The field equations of motion from the action (2.6) are consistent after to impose

$$\tilde{F}_5 = \star\tilde{F}_5 \ , \tag{2.9}$$

where $\star$ is the Hodge star operator in the ten-dimensional theory. One has to impose the self-duality constraint after derive the equations of motion from the type IIB action. This must be imposed as an additional constraint; it cannot be imposed on the action then wrong equations of motion result [2].

We have written the NSNS sector (2.1) of the actions (2.3) and (2.6) in the string frame, meaning that appears an exponential dilaton dependence $e^{-2\Phi}$ in front of the Ricci scalar R. However, for many reasons involving supergravity theory and at the same time to make apparent a global symmetry $SL(2,\mathbf{R})$ it is more convenient to work in Einteins frame. It is possible to get Einstein frame by performing the Weyl transformation (Weyl rescaling)[3, 8, 6]

$$G_{E,MN} \equiv e^{-2\Phi} G_{MN} \ , \tag{2.10}$$

then the NSNS action transform as [3]

$$S_{\text{NS}} = \frac{1}{2\kappa_{10}^2} \int d^{10}X \sqrt{-G} e^{-2\Phi} R \rightarrow \frac{1}{2\kappa_{10}^2} \int d^{10}X \sqrt{-G_E} \left(R_E - \frac{9}{2} \partial_M \Phi \partial^M \Phi \right) . \tag{2.11}$$

Also in type IIB string theory, it is convenient to define the combinations

$$G_3 \equiv F_3 - \tau H_3 \ , \tag{2.12}$$

$$\tau \equiv C_0 + i e^{-\Phi} \ , \tag{2.13}$$

in terms of which the action (2.6), written in Eintein frame, takes the form manifestly invariant under $SL(2,\mathbf{R})$ transformations [3, 6]

$$\begin{aligned} S_{\text{IIB}} = & \int d^{10}X \sqrt{G_E} \left[R_E - \frac{\partial_M \tau \partial^M \tau}{2(\text{Im}(\tau))^2} - \frac{|G_3|^2}{2\text{Im}(\tau)} - \frac{|\tilde{F}_5|^2}{4} \right] \\ & - \frac{i}{8\kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\text{Im}(\tau)} \ . \end{aligned} \tag{2.14}$$

Normally, this equation is the starting point for discussion of type IIB flux compactification in the search for supersymmetric vacuum $\mathcal{N} = 1$ [8] as well as to address the problem of hierarchies from perspective of wrapped compactifications [6].

The superstring theories are consistent in ten dimensions and connected by different dualities that allow us to explore different regimes of the theories. Nevertheless in the cosmological likewise phenomenological context need to connect with the physics in the four-dimensional spacetime. For this, the best-understood mechanism is string theory compactifications, and for more realistic descriptions, we consider the compactification in Calabi-Yau manifolds. In addition, in the light of preserving the minimum of supersymmetry in the four-dimensional theory resulting from the compactification process, the proposed Calabi-Yau manifold is the best understood which the majority of the cases contain an extension of the Standard Model. The reason for this is due to the simplicity of calculating the effects of the internal space in the four-dimensional physics that finally is our purpose. The theories mentioned above have supersymmetry $\mathcal{N} = 2$ in ten dimensions and when we perform the dimensional reduction, e.g. Kaluza-Klein in a Calabi-Yau [13, 14, 15, 16] we have a four-dimensional low-energy theory with supersymmetry $\mathcal{N} = 2$. However, for many reasons need configurations with supersymmetry $\mathcal{N} = 1$ in four dimensions. There are compactifications in Calabi-Yau orientifolds which leads to four-dimensional low energy effective theory compatible with $\mathcal{N} = 1$ supergravity [13, 14, 15, 17].

In any of these compactifications, the four-dimensional low-energy action appears with a number of massless scalar fields without potential. These would lead to long-range scalar forces unobserved in nature. Furthermore, the couplings of other fields depend on their vacuum expectation values (VEV's), an example is a dilaton Φ which describe the interaction strengths between the strings. As a consequence, it difficult make any predictions in these scenarios since the VEV of the moduli can take any vacuum expectation values. Therefore, it is important to realize that a mechanism that generates a potential for these free parameter is necessary, fixing (or "stabilizing") their VEV's . The mechanism within perturbative string theory that we know today is via fluxes explained e.g. in [6, 13, 14, 15, 17, 18, 19]. It has been shown that non-pertubative mechanisms work out for this purpose. We are going to talk about this in the following sections.

2.2 Compactification on Calabi-Yau Manifolds

Here we briefly describe the mechanism to connect our ten-dimensional description with the four-dimensional physics considering a background

$$\mathcal{M}_4 \times X_3 \ , \tag{2.15}$$

where $\mathcal{M}_4$ is the four-dimensional Minkowski-space and X_3 is a compact Calabi-Yau manifold of complex dimension three. This manifold with a Levi-Civita connection defines a transformation group[1] which is called the holonomy group and it is who determines the amount of supersymmetry left intact by the background. The four-dimensional low-energy effective action can be determined via Kaluza-Klein reduction of the ten-dimensional supergravity actions in the product space background (2.15). At the same time compactification in the internal compact space X_3 choosen to be a Calabi-Yau threefold, leads to a simplification for the ten-dimensional equations of motion, e.g the ten-dimensional equation of motion for a p-form field B_p

$$\Delta_{10} B_p = \left(d^\dagger d + d d^\dagger\right) B_p = 0 \ , \tag{2.16}$$

where (2.16) encode the equation of motion and a choice of gauge. Thanks to the product structure of the background (2.15), the spacetime and internal Beltrami operator split, additionally the ten spacetime coordinates split into four spacetime coordinates plus six real coordinates (or three complex coordinates) thus the internal forms in this separation are eigenfunction of the internal Beltrami operator of the CY manifold

$$\Delta_{10} B_p = (\Delta_4 + \Delta_6) B_p \ , \tag{2.17}$$

and the massless modes of the four-dimensional theory correspond to the harmonic forms of Δ_6 in the low-energy approximation. These zero modes (or harmonic forms) correspond to elements of the Dolbeault cohomology groups on X_3. More precisely the cohomology groups of X_3 decompose in subspaces which are more refined cohomology

[1] More in general an m-dimensional Riemannian manifold M with an affine connection defines a group of transformation called holonomy group and in a complex manifold M, which is, that admit complex structure also is possible to talk about its holonomy group.

groups which is, the Dolbeault cohomology groups

$$H^k = \bigoplus_{p+q=k} H^{p,q} \quad . \tag{2.18}$$

Here (p,q) denotes the number of holomorphic and anti-holomorphic differentials of the harmonic forms. The dimension of the (p,q)-cohomology group are called the Hodge number and it is denoted by $h^{p,q} = \text{dim}H^{p,q}(Y)$. If we consider to a CY threefold then the non-trivial Hodge numbers are

$$h^{1,0} = h^{0,1} = h^{2,0} = h^{0,2} = h^{3,1} = h^{1,3} = h^{3,2} = h^{2,3} = 0 \ ,$$

$$h^{0,0} = h^{3,0} = h^{0,3} = h^{3,3} = 1 \ ,$$

$$h^{2,1} = h^{1,2} \ ,$$
$$h^{1,1} = h^{2,2} \ .$$

We see that $h^{1,2}$ and $h^{1,1}$ are the non vanishing Hodge numbers in the CY threefold. Apart of the zero modes in the reduction of the bosonic supergravity actions, that is, the scalar and p-form fields corresponding to harmonic forms on Y, we have that deformations of the CY metric which does not modify the condition of Ricci-flatness (hermitian metric with vanishin Ricci tensor) correspond to elements that lie in the cohomology groups of X_3 and those are manifested in the low-energy effective action as free parameter. This moduli space $\mathcal{M}$ of metric deformations can be described into a product space which locally splits into the space of Kähler deformations of the hermitian metric and in complex structure deformations of the same metric. In other words, the moduli space $\mathcal{M}$ of CY manifolds is locally a direct product of $\mathcal{M}_{1,1}$ and the space $\mathcal{M}_{2,1}$, both are Kähler manifolds, spanned by the complex structure deformations, which deformations into $\mathcal{M}_{1,1}$ are parameterized by the harmonic (1,1)-forms and deformations into $\mathcal{M}_{2,1}$ correspond to harmonic (2,1)-forms. In what follows we briefly summarize the description of moduli space $\mathcal{M}$ of CY manifolds, for more detail, see [3, 20].

2.2.1 The Special Geometry $H^3(X)$

The mathematical tools used to describe CY moduli spaces known as special geometry is presented here. The complex structure deformation can be expanded in harmonic real (2,1)-forms of $H^3(X)$. From (2.18) one finds that the holomorphic (3,0)-form Ω is the representative of $H^{3,0}$ and that $H^{0,3}$ correspond to the anti-holomorphic (0,3)-form $\bar{\Omega}$. One can also choose the elements of $H^{2,1} \oplus H^{1,2}$ to be the real basis $\left(\alpha_I, \beta^J\right)$ where $I, J = 0, \ldots, h^{1,2}$ and specifying a basis of three cycles of the Calabi-Yau $\left\{A^I, B_J\right\}$, with intersection numbers satisfying $A^I \cap B_J = -B_J \cap A^I = \delta_I^J$ and otherwise zero, thus together they can also be chosen to satisfy [3, 9, 12, 15, 21, 22]

$$\int_{A^I} \alpha_J = \delta_J^I\,, \quad \int_{B_I} \beta^J = -\delta_I^J, \quad \int_{X_3} \alpha_I \wedge \beta^J = \delta_I^J, \quad \int_{A^I} \beta_J = 0 = \int_{B_I} \alpha^J\ . \tag{2.19}$$

The unique holomorphic (3,0)-form Ω can be expressed in this basis, it follows that in term of it

$$\Omega = X^I \alpha_I - \mathcal{F}_I \beta^I\ , \tag{2.20}$$

where $X^I, \mathcal{F}_I$ are the periods of Ω, defined by

$$X^I = \int_{A^I} \Omega\ , \qquad \mathcal{F}_I = \int_{B_I} \Omega\ , \tag{2.21}$$

where the X^I are projective coordinates in the moduli space of complex structures and the $\mathcal{F}_I$ are functions of X^I and determined by a homogeneous function (in a proper symplectic frame), i.e. a prepotential of degree two $\mathcal{F}(X)$ as

$$\mathcal{F}_I = \frac{\partial \mathcal{F}}{\partial X^I} \equiv \partial_I \mathcal{F}\ . \tag{2.22}$$

The periods can be conveniently combined into a period vector $\Pi^t = \left(X^I, \mathcal{F}_I\right)$ and if one expresses $X^I = (X^0, X^i)$, the deformations of the complex structure $u^i,\ i = 1, \ldots, h^{1,2}$ which parameterize $H^{1,2}(Y_3)$ are related to the coordinate X^I via $u^i = X^i/X^0$ or in other words one could choose $X^I = (1, u^i)$ establishing a gauge $X^0 = 1$, then the period vectors can be expressed as [23, 21]

$$\Pi = \begin{pmatrix} X^0 \\ X^i \\ \mathcal{F}_0 \\ \mathcal{F}_i \end{pmatrix} = \begin{pmatrix} 1 \\ u^i \\ 2\mathcal{F} - u^i\mathcal{F}_i \\ \mathcal{F}_i \end{pmatrix} , \tag{2.23}$$

where the second equality is normalized choosing $X^0 = 1$ as it was mentioned above. The metric $g_{a\bar{b}}$ on the space of complex structure deformation $\mathcal{M}_{2,1}$ is characterized by a Kähler metric given by $g_{a\bar{b}} = \partial_a \overline{\partial}_{\bar{b}} K$ and the Kähler potential is

$$\begin{aligned} K &= -\log\left(-\mathrm{i}\int \Omega \wedge \overline{\Omega}\right) = -\log\left[\mathrm{i}(\bar{X}^K \mathcal{F}_K - X^K \bar{\mathcal{F}}_K)\right] \\ &= \log\left[2\mathrm{i}\mathrm{Im}\left(\bar{X}^I \mathcal{F}_I\right)\right] . \end{aligned} \tag{2.24}$$

Defining the symplectic matrix $\Sigma = \left(\begin{smallmatrix} 0 & I \\ -I & 0 \end{smallmatrix}\right)$ the Kähler potential (2.24) is rewritten as

$$K = -\log\left(-\mathrm{i}\int \Omega \wedge \overline{\Omega}\right) = -\log\left[\mathrm{i}\Pi^\dagger \cdot \Sigma \cdot \Pi\right] , \tag{2.25}$$

where " $\cdot$ " is the ordinary scalar product. Note that in the above expression there is freedom to normalize the X^0 component of the period vector consistent with the equations defined in (2.19). Later in this thesis this gauge freedom in component X^0 will depend on the size of the torsional cycles of the manifold which will be generalized.

2.2.2 The Special Geometry $H^2(X)$

Harmonic (1,1)-forms and its dual (2,2)-forms being part of $(\omega_i, \tilde{\omega}^i)$ and satisfying the normalization condition

$$\int_{X_3} \omega_i \wedge \tilde{\omega}^j = \delta_i^j , \tag{2.26}$$

denote the elements $\omega_i \in H^{1,1}(X)$ and those $\tilde{\omega}^i \in H^{2,2}$ and $i = 1, \ldots, h^{1,1}$. Usually it is combined the Kähler form J and the two-form B_2 since after a compactification there arise so many $h^{1,1}$ zero modes in low dimensions as Kähler deformations. Hence can use the basis $\omega_i \in H^{1,1}$ to expand J and B_2 as follows

$$J = v^i \omega_i \ , \qquad B_2 = b^i \omega_i \ . \tag{2.27}$$

Now combine them in the complexified Kähler form

$$B_2 + \mathrm{i} J = (b^i + \mathrm{i} v^i)\omega_i \equiv t^i \omega_i \ , \tag{2.28}$$

given rise to $h^{1,1}$ massless complex scalar fields. Also we define the following integrals

$$\mathcal{K} = \frac{1}{6}\int_{X_3} J \wedge J \wedge J \ , \qquad \mathcal{K}_{ijk} = \int_{X_3} \omega_i \wedge \omega_j \wedge \omega_k \ , \tag{2.29}$$

where $\mathcal{K}$ is the classical volume of the CY X, and it is used the Kähler form expanded in terms of the basis ω_i

$$J = v^i \omega_i \ . \tag{2.30}$$

The deformation space of the complexified Kähler form is Kähler with metric $g_{ij} = \partial_i \bar{\partial}_j K$ and given by [3, 20, 18, 16]

$$g_{ij} = \partial_i \bar{\partial}_j (-\log 8\mathcal{K}) \ . \tag{2.31}$$

Furthermore, the Kähler potential can be written in terms of a prepotential $\mathcal{F}$ in analogy to the prepotential of the complex structure moduli space

$$e^{-K} = \mathrm{i}(\bar{X}^K \mathcal{F}_K - X^K \bar{\mathcal{F}}_K) \ , \quad \mathcal{F}_I = \frac{\partial \mathcal{F}}{\partial X^I} \equiv \partial_I \mathcal{F} \ , \tag{2.32}$$

with

$$\mathcal{F} = \frac{1}{3!}\frac{\mathcal{K}_{ijk} X^i X^j X^k}{X^0} \ . \tag{2.33}$$

The coordinates X^I with $I = 0, \dots, h^{1,1}$ are defined in terms of the special coordinate t^i via $X^I = (1, t^i)$ choosing the normalization $X^0 = 1$. And therefore t^i parametrizes the deformations of the complexified Kähler form.

2.3 Flux Compactification

In flux compactifications of CY manifolds (in fact, it requires a CY orientifold) the effective field theory corresponding to a standard $\mathcal{N} = 1$ supergravity theory and with the presence of three-form fluxes, one generates a superpotential for the CY moduli, i.e. it possible the stabilisation of axion-dilaton and complex structure moduli which we denote as $\tau = C_0 + \mathrm{i}/g_\mathrm{s}$ where $g_\mathrm{s} = e^\Phi$ and $u^i\ ,\ \ i = 1,\ldots,h^{1,2}$. Then the R-R-fluxes and NS-NS-fluxes, $F_3 = \mathrm{d}C_2$ and $H_3 = \mathrm{d}B_2$ respectively and it is usual to combine them into $G_3 = F_3 - \tau H_3$, and it is induced the following superpotential [6, 24]

$$W_{\mathrm{flux}}(u^i,\tau) = \int_{X_3} G_3 \wedge \Omega\ . \tag{2.34}$$

In this point we make to note that this superpotential (2.34) depends on the axion-dilaton and complex structure moduli u^i, but it does not depend on Kähler moduli which implies that we have to resort to another mechanism for stabilization of this moduli.

The fluxes satisfy a quantization condition [21]

$$\begin{aligned} \int_{A^I} F_3 &= -f_A^I \in \mathbb{Z}\ , \qquad \int_{B_I} F_3 = -f_I^B \in \mathbb{Z}\ , \\ \int_{A^I} H_3 &= -h_A^I \in \mathbb{Z}\ , \qquad \int_{B_I} H_3 = -h_I^B \in \mathbb{Z}\ . \end{aligned} \tag{2.35}$$

Here the minus signs have been introduced for convenience. If we define the symplectic flux vectors $f^t = (f_A^I, f_I^B)\ ,\ h^t = (h_A^I, h_I^B)$, they can be used to write the flux superpotential (2.34) in a compact way using

$$F_3 = -(f_A^I \alpha_I - f_I^B \beta^I)\ , \qquad H_3 = -(h_A^I \alpha_I - h_I^B \beta^J)\ , \tag{2.36}$$

and the relations (2.19) and $\Sigma = \left(\begin{smallmatrix} 0 & I \\ -I & 0 \end{smallmatrix}\right)$ as follows

$$\begin{aligned} W(u^i, \tau) &= \int_{X_3} \left[-(f^I_A \alpha_I - f^B_I \beta^I) + \tau(h^I_A \alpha_I - h^B_I \beta^J)\right] \wedge \left[X^J \alpha_J - \mathcal{F}_J \beta^J\right] \\ &= \left[(f^I_A - \tau h^I_A)\mathcal{F}_I - (f^B_I - \tau h^B_I) X^I\right] \\ &= (f - \tau h)^t \cdot \Sigma \cdot \Pi \ . \end{aligned} \tag{2.37}$$

The Kähler potential for the different moduli (and if in the expression that follows we consider one Kähler modulo) become in the sum of terms after a compactification depending on the different moduli and take the form

$$\mathcal{K} = -\log\left[-\mathrm{i}(\tau - \bar{\tau}]\right) - \log\left[\mathrm{i}\Pi^\dagger \cdot \Sigma \cdot \Pi\right] - \log\left[-\mathrm{i}(\rho - \bar{\rho})\right] \ . \tag{2.38}$$

Now, the above expressions (2.37) and (2.38) combine in the formula for the potential in $\mathcal{N} = 1$ supergravity as

$$V = \mathrm{e}^{\mathcal{K}} \left(K^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2 \right) \ , \tag{2.39}$$

where I, J run over indices labeling the complex structure and Kähler moduli as well as the axion-dilaton. $D_I W$ is the Kähler covariantized derivative $D_I W = \partial_I W + W \partial_I \mathcal{K}$. At this point something very interesting happen. Because of W is independent of the Kähler moduli at tree level using (2.37) and (2.38), we have that the term

$$K^{I\bar{J}} D_I W \overline{D_J W} = K^{i\bar{j}} \partial_i \mathcal{K} \overline{\partial}_{\bar{j}} \mathcal{K} |W|^2 = 3|W|^2 \tag{2.40}$$

where $K_{i\bar{j}} = \partial^2 \mathcal{K} / \partial_i \rho \partial_{\bar{j}} \rho$ and $K^{i\bar{j}} = (K_{i\bar{j}})^{-1}$ (note that I, J only run over the i, j Kähler moduli there) precisely cancels the term $-3|W|^2$ in (2.39). Therefore, one can express this tree-level flux potential as

$$V = \mathrm{e}^{\mathcal{K}_{\text{sc}} + \mathcal{K}_\tau} \left(K^{a\bar{b}} D_a W \overline{D_b W} \right) \ , \tag{2.41}$$

where a and b run over axion-dilaton and complex-structure moduli only and where $\mathcal{K}_{\text{sc}}$ denote the Kähler potential of complex structure and $\mathcal{K}_\tau$ denote the Kähler potential of axion-dilaton.

Notice that this potential known as no-scale potential in $\mathcal{N} = 1$ supergravity is positive semi-defined and precisely the minimum can be located in $V_{\text{no-scale}} = 0$, such that for generic fluxes choice this minimum corresponds to a supersymmetric minimum with the dilaton and the complex structure moduli stabilized. More precisely, we have that the supersymmetric minimum corresponds to points of the moduli space that give a solution to $V = 0$, that is, we need to solve the equations

$$D_\tau W = D_i W = 0 \ . \tag{2.42}$$

Hence, it seems clear that we are solving $h^{2,1} + 1$ equations in $h^{2,1} + 1$ variables and therefore that generic fluxes choices will fix all the complex structure moduli as well as axion-dilaton, more explicitly for the axion-dilaton [23]

$$\tau = \frac{f \cdot \bar{\Pi}}{h \cdot \bar{\Pi}} \ , \qquad \bar{\tau} = \frac{f \cdot \Pi}{h \cdot \Pi} \ , \tag{2.43}$$

and the complex structure moduli

$$(f - \tau h) \cdot (\partial_i \Pi + \Pi \partial_i \mathcal{K}) = (f - \tau h) \cdot \partial_i \Pi - (f - \tau h) \cdot \Pi \frac{\Pi^\dagger \cdot \Sigma \cdot \partial \Pi}{\Pi^\dagger \cdot \Sigma \cdot \Pi} = 0 \ , \tag{2.44}$$

where we used (2.25) and have a set of $h^{2,1}$ equations that we should solve for the $h^{2,1}$ variables with all moduli stabilized in the minimum of the potential (2.41) which correspond to zero.

Also, it is important to mention that in orientifold compactifications the cancellation requirement of tadpoles implies that the allowed flux choices are stringently constrained and hence the number of flux vacua, in that way the enormous number of consistent vacuum come from flux compactification may be reduced, for more detail see [6, 25].

At this point, as mentioned above the Kähler moduli is not stabilized by the presence of fluxes in our compactification manifold, hence another mechanism is necessary to stabilize this moduli and then have under control these free parameters that are manifested in our low-dimensional theory. For this purpose non-perturbative effects are used, e.g., arising either Euclidean D3-brane instantons or gaugino condensation on stacks of D7-brane wrapping a four-cycle in the internal space. In the seminal work of KKLT these non-perturbative corrections were added to the superpotential which allowed to stabilize the Kähler modulus in their model, we will specify more on this model in Chapter 4.

3. Generalized Compactifications

3.1 Half Flat Compatification

The background geometry (2.15) undergo slight modifications when one allows the presence of p-form field strengths, and take a certain non-trivial background value so that fluxes associated thread appropriate cycles in the internal Calabi-Yau manifold and therefore the background geometry changes to a warped geometry. These background fluxes configuration are allowed to have certain component that are restricted by Poincare invariance and the Bianchi identities, and the metric written as the product metric of a four-dimensional Minkowskian spacetime and six-dimensional Calabi-Yau manifold is replaced by a warped-metric with a warp factor that depend on the coordinate of the internal space. With a convenient parameterization the warped metric normally considered [3, 6]

$$ds_{10} = \eta_{\mu\nu}dx^\mu dx^\nu + g_{mn}dy^m dy^n \to e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{mn}dy^m dy^n \,, \quad (3.1)$$

in terms of four-dimensional coordinates x^μ and coordinates y^m on the six-dimensional internal space. From the equations of motion one can see that the flux parameters contribute to the energy-momentum tensor and as a consequence the geometry backreacts and it is induced a non-trivial warp-factor in terms of the fluxes. However preserving the CY geometry in the large volume approximation we are considering here they appear as continuous parameters which modify the low energy supergravity. In the low-energy approximation the homology cycles are so that the flux parameters are effectively continuous and represent small perturbations of the original Calabi–Yau compactification that otherwise could contribute as torsional elements in the connection, see section 3.1.1.

In the above discussion, a important role has been played by topological invariants of the compactification manifold. In particular, we have been able to derive rather general features of the four-dimensional low energy action thanks to the fact that each object of the compactification corresponds to a topological classes of the compactification manifold, in particular an important role is played by the (co)homology in the four-dimensional low-energy physics. Given these facts, in [26] they wondered if that is all the topological information of complex Kähler manifold X that is relevant for the four-dimensional effective action coming from the (co)homology.[1] More precisely the homology group with integers coefficients $H_p(X, \mathbb{Z})$ contains more information than $H_p(X, \mathbb{R})$, cohomology with real coefficients, the difference being the torsion homology groups Tor $H_p(X, \mathbb{Z})$. Actually the most general form of $H_p(X)$, i.e the universal coefficient theorem is

$$H_p(\mathcal{M}_6, \mathbb{Z}) = \underbrace{\mathbb{Z} \oplus \cdots \oplus \mathbb{Z}}_{b_p} \oplus \mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_n} \ . \tag{3.2}$$

Here $b_p \equiv \dim H_p(X, \mathbb{R})$ stands for the pth Betti number of X, which also counts the number of harmonic p-forms of X, in other words is the dimension of the pth group of homology $H_p(X, \mathbb{R})$ and at the same time is the dimension of the pth cohomology group via de Rahm theorem, i.e., $H_p(X, \mathbb{R})$ and $H^p(X, \mathbb{R})$ are dual vector space. See [26, 27] for more detail in these torsional elements.

In what follows we will motivate the appearance of half-flat manifolds, which in the context of mirror symmetry allowed to show that the compactification of type IIB theory in the presence of NSNS fluxes in an internal Calabi-Yau space is dual to Type IIA theory compactified in a half-flat manifold [9]. In the ref [10] it was shown that the opposite situation, i.e., type IIA in the presence of NSNS fluxes compactified in Calabi-Yau threefolds is dual to type IIB compactified in a half-flat manifold.

[1]Treat the problem of mass mixing of gauge symmetries $U(1)$ of open and closed string, in particular, among other things, they studied RR $U(1)$ gauge symmetries arising from KK reduction of RR closed string fields and their possible mixing with D-brane gauge bosons and found that such mixing are posible e.g. via mass terms induced by Stückelberg mechanism which appears whenever D-branes wrap torsional p-cycles in the compactification manifold.

3.1.1 Half-Flat Manifold

Briefly we define the half-flat manifolds following the references [9, 15, 16]. Then we motive its appearance in the context of mirror symmetry of type II theories.

In relation with supersymmetry a non-vanishing globally well defined spinor have to exist. Manifolds with $SU(3)$ structure admit one globally defined spinor η. And more in general a well-defined non-vanishing spinor exist only on manifolds that its structure group is reduced. In addition if we demand to preserve supersymmetry in a Minkowskian ground state η it has to be convariantly constant with respect to the Levi-Civita connection ∇, which corresponds to a particular case of $SU(3)$-structure manifolds, i.e., the holonomy group has to be $SU(3)$ and therefore the manifold is a Calabi-Yau threefold. However, in general the covariant derivative ∇ does not need to vanish on η, this means that generically the Levi-Civita connection no longer has $SU(3)$-holonomy, then

$$\nabla\eta = \kappa \cdot \eta \ , \tag{3.3}$$

where κ is the contorsion which is related to torsion T, the torsion is defined as $T_{mn}{}^p = \kappa_{[mn]}{}^p$ with indices $m = 1, \ldots, 6$, running over the real dimension of internal space. When we consider the action of κ on the spinor η and since from the point of view of the irreducible representations η is an $SU(3)$ singlet, it is found that the failure of the Levi-Civita connection to vanish on η is measured by a piece of the contorsion which is called the intrinsic contorsion and hence, from (3.3), we have that

$$(\nabla + \kappa^0)\eta = \nabla^{(T)}\eta = 0 \ , \tag{3.4}$$

where we have defined the covariant derivative operator $\nabla^{(T)}$ with torsion in such a way $\nabla^{(T)} = \nabla + \kappa^0$ vanishes on η.

The corresponding intrinsic torsion (or equivalently the contorsion, as we have seen torsion and contorsion are actually equivalent i.e. torsion is defined as $T_{mn}{}^p = \kappa_{[mn]}{}^p$) can be decomposed in $SU(3)$-representations labelled W_k, $k = 1 \ldots, 5$ all of which appear in the exterior derivative of the Kähler and the holomorphic form, as

$$\begin{aligned} \mathrm{d}J &= \frac{3}{2}\mathrm{Im}(\bar{W}_1\Omega) + W_4 \wedge J + W_3 \ , \\ \mathrm{d}\Omega &= W_1 J^2 + W_2 \wedge J + \bar{W}_5 \wedge \Omega \ . \end{aligned} \tag{3.5}$$

The manifolds known as half-flat manifolds correspond to

$$T \in \mathrm{Re}W_1 + \mathrm{Re}W_2 + W_3 \ , \tag{3.6}$$

what is equal to having the following vanishing torsion class

$$\mathrm{Im}W_1 = \mathrm{Im}W_2 = W_4 = W_5 = 0 \ , \tag{3.7}$$

and which can be written alternatively in term of the conditions

$$J \wedge \mathrm{d}J = 0 \ , \qquad \mathrm{Imd}\Omega = 0 \ . \tag{3.8}$$

These half-flat manifolds are not complex, not Kähler and not Ricci-flat, and therefore not Calabi-Yau.

Now the mirror duality that exists between compactifications of type II theories is a well-known fact in the absence of fluxes, that is, the type IIB theory compactified in a manifold X_3 is dual to the type IIA theory compactified in a manifold Y_3, in order to do this one requires that for every X_3 exit a mirror manifold with Hodge numbers

$$h^{1,1}(X_3) = h^{2,1}(Y_3) \ , \qquad h^{2,1}(X_3) = h^{1,1}(Y_3) \ . \tag{3.9}$$

Therefore the theories obtained from the pair (X_3, Y_3) are equivalent [10]

$$\mathcal{L}_{\mathrm{IIB}}(X_3) \equiv \mathcal{L}_{\mathrm{IIA}}(Y_3) \ , \tag{3.10}$$

which is known as mirror symmetry and therefore the theories are mirror symmetric. However, as mentioned before, the presence of fluxes is essential if we want to stabilize (part of) the massless modes (moduli space) that are manifested at low-dimension.

On the one hand in type IIA the p-form field strengths of the RR sector, (see Section 2.1) lie in the even cohomology group of the CY threefold. On the other hand for the type IIB one encounters odd p-form field strengths and therefore written in terms of element of odd cohomology groups of internal space. So one can argue that under mirror symmetry which exchanges the even cohomology for the odd cohomology of two internal spaces, appears to be clear the duality in the RR sector. However, in both type IIA and type IIB theory we can have the presence of NS-NS fluxes H_3 however mirror symmetry in this case is lees clear as none of the two type II theories contain even form field strengths in the NS-NS sector only the H_3 flux. With the purpose of solving this problem and guarantee mirror symmetry of the two type II theories in the presence of NS-NS fluxes, [9] used a half-flat manifold as compactification manifold, where, unlike the standard CY, the Levi-Civita connection fails to preserve Ω and J, i.e. are no longer closed. Therefore they found that the non-closed part of the holomorphic form Ω plays precisely the role of an NS four-form thus the missing NS-fluxes are purely geometrical and arise directly from the change in the compactification geometry. Moreover, in order of mirror symmetry works, precise relation must be maintained. In particular the moduli space of metrics of the half-flat manifold should coincide with the moduli space of the corresponding CY manifold. However, the key point here is that although (α_I, β^I) form a basis for Ω and the ω_i of $H^{1,1}(Y)$ form a basis for J they are not, in general, harmonic, and thus there are not bases for $H^{2,1}(Y_{\text{hf}})$ and $H^{1,1}(Y_{\text{hf}})$ respectively. However in the limit of large complex structure (the limit of small intrinsic torsion[2]) locally the usual basis $(\alpha_I, \beta^J) \in H^{2,1}(Y)$ and $(\omega_i, \tilde{\omega}^j) \in H^{1,1}$ must be used to perform the usual moduli expansion

$$\Omega = X^I \alpha_I - \mathcal{F}_I \beta^J \ , \qquad J = v^i \omega_i \ , \tag{3.11}$$

and the forms (α_I, β^J) and $(\omega_i, \tilde{\omega}^j)$ should have the same relations (2.19) and (2.26), respectively, as on the CY cases. However, as it was found in [9] the NSNS fluxes mirror was provided by RedΩ and in order to have this and (3.11) then

$$d\alpha_0 = e_i \tilde{\omega}^i \ , \qquad d\alpha_i = d\beta^J = 0 \ , \tag{3.12}$$

[2]Do not confuse geometric torsion, referring to the metric and more precisely to the connection in the manifold, with that of the torsion group, a topological concept that does not depend on the metric.

where α_i and β^J are closed under the action of the differential operator d. And this operator d is a map $\Omega^p(X)$ to $\Omega^{p+1}(X)$ where $\Omega^p(X)$ is the vector space of p-forms.

In addition consistency with both (2.19) and (2.26) thus

$$\mathrm{d}\omega_i = e_i\beta^0 \ , \qquad \mathrm{d}\tilde{\omega}^i = 0 \ . \tag{3.13}$$

In what follows, these forms that are no longer harmonic imply the existence of torsional component in the (co)homology of Y_{hf}.

3.1.2 Torsional Elements

As pointed out and shown by the authors of [11] the facts mentioned above lead to the existence of torsional components in the (co)homology of half-flat manifold. Taking the zero-components of the symplectic three-form basis (α_I, β^J) satisfying

$$\begin{aligned} \mathrm{d}\alpha_0 &= e_i\tilde{\omega}^i \ , \\ \mathrm{d}\omega_i &= e_i\beta_0 \ , \end{aligned} \tag{3.14}$$

then the cohomology group of the half-flat manifold are as shown in the Table 1 [11]

	$H^n(Y_3,\mathbb{Z})$	Tor$H^n(Y_3)$	exact mod k	non-closed
$n=0$	$\mathbb{Z}$	–	–	–
$n=1$	–	–	–	–
$n=2$	$\mathbb{Z}^{h^{1,1}-1}$	–	–	$n^i\omega_i \equiv \hat{\omega}_2$
$n=3$	$\mathbb{Z}^{2h^{2,1}}$	$\mathbb{Z}_k$	$n^in_i\beta^0 \equiv \beta^{0,tor}$	$\hat{\alpha}_0$
$n=4$	$\mathbb{Z}^{h^{1,1}-1}$	$\mathbb{Z}_k$	$n^i\tilde{\omega}_i \equiv \omega_4^{tor}$	–
$n=5$	–	–	–	–
$n=6$	$\mathbb{Z}$	–	–	–

Table 1. Cohomology group Y_3, taken from Ref. [11]

.

Let us note that a correspondence was established between the elements (α_0, β^0) of the simplectic three-form basis of $H^3(Y)$ used in the description of the moduli space of deformations of complex structure in the section 1.1.1, with the torsional elements of the cohomology of the half-flat manifold. More specifically [11, 26] it is possible to define a basis of three-forms in the half-flat manifold as $(\hat{\alpha}_0, \beta^{0,tor})$ so that the pair $\left(\Sigma_3^{tor}, \hat{\Pi}_3\right)$ using (3.14) and Poincare duality are conformed by a three-cycle and a three-chain with

$$\begin{aligned} k\Sigma_3^{tor} &= \partial\hat{\Pi}_4 \ , \\ \partial\hat{\Pi}_3 &= k\Sigma_2^{tor} \ , \end{aligned} \tag{3.15}$$

such that the following relation is holding

$$\begin{aligned} &\int_{\Sigma_3^{tor}} \hat{\alpha}_0 = -\int_{\hat{\Pi}_3} \beta^{0,tor} = \int_{Y_3} \hat{\alpha}_0 \wedge \beta^{0,tor} = 1 \ , \\ &\int_{\Pi_4} \omega_4^{tor} = -\int_{\Sigma_2^{tor}} \hat{\omega}_2 = \int_{Y_3} \omega_4^{tor} \wedge \hat{\omega}_2 = 1 \ , \end{aligned} \tag{3.16}$$

where it was used

$$\int_{\Sigma_3^{tor}} \hat{\alpha}_0 = \frac{1}{k}\int_{\partial\hat{\Pi}_4} \hat{\alpha}_0 = \frac{1}{k}\int_{\hat{\Pi}_4} \mathrm{d}\hat{\alpha}_0 = \int_{\hat{\Pi}_4} \omega_4^{tor} = 1 \ . \tag{3.17}$$

Now, using this structure it is possible to write down a expression for the holomorphic form which has information about torsional cohomology. Additionally we develop part of work of section 1.1.1 using this torsional structure

$$\Omega_3 = \Omega_3^0 + \tilde{\Omega}_3 = X^i\alpha_i - \mathcal{F}_i\beta^i + X^0\hat{\alpha}_0 - \mathcal{F}_0\beta^{0,tor} \ , \tag{3.18}$$

with $\tilde{\Omega}_3$ corresponding to the components expanded in the basis $(\hat{\alpha}_0, \beta^{0,tor})$ referred to torsional structure, and where the periods are given by the integrals

$$X^I = \begin{pmatrix} X^0 \\ X^i \end{pmatrix} = \begin{pmatrix} \int_{\Sigma_3^{tor}} \tilde{\Omega}_3 \\ \int_{A^i} \Omega_3^0 \end{pmatrix} \ , \tag{3.19}$$

$$\mathcal{F}_I = \begin{pmatrix} \mathcal{F}_0 \\ \mathcal{F}_i \end{pmatrix} = \begin{pmatrix} \int_{\hat{\Pi}_3} \tilde{\Omega}_3 \\ \int_{B^i} \Omega_3^0 \end{pmatrix} \ . \tag{3.20}$$

Moreover we organize this information in a period vector that now corresponds to a period vector of a half-plane manifold whose moduli space is at least locally equal to the moduli space of a CY, where the deviation is measured by the presence of torsional classes and therefore the value of X^0 and $\mathcal{F}_0$ will depend on the size of the torsional cycles, then

$$\Pi_{\text{hf}} = \begin{pmatrix} \tilde{X}^0 \\ X^i \\ \tilde{\mathcal{F}}_0 \\ \mathcal{F}_i \end{pmatrix} \ , \tag{3.21}$$

where X^0 and $\mathcal{F}_0$ are determined for the torsional component and X^i and $\mathcal{F}_i$ are the projective coordinates that describe the complex structure deformation and the superpotential, respectively. In addition the fluxes can be decomposed in the symplectic basis with the use of torsional components and satisfy the quantization conditions

$$\begin{aligned} \int_{A^I} F_3 &= \int_{\Sigma_3^{tor}} F_3 + \int_{A^i} F_3 = -f_\Sigma^{tor} + f_A^i \in \mathbb{Z} \ , \\ \int_{A^I} H_3 &= \int_{\Sigma_3^{tor}} H_3 + \int_{A^i} F_3 = -h_\Sigma^{tor} + h_A^i \in \mathbb{Z} \ , \end{aligned} \tag{3.22}$$

and

$$\begin{aligned} \int_{B^I} F_3 &= \int_{\hat{\Pi}_3} F_3 + \int_{B^i} F_3 = -f_\Pi^{tor} + f_B^i \in \mathbb{Z} \ , \\ \int_{B^I} H_3 &= \int_{\hat{\Pi}_3} H_3 + \int_{B^i} F_3 = -h_\Pi^{tor} + h_B^i \in \mathbb{Z} \ . \end{aligned} \tag{3.23}$$

Therefore the fluxes are

$$\begin{aligned} F_3 &= -(f_\Sigma^{tor}\hat{\alpha}_0 + f_A^i \alpha_i - f_\Pi^{tor}\beta^{0,tor} - f_i^B \beta^i) \ , \\ H_3 &= -(h_\Sigma^{tor}\hat{\alpha}_0 + h_A^i \alpha_i - h_\Pi^{tor}\beta^{0,tor} - h_i^B \beta^i) \ , \end{aligned} \tag{3.24}$$

and the flux superpotential can be determined considering the torsional structure as

$$W = \left[(f_A^i - \tau h_A^i)\mathcal{F}_i - (f_i^B - \tau h_i^B)X^i + (f_\Sigma^{tor} - \tau h_\Sigma^{tor})\mathcal{F}_0 - (f_\Pi^{tor} - \tau h_\Pi^{tor})X^0\right] \ , \tag{3.25}$$

where $X^i \ , \ \ i = 1, \ldots, h^{1,2}$ parameterize the complex structure deformation and $\mathcal{F}_i = \partial_i \mathcal{F}$ is the derivative of a holomorphic prepotential $\mathcal{F}(X)$.

The Kähler potential is

$$
\begin{aligned}
K_{\rm cs} &= -\log\left(\mathrm{i}\int \Omega^0 \wedge \bar{\Omega}^0 + \int \tilde{\Omega} \wedge \bar{\tilde{\Omega}}\right) \\
&= -\log\left[\mathrm{i}(\bar{X}^i \mathcal{F}_i - X^i \bar{\mathcal{F}}_i) + i(\bar{X}^0 \mathcal{F}_0 - X^0 \bar{\mathcal{F}}_0)\right] \\
&= -\log\left[2\mathrm{iIm}(\bar{X}^i \mathcal{F}_i) + 2\mathrm{iIm}(\bar{X}^0 \mathcal{F}_0)\right] \ .
\end{aligned}
\tag{3.26}
$$

Notice the Kähler potential contains some information coming from torsional cohomology since it is related to the geometry of the internal space.
At tree-level, the Kähler sector satisfies the no-scale property $K^{\rho\bar{\sigma}}\partial_\rho\partial_{\bar{\sigma}}K = 3$, and therefore the scalar potential of the effective supergravity action reads

$$
V = \mathrm{e}^{\mathcal{K}_{\rm cs}+\mathcal{K}_\tau}\left[K^{i\bar{j}} D_i W D_{\bar{j}} W + K^{\tau\bar{\tau}} D_\tau W D_{\bar{\tau}} W\right] \ . \tag{3.27}
$$

If we consider points where the axion-dilaton/complex structure configuration preserve supersymmetry, namely those satisfying

$$
D_\tau W = 0 \ , \qquad D_i W = 0 \ . \tag{3.28}
$$

Evaluating these conditions we obtain a supersymmetric minimum as explained in the previous chapter. Therefore, based on the previous discussion about torsional elements in the next chapter explore the KKLT model.

4. KKLT and Torsion Cycles

4.1 KKLT

Kachru, Kallosh, Linde and Trivedi [5] found the first explicit realization of four-dimensional de Sitter space as a solution to the low-energy equations of string theory with the axion-dilaton and complex structure moduli stabilized in a minimum supersymmetric of the potential and they applied nonpertuvative effects to stabilize the unique volume Kähler modulo considered in their model. Finally, the addition of an anti-D3-brane provides an extra source of positive potential energy which depends on the Kähler modulus, and hence uplifting the minimum to a four-dimensional de Sitter vacuum. This is a significant achievement given the importance that de Sitter space has acquired from the recent data on the acceleration of the universe and also for its close relation with the inflationary scenario and different conjectures limiting the existence of these type of solutions.

Here we quickly review the KKLT model. In order to stabilize ρ one adds nonperturbative corrections, e.g. Euclidean D3-brane instanton or gaugino condensation on stacks of D7-brane that wrap a 4-cycle in the internal space. Both effects lead to the superpotential

$$W = W_0 + Ae^{\mathrm{i}a\rho} \ . \tag{4.1}$$

Here W_0 is a constant at tree-level and can viewed as the remnant from (2.34) after stabilization of axion-dilaton/complex structure moduli with (2.43) and (2.44), in consequence it depends on a generic selection of the fluxes. And the parameters A and a can be chosen as appropriate constants. Using (2.39), the tree-level Kähler potential $\mathcal{K} = -3\log\left[-\mathrm{i}(\rho - \bar{\rho})\right]$ and assuming for simplicity real A and W_0 as well as Re(ρ)

$= 0$ one find the potential

$$V = \frac{aAe^{-a\sigma}}{2\sigma^2}\left[\frac{1}{3}\sigma aAe^{-a\sigma} + W_0 + Ae^{-a\sigma}\right] , \tag{4.2}$$

with Im(ρ) = iσ. At a supersymmetric vacuum $D_\rho W = 0$

$$D_\rho \left(W_0 + Ae^{\mathrm{i}a\rho}\right) = 0, \qquad D_\rho = \partial_\rho + \mathcal{K}_\rho, \qquad \mathcal{K}_\rho = \partial_\rho \mathcal{K} , \tag{4.3}$$

then minimum lies at

$$W_0 = -Ae^{-a\sigma_0}\left(1 + \frac{2}{3}a\sigma_0\right) . \tag{4.4}$$

Therefore the minimum of the potential corresponds to a supersymmetric AdS minimun

$$\begin{aligned} V &= e^{\mathcal{K}_\rho}(K^{\rho\bar{\rho}}D_\rho W\overline{D_\rho W} - 3|W|^2) \\ &= -\frac{a^2A^2e^{-2a\sigma_0}}{6\sigma_0} . \end{aligned} \tag{4.5}$$

As an example they considered constants with values $W_0 = -10^{-4}$, $A = 1$, $a = 0.1$. In the Figure 1 we show an image representing the minimum of the potential (4.2)

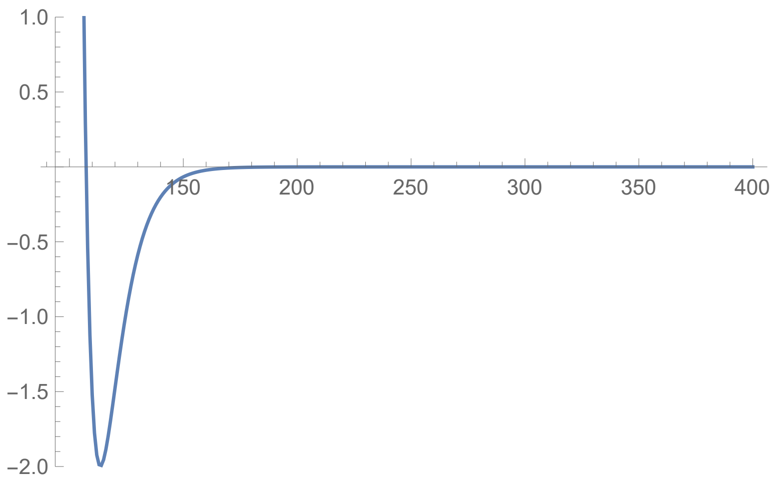

Figure 1. Potential (multiplied by 10^{15}). There is an AdS minimum.

Up to now we have stabilized all moduli in a supersymmetric minimum of negative vacuum energy density. First at tree-level in a wrapped compactification [6] stabilized a dilaton/complex structure moduli and then introduce nonpertuvative correction for stabilizing the Kähler volume modulo. Now [5] uplift the minimum by incorporating an anti-D3-brane in the resulting warped geometry of the compactification. This adds a contribution of the scalar potential and (4.2) is as follows[1]

$$V = \frac{aAe^{-a\sigma}}{2\sigma^2}\left[\frac{1}{3}\sigma aAe^{-a\sigma} + W_0 + Ae^{-a\sigma}\right] + \frac{D}{\sigma^3}\ . \tag{4.6}$$

By fine-tuning D it is possible to have the dS minimum and we note that it is very close to zero, Figure 2. As before we use the model with $W_0 = -10^{-4}$, $A = 1$, $a = 0.1$, $D = 3\times 10^{-9}$. The new minimum obtained via the potential contributed by anti-D3-

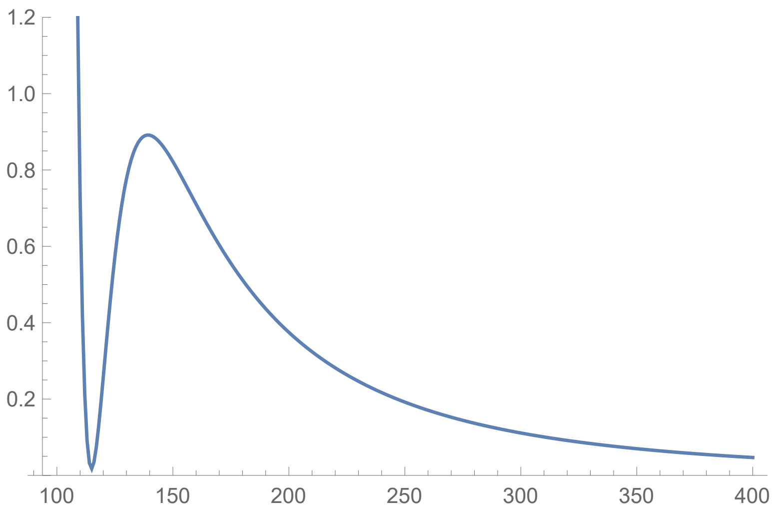

Figure 2. Potential (multiplied by 10^{15}). There is a dS minimum.

brane is a metastable dS minimum, with supersymmetry broken by the presence of the term coming from anti-D3-branes. Furthermore, this minimum is only metastable in the sense that the decompactification limit $\sigma \longrightarrow \infty$ necessarily has zero, it is expected to decay to a Minkowski vacuum consistent with the age of our universe [5].

[1]Here we used the original term D/σ^3 from the anti-D3-brane proposed by [5], however this term it was refined to D/σ^2 by considering a contribution coming form the warp factor in the warpped compactifications, see e.g. [28].

4.2 KKLT with Torsion Cycles

In this section we explore the effects of the superpotential resulting from wrapping D3-branes in torsional cycles using the results of the reference [11]. In that reference, the effects of torsional (co)homology present in a half-plane manifold D3-branes wrapping torsional cycles were explored and found that a supersymmetric black hole has quantum hair at quantum level. They also find extra degrees of freedom due to these torsional groups associated to the mass, entropy and temperature of the supersymmetric black hole, and the change associated with these parameters when k of the D3-branes reaches the order of the discrete group, i.e., the torsional group.

The superpotential (4.1) additionally to nonperturbative contribution will have a contribution that depend on D3-branes wrappind torsional cycles, we generically denote this contribution as $\widetilde{W}$

$$W = W_0 + Ae^{-\mathrm{i}a\rho} + \widetilde{W} \ , \tag{4.7}$$

where the contribution to the superpotential W is [11]

$$\widetilde{W} = -\left(\frac{h^0}{k}\mathrm{mod}1\right)\frac{1}{C} \ . \tag{4.8}$$

The equation (4.8) say that if $h^0 = k$ then this contribution to superpotential it is vanishing, where h^0 is the zero component of the flux vector NSNS and k is the order of torsional group. C is a matrix that depend of wedge product of the basis (α_I, β^I) and in general of the moduli of complex structure.

Using (4.7) and the Kähler potential in (2.39) and assuming that the Kähler moduli, the volume modulus we are considering remain unchanged we have that

$$\begin{aligned} D_\rho W &= \partial_\rho W + W(\partial_\rho \mathcal{K}) \\ &= \mathrm{i}Aae^{a\sigma} + \frac{3\mathrm{i}}{2\sigma}\left(W_0 + \widetilde{W} + Ae^{-a\sigma}\right) \ , \end{aligned} \tag{4.9}$$

and our potential be as follow inserting (4.7) and (4.9)

$$
\begin{aligned}
V &= e^{\mathcal{K}_\rho}(K^{\rho\bar{\rho}} D_\rho W \overline{D_\rho W} - 3|W|^2) \\
& \frac{1}{(2\sigma)^3}\left[-\frac{1}{3}(\mathrm{i}2\sigma)^2 \left(Aae^{-a\sigma} + \frac{3}{2\sigma}\left(W_0 + \widetilde{W} + Ae^{-a\sigma}\right)\right)\right. \\
& \times \left(Aae^{-a\sigma} + \frac{3}{2\sigma}\left(W_0 + \overline{\widetilde{W}} + Ae^{-a\sigma}\right)\right) \\
& \left. -3\left(W_0 + \widetilde{W} + Ae^{-a\sigma}\right)\left(W_0 + \overline{\widetilde{W}} + Ae^{-a\sigma}\right)\right] \\
&= \frac{1}{(2\sigma)^3}\left[-\frac{4}{3}\sigma^2\left(A^2a^2e^{-2a\sigma} + \frac{9}{4\sigma^2}\left(W_0 + \widetilde{W} + Ae^{-a\sigma}\right)\right)\right. \\
& \times \left(W_0 + \overline{\widetilde{W}} + Ae^{-a\sigma}\right) + \frac{3Aae^{-a\sigma}}{2\sigma}\left(W_0 + \widetilde{W} + Ae^{-a\sigma}\right) \\
& + \frac{3Aae^{-a\sigma}}{2\sigma}\left(W_0 + \overline{\widetilde{W}} + Ae^{-a\sigma}\right) \\
& \left. -3\left(W_0 + \widetilde{W} + Ae^{-a\sigma}\right)\left(W_0 + \overline{\widetilde{W}} + Ae^{-a\sigma}\right)\right] ,
\end{aligned} \tag{4.10}
$$

and assuming likewise KKLT W_0, A, a real constant, thus

$$
V = \frac{aA\mathrm{e}^{a\sigma}}{2\sigma^2}\left(\frac{1}{3}\sigma A\mathrm{e}^{-a\sigma} + W_0 + A\mathrm{e}^{-a\sigma^2}\right) + \frac{aA\mathrm{e}^{-a\sigma}}{4\sigma^2}\left(\widetilde{W} + \overline{\widetilde{W}}\right) . \tag{4.11}
$$

We note on the one hand that the first term on the right hand side of (4.11) is the KKLT potential with the dilaton/complex structure moduli stabilized at the tree level under the choice of RR and NSNS fluxes and including non-perturbative effects, the only volume modulus in the model stabilized. On the other hand, the second term on the right hand side of (4.11) is the contribution of the torsional part, i.e., of the D3-branes wrapping torsional cycles.

Since the moduli of complex structures were fixed by the fluxes to appropriate scale we can integrate them out and view $\widetilde{W}$ as a contribution to the potential. In addition this contribution does not depend on the volume Kähler modulo only depend of the number D3-branes wrapping torsional cycles and in consequence the condition of supersymmetry depend on these torsional branes, while $\frac{h^0}{k}$ mod 1 does not saturate. In this point we also stress that in the supergravity limit of type IIB string, to ensure a computation only valid in the large volume limit thus $\rho \gg 1$ [5]. In addition to this the α' corrections to the Kähler potential are neglected, however for a critical assessment of such concern of α' effect , see [12]. From the above it follows that $W_0 \ll 1$ (recall we adjust $W_0 = -10^{-4}$ as in KKLT model) in equation (4.4) such that $\rho \gg 1$ it is hold thus in consequence $a\sigma > 1$ [5]. From the discrete adjustment that the fluxes admit, it is expected to achieve these conditions with some precision. We observe that (4.11) has the same structure as (4.2) except for a contribution to W_0 making it a W_{eff} in the sense that the contribution of torsional branes is included, and for simplicity we take in accordance with the choice of constants of the KKLT model $\widetilde{W}$ to be real. In addition, for consistency with the KKLT model we take this contribution such that W_0 is not severely modified and therefore we have that the minimum is AdS with a contribution of energy coming from torsional D3-brane. During these torsional D3-branes does not reach k and therefore does not saturated the relation $\frac{h^0}{k}$ mod 1, the D3-branes wrapping torsional cycles are stable BPS state [11, 26] since the Kähler form J is non-close on the half-flat manifold and therefore the charge computed through dJ on a three-chain does not vanish so that the corresponding AdS vacuum preserve supersymmetry. However, once the number of torsional branes reach the number k, the torsional three-cycles become trivial in homology departing the system from the stable BPS state and therefore during this stage the vacuum is not supersymmetric. We can write dawn the potential during the torsional D3-branes are present

$$V = \frac{aAe^{-a\sigma}}{2\sigma^2}\left[\frac{1}{3}\sigma aAe^{-a\sigma} + (W_0 + \widetilde{W}) + Ae^{-a\sigma}\right] \ , \tag{4.12}$$

and this has the same structure of the potential in Figure 1, an AdS minimum. However once we reach k torsional branes and after this setup radiate its energy decay to the vacuum (4.2).

We would like to comment that in [29] was addressed the problem of finding $W_0 \ll 1$ in the limit of a large complex structure and this process was generalized in [30] in the context of CY compactifications and this lead the problem of fine-tuning of the different constants.

According to Table 1 and equations (3.15), the above opens the window of exploring 2-cycles of torsion and again exploring its consequences in the AdS vacuum without resorting to anti-D3-branes and we are working on it.

5. Conclusions

In this thesis, we investigated the possibility of finding a de Sitter vacuum without resorting to anti-D3-branes as in the KKLT model. Instead of we use torsional elements, i.e. the (co)homology groups present in a halt-flat manifold. We explore the effect of a superpotential coming from these torsional elements calculated in [11]. We find that the remnant superpotential W_0 after the stabilization of the moduli of complex structure and the dilaton via the RR and NSNS fluxes, appears with an additional contribution making W_0 it become into a W_{eff} in the sense that it contains this contribution of D3-branes wrapping torsional cycles . For reasons of consistency in the KKLT model imposed by the large volume limit, i.e. the supergravity limit it has to be $W_{\text{eff}} \ll 1$ and therefore the AdS structure for the potential continues to be of negative energy, therefore we conclude that the potential continues to be anti de Sitter during these torsional branes are present, in addition this minimum is a stable $\mathcal{N} = 1$ supersymmetric vacuum.

However the supersymmetry depends on the contribution $\widetilde{W}$, i.e. once we add k torsional branes this contribution disappear and the AdS minimum goes from a (meta)stable state to the original state (4.2), but during the transition the system is not (meta)stable because of the torsional brane are not represented by stable BPS state but we remark that both state (4.2) and (4.12) are minimum (meta)stables with objects being BPS states.

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