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Explorando la sensibilidad futura a no-unitariedad en un haz de neutrinos

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Resumen

En la búsqueda de una explicación para las masas de neutrinos, muchas propuestas teóricas consideran nuevos estados pesados de neutrinos que implican una matriz de mezcla de tres neutrinos no unitaria, produciendo nuevos efectos en la probabilidad de oscilaciones. Si estos efectos fueran medibles, se podría tener evidencia indirecta de nueva física.

En este trabajo estudiamos una propuesta experimental y obtenemos su sensibilidad a neutrinos ligeros, considerando tanto interacción neutrino-nucleón como neutrino-electrón. Usando los resultados de neutrinos estériles ligeros y el formalismo de la no-unitariedad (en la aproximación de distancia cero) calculamos la sensibilidad esperada a uno de los elementos de la matriz de mezcla no-unitaria. Estos cálculos se realizaron tanto para el caso de la interacción neutrino-nucleón como para neutrino-electrón.

Abstract

In the search for an explanation of the neutrino mass, many theoretical proposals consider new heavy neutrinos states that imply that a three neutrino mixing matrix is non-unitary. This would lead to new effects on the oscillation probability that, if measurable, could be indirect evidence of new physics.

In this work, we will study an experimental proposal and obtain its sensitivity to light sterile neutrinos for neutrino-nucleon and neutrino-electron interaction. Using the light sterile neutrino results and the non-unitary formalism (in the zero-distance approximation) we compute the expected future sensitivity to an element of the non-unitary oscillation matrix. We perform these computations both for the neutrino-nucleon and the neutrino-electron interaction case.

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Chapter 1

Introduction

As scientists, we search for models or theories that can describe, with good accuracy, the observed data in the experiments. In this sense, one of the most essential and successful models in physics is the Standard Model (SM). The SM describes three of four interactions: electromagnetic, weak, and strong force, with the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ as their symmetry group. The SM was born in '60-'70s by the work of many important physicists like Feynman, Tomonaga, Schwinger, and many others. They formulated quantum electrodynamics (QED). After some years Sheldon Glashow [8], Steven Weinberg [30], and Abdus Salam [27] unifiedthe electrodynamics with weak interaction in the Electro-Weak theory (E-W). Another essential contribution to SM is the quark model by Gell-Mann, which is the first step to the quantum chromodynamics theory (QCD). All these theories compose the SM and are the most consistent models due to their prediction level and the accuracy with the experimental observations.

One of the most important SM predictions is the existence of a boson that gives mass to all the fermions. The scalar, known as the Higgs boson, also gives mass to the Z and $W^{\pm}$ bosons through the Higgs mechanism. The Z and $W^{\pm}$ bosons are a consequence of the gauge structure of the SM. In 2012, in the Large Hadron Collider, the Higgs boson was successfully discovered [18] [1]. With this discovery, the SM gained even more reliability in their prediction and their accuracy. We discuss the SM with more details in the next section.

Although the SM is currently the most reliable model, it does not mean that it is a perfect model or a fundamental theory that describes everything. The SM has many problems that the model cannot answer. One of the problems of the SM that we can notice immediately is that, as we said in the first paragraph, the SM only describes 3 out of 4 fundamental forces but what happens with gravity? In history have been many attempts to create a quantum field theory of gravity.

In other words, we want to find a particle associated with the gravity force. However, all these attempts have failed because the gravity is not renormalizable, which means that the theory has divergences that do not have physical sense. The incompatibility of gravity with a quantum field theory is one of the problems of SM, but theories like loop quantum gravity (LQG) [26] and string theory [28] try to describe it.

Besides the lack of a quantum field theory to the gravity, there are many other problems in the SM. A few examples are the asymmetry of the matter in the universe [11], the dark matter [11], the hierarchy problem [4], and the nature of the neutrino mass . In this work, we focus our attention on the nature of the neutrino mass.

Therefore, the SM is far from being a perfect model. These problems suggest that we need an extension of SM to complement it. As we said above, we focus on the neutrino mass. This problem began when the measurement of atmospheric neutrinos did not match with the result predicted by SM. This was solved with the mechanism of neutrino oscillation that allows passing from one neutrino flavor to another. It should be noted that only three neutrino flavors have been discovered (electron, muon, and tau flavor), and the oscillation of these three neutrino flavors are described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.

The confirmation of the neutrino oscillation did not arrive until 2002, when the Super-Kamiokande collaboration and Sudbury Neutrino Observatory (SNO) had enough experimental data to support the neutrino oscillation theory. For this confirmation, in 2015, Takaaki Kajita [12] and McDonald [19] were recognized with the physics Nobel prize. The discovery of neutrino oscillation has consequences on our understanding of neutrino. The oscillation process requires that the neutrinos have mass, and in the SM, the neutrino is a massless particle. The neutrino oscillation evidence shows that the neutrino has mass but we do not know their values and their nature.

There are many extensions of SM to try to describe the mass of neutrino. These extensions use either Dirac and Majorana mass terms. Where Dirac mass is the mass of all charged fermions and Majorana mass is the mass of the particles that are their antiparticles and chargeless. Majorana mass terms are used in the seesaw mechanism, one of the most important SM extensions. The seesaw mechanism used neutral heavy leptons with Majorana mass as a messenger to transport mass to the active neutrinos. This work will use the effects of these neutral heavy leptons that can appear in a non-unitary oscillation matrix.

Currently, there are many experiments to try to detect new neutrino or indirect evidence of them. For this reason, we focus on a proposed experiment with a new neutrino source using Kaon decays [5], [16] to get constraints for light- sterile neutrino and for the neutral heavy leptons. In chapter 2, we describe theSM content. Then in chapter 3, we talk about of neutrino oscillations and thetypes of neutrino experiments. We dedicate chapter 4 to explaining the non- unitary oscillation matrix formalism and describing one important effect calledthe zero-length effect. Chapter 5 is devoted to describing our attempt to replicate the results of [5], using some approximations, to get similar results as a test ofour computations. We also used the neutrino-electron scattering to get original constraints. In chapter 6 we get the light sterile constraints and the heavy leptons constraints. Finally, in chapter 7, we give the conclusions and perspectives to improve the constraints.

Chapter 2

Standard model

The standard model of elementary particle physics studies three of four fundamental forces: strong, weak, and electromagnetic force. Gauge theories describe these forces with two essential ingredients: quantum field theories and an internal symmetry that governs its dynamics [23]. The quantum field theories that belong to the SM are the Quantum Chromodynamics (QCD) and the Electro-weak interaction, which is the unification of weak interaction and Quantum Electrodynamics (QED). The symmetry groups that accompany the quantum field theories are $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, where the label c is the charge color label, and L means that the weak force only interacts with particles with left chirality. Finally, the Y means the hypercharge. The generators of these symmetry groups are the carries of the 3 fundamental forces, which are known as bosons. For the case of $SU(3)_c$, the eight generators are the gluons that transport the strong force. The four generators of $SU(2)_L \otimes U(1)_Y$ are the A^i_μ (with $i = 1, 2, 3$), and B_μ [25]. After the Spontaneous symmetry breaking, the boson A^3_μ and B_μ mix to get the Z^0 and A_μ. The mixing of A^1_μ and A^2_μ gives us the boson $W^\pm$. All these bosons transport the force to all the particles in the SM. The SM has 12 fundamental fermions divided into leptons and quarks, which we will discuss in the next section.

2.1 The SM particle content

As we have seen above, the SM particles are divided into bosons and fermions. The bosons are the particles that transport the different forces to the fermions and obey the Bose-Einstein statistics. This statistics allows the bosons to be in the same quantum state than another boson. On the other hand, the fermions obey the Fermi-Dirac statistics. In other words, the fermions can not occupy the same quantum state that another fermion. This restriction is known as the Pauli exclusion principle. As we said, the fermions are divided into leptons and quarks.

In the lepton section, there are only 6 fermions (and their respective antiparticles). They are: electron (e), electron neutrino (ν_e), muon (μ), muon neutrino (ν_μ), tau(τ), tau neutrino (ν_τ). We can see some parameters of these leptons in the table (2.1).

	Leptons			
	Particles	Q	mass [GeV]	I_{W_3}
First family	Electron (e)	-1	0.511 x 10^3	$-\frac{1}{2}$
	Electron neutrino (ν_e)	0	$<10^{-9}$	$+\frac{1}{2}$
Second family	Muon (μ)	-1	105.658x10^{-3}	$-\frac{1}{2}$
	Muon neutrino (ν_μ)	0	$<10^{-9}$	$+\frac{1}{2}$
Third family	Tau (τ)	-1	1.777	$-\frac{1}{2}$
	Tau neutrino (ν_τ)	0	$<10^{-9}$	$+\frac{1}{2}$

Table 2.1: The fundamental leptons are divided in their respective family, where Q is the electric charge and I_{W3} is the third component of weak isospin. All the information is in accordance with the particle data group (PDG) [29].

The electron, muon, and tau have the same electric charge and quantum numbers, such as weak isospin. The only difference between these three particles is the mass. The muon is approximately 200 times heavier than the electron and the tau is heavier than the muon. Family by family, the charged leptons have more mass than the lepton of the previous family. The quarks fulfill this feature too. Another important feature is that the neutrino does not have an electric charge, and for this reason, the neutrino can not take part in electromagnetic interaction only in the Weak interaction.

In the case of the quark, there are 6 fermions (and their respective antiparticles). They are up (u), down (d), charm (c), strange (s), top (t), bottom (b) and, as well as in the case of leptons, the quarks are divided into families as shown in the table (2.2). We notice that the electric charge is a fraction of the electric charge of the electron. Also, an important feature is that the quarks are the only fermions that have a color charge, and for this reason, the quark takes part in the strong interaction.

	Quarks			
	Particles	Q	mass [GeV]	I_{W_3}
First family	Up (u)	$+\frac{2}{3}$	2.2×10^{-3}	$+\frac{1}{2}$
	Down (d)	$-\frac{1}{3}$	4.7×10^{-3}	$-\frac{1}{2}$
Second family	Muon Charm(c)	$+\frac{2}{3}$	1.27	$+\frac{1}{2}$
	Strange (s)	$-\frac{1}{3}$	96×10^{-3}	$-\frac{1}{2}$
Third family	Top (t)	$+\frac{2}{3}$	173.21	$+\frac{1}{2}$
	Bottom (b)	$-\frac{1}{3}$	4.18	$-\frac{1}{2}$

Table 2.2: The quarks are divided in their respective family, where Q is the electric charge and I_{W3} is the third component of weak isospin, all the information is in accordance with particle data group (PDG) [29].

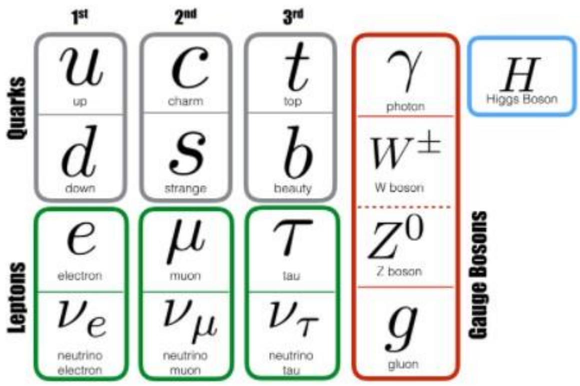

Figure 2.1: A box of the standard model and their principal particles [31].

These fermions and bosons are the fundamental particles of the standard model. They are depicted in fig. (2.1). However, these fundamental fermions are not all the particles that exist. Some particles, like the baryons and mesons, are made up of these fundamental particles. The baryons are composed of three quarks, some of the common baryons are the proton and neutron. In contrast, the mesons are formed by one quark and one anti-quark like the pion and the kaon. All these composed particles made of quarks are called Hadrons and, as we can see, are massive particles.

2.1.1 Dynamics of spin 1/2 particles

All the leptons have spin one half while all the bosons have integers number of spin. This feature is important because the description of a particle with spin $\frac{1}{2}$ without interaction (free particle) is given by Dirac equation

$$(i\hbar\gamma^{\mu}\partial_{u} - mc)\psi = 0, \qquad (2.1)$$

or in natural units

$$(i\gamma^{\mu}\partial_{u} - m)\psi = 0. \qquad (2.2)$$

This equation of motion comes out of the Lagrangian density [25]

$$\mathrm{L}_{Dirac} = \bar{\psi}(i\gamma^{\mu}\partial_{u} - m)\psi. \qquad (2.3)$$

Where

$$\bar{\psi} \equiv \gamma^{0}\psi^{\dagger}, \qquad (2.4)$$

and the γ represent the four gamma matrices, which are:

$$\gamma^{0} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (2.5)$$

$$\gamma^{i} = \begin{pmatrix} 0 & \sigma^{i} \\ -\sigma^{i} & 0 \end{pmatrix}. \qquad (2.6)$$

This representation of the γ matrices is called the Weyl or chiral representation, also σ^{i} refers to the Pauli matrices, is well known that Pauli matrices are 2×2, therefore the γ matrices have 4×4 dimension. For the aforementioned ψ must have 1×4 dimension. It is called spinor and in the chiral representation is:

$$\psi = \begin{pmatrix} \psi_{L} \\ \psi_{R} \end{pmatrix}. \qquad (2.7)$$

2.2 Introduction to the electro-Weak theory

The Weak theory and the QED were described as not related theories. However, the physicist noticed that we could unify both theories using the combination of spontaneous symmetry breaking with a local gauge. This combination is called the Higgs mechanism [13]. The Higgs mechanism broke the $U(1)$ symmetry.

After this, the bosons (A^i_μ and B_μ) are combined to get the physical fields as follow:

$$W^{\pm} = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu) \qquad \textit{with mass} \quad m_w = g\frac{v}{2}, \tag{2.8}$$

$$Z^0_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(gA^3_\mu - g'B_\mu) \qquad \textit{with mass} \quad m_z = \sqrt{g^2 + g'^2}, \tag{2.9}$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g'A^3_\mu + gB_\mu) \qquad \textit{with mass} \quad m_A = 0. \tag{2.10}$$

After the Higgs mechanism, the bosons $W^{\pm}$, Z^0 get mass. Also, the coupling constants are related to the mixing weak angle:

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}, \tag{2.11}$$

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}}, \tag{2.12}$$

$$\sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}. \tag{2.13}$$

We can rewrite the equations (2.9) and (2.10) in terms of the mixing weak angle, using the equations (2.11-2.13),

$$\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix}. \tag{2.14}$$

We observe that the coupling of all the bosons is described by the electron charge and the weak mixing angle. We can separate the couplings of the charged bosons and put only the Lagrangian of the interaction between fermions and $W^{\pm}$. The Lagrangian takes the next form :

$$\mathcal{L}_W = \frac{g^2}{m_W^2} J^{\mu -}_W J^{+}_{\mu W}. \tag{2.15}$$

The equation (2.15) is called the charged current interaction (CC) and the current $J_W^{\mu-}$ is given by [14]:

$$J_\mu^{\prime\dagger} = \sum_f \bar{\nu}_f \gamma_\mu (1 - \gamma^5) f. \tag{2.16}$$

We can write the Lagrangian to the Z^0 interaction as well as in the CC case,

$$\mathcal{L}_Z = \frac{g^2}{2m_Z^2} J_Z^{\mu} J_{\mu Z}, \tag{2.17}$$

$$= \frac{4G_F}{\sqrt{2}} \left(\sum_f \bar{f} \gamma^\mu (T^3 - \sin^2\theta_w Q) f \right)^2. \tag{2.18}$$

Where T^3 is the third component of weak isospin and Q is the electric charge. The equation (2.17) is called the neutral current interaction (NC). Sometimes is useful to put these two contributions in terms of the currents. We define the neutral current weak interaction as:

$$J^{\mu a} = \sum_f \bar{f} \gamma^\mu T^a f. \tag{2.19}$$

And the sum of the CC and NC interaction in terms of the currents is:

$$\mathcal{L}_W + \mathcal{L}_Z = \frac{4G_F}{\sqrt{2}} \left[(J^{\mu 1})^2 + (J^{\mu 2})^2 + (J^{\mu 3} - \sin^2\theta_w J^\mu_{EM})^2 \right]. \tag{2.20}$$

Where J^μ_{EM} is the electromagnetic current. In figs. (2.2) and (2.3), we can see examples of CC and NC interactions, respectively.

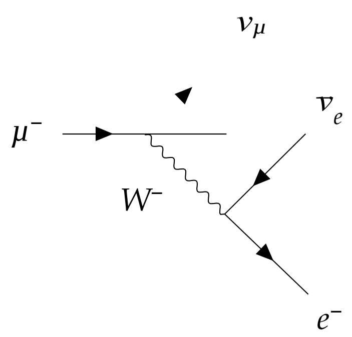

Figure 2.2: Example of CC interaction

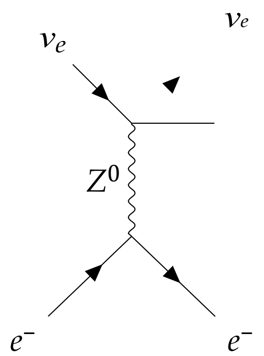

Figure 2.3: Example of NC interaction

Chapter 3

Neutrino oscillation

Pontecorvo was the first to postulate neutrino oscillations in the late 1950s as an analogy to kaon oscillations. However, since this proposal was previous to the SM, the proposed oscillation was from neutrino to anti-neutrino in the first family of fermions. The physicist had to wait for a more precise theory until the electro-weak unification appeared. With this theory, it was easier to describe the neutrino oscillation as a consequence of considering the leptonic charge currentas a superposition of massive neutrinos. Certain conditions are required, like that the energies and momentum of the particles in the neutrino production processare not measured with a degree of accuracy, allowing the determination of the massive neutrino emitted [7]. In other words, we can write a neutrino flavorstate in terms of a sum of massive neutrinos states with a certain weight. In this chapter, we will present a brief summary of the neutrino oscillation formalism.

3.1 Oscillation probability

To start our discussion on neutrino oscillations, we can define a neutrino state with a flavor α as a superposition of the called massive neutrinos that we canwrite as follow:

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k}|\nu_k\rangle, \tag{3.1}$$

where $U^*_{\alpha\kappa}$ is a rotation operator. The massive neutrinos and the flavor states obey the unitary conditions

$$\begin{aligned} \langle\nu_k|\nu_j\rangle &= \delta_{kj}, \\ \langle\nu_\alpha|\nu_\beta\rangle &= \delta_{\alpha\beta}. \end{aligned} \tag{3.2}$$

Currently, we only know three flavor states; therefore, we know that we have three massive states. If we find more than three neutrino states, they probably are sterile. The sterile states do not participate in the weak interaction. The massive states are eigenstates of the Hamiltonian, so the massive neutrino satisfies the Schrödinger equation,

$$i\frac{d}{dt}|\nu_k(t)\rangle = \hat{H}|\nu_k\rangle. \tag{3.3}$$

And its temporal evolution is:

$$|\nu_k(t)\rangle = e^{-iE_k t}|\nu_k\rangle. \tag{3.4}$$

We can obtain the temporal evolution of flavor state as follows:

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k} e^{-iE_k t}|\nu_k\rangle. \tag{3.5}$$

Considering that a massive neutrino is a superposition of flavor states, we can use the conjugate operator U to obtain:

$$|\nu_k\rangle = \sum_\alpha U_{\alpha k}|\nu_\alpha\rangle. \tag{3.6}$$

And inserting this last equation in the (3.5) we have:

$$|\nu_\alpha(t)\rangle = \sum_{\beta=e,\mu,\tau} \sum_k U^*_{\alpha k}\, e^{iE_k t} U_{\beta k} \quad |\nu_\beta\rangle. \tag{3.7}$$

Obtaining the temporal evolution of a given flavor state as a superposition of all the flavor states. To get the probability we need the projection of the flavor states:

$$A_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta|\nu_\alpha\rangle. \tag{3.8}$$

Using equations (3.6) and (3.8), we get the probability amplitude:

$$A_{\nu_\alpha \to \nu_\beta} = \sum_k U^*_{\alpha k}\, U_{\beta k}\, e^{-iE_k t}. \tag{3.9}$$

And the probability is written as:

$$P_{\nu_\alpha \to \nu_\beta}(t) = \sum_{k,j} U^*_{\alpha k}\, U_{\beta k} U_{\alpha j} U^*_{\beta j}\, e^{-i(E_k - E_j)t}. \tag{3.10}$$

We use now the dispersion relation:

$$E = \sqrt{(mc^2) + (pc)^2}. \tag{3.11}$$

That in the ultrarelativistic approximation becomes

$$E = |\vec{p}|. \tag{3.12}$$

Other form to approximate the dispersion relation is using the Taylor series:

$$E \approx |\vec{p}| + \frac{m_k^2}{|\vec{p}|}.$$

Using this result with the ultrarelativistic approximation, we obtain the following result:

$$E_k \approx E + \frac{m_k^2}{2E}. \tag{3.13}$$

Therefore,

$$E_k - E_j \approx \frac{\Delta m_{kj}^2}{2E}. \tag{3.14}$$

Using this energy difference in Eq. (3.5), we have

$$P_{\nu_\alpha \to \nu_\beta}(t) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \, e^{-i\left(\frac{\Delta m^2}{2E}\right)t}. \tag{3.15}$$

It is important to note that neutrino oscillation experiments don't measure time evolution. The parameter that the experiment can control and know is the distance, L, between the source of neutrinos and the detector. If we assume that neutrinos propagate nearly at the speed of light, we can approximate $L = t$, and the probability will be:

$$P_{\nu_\alpha \to \nu_\beta}(t) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \, e^{-i\left(\frac{\Delta m^2 L}{2E}\right)}. \tag{3.16}$$

We can also define the oscillation phase as:

$$\Phi_{ph} = -\frac{\Delta m^2 L}{2E}. \tag{3.17}$$

An important observation is that neutrino oscillation needs neutrino masses different from zero to produce the oscillation. The relevant parameters in an experiment are the distance between source and detector and the source energy. An important property of Operator U is:

$$UU^\dagger = 1, \tag{3.18}$$

another form of the above identity is:

$$\sum_k U_{\alpha k} U^*_{\beta k} = \delta_{\alpha\beta}. \tag{3.19}$$

These properties allow us to put the probability in a useful way:

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \sum_k |U_{\alpha k}|^2|U_{\beta k}|^2 + 2\,\mathrm{Re}\left[\sum_{i>j} U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j}\, e^{-2\pi i\left(\frac{L}{L_{osc}}\right)}\right]. \qquad (3.20)$$

Where L_{osc} is:

$$L_{osc} = \frac{4\pi E}{\Delta m^2_{kj}}. \qquad (3.21)$$

Using the square of (3.19) it is obtained:

$$\sum_k |U_{\alpha k}|^2|U_{\beta k}|^2 = \delta_{\alpha\beta} - 2\sum_{k>j}\mathrm{Re}(U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j}). \qquad (3.22)$$

with this result we can put the oscillation probability in terms of real and imaginary part of the multiplication of four operators U:

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \delta_{\alpha\beta} - 2\sum_{k>j}\mathrm{Re}(U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j})\left[1-\cos\left(\frac{\Delta m^2_{kj}L}{2E}\right)\right]$$
$$+ 2\sum_{k>j}\mathrm{Im}(U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j})\left[\sin\left(\frac{\Delta m^2_{kj}L}{2E}\right)\right], \qquad (3.23)$$

with trigonometric properties, we have:

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \delta_{\alpha\beta} - 4\sum_{k>j}\mathrm{Re}(U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j})\left[\sin^2\left(\frac{\Delta m^2_{kj}L}{4E}\right)\right]$$
$$+ 2\sum_{k>j}\mathrm{Im}(U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j})\left[\sin\left(\frac{\Delta m^2_{kj}L}{2E}\right)\right]. \qquad (3.24)$$

It has two cases, the first one is when $\alpha = \beta$ it is called transition probability. The second case is when $\alpha = \beta$ usually called survival probability. The survival probability is:

$$P_{\nu_\alpha\to\nu_\alpha}(L,E) = 1 - 4\sum |U_{\alpha k}|^2|U_{\alpha j}|^2 \sin^2\left(\frac{\Delta m^2_{jk}L}{4E}\right)$$

α α $k>j$ α (3.25) α

An easy way to approximate the neutrino oscillation is using only two flavors of neutrinos. This approximation is called the “two neutrino mixing” limit case. Despite we have three neutrino flavors, it is a good approximation to use only two of them because many experiments are not sensitive to the three-neutrino mixing. Therefore, we can use an effective model in order to analyze the data with the two-neutrino mixing limit. In this effective model, the operator U is a 2x2 matrix

$$\mathrm{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{3.26}$$

we realize that only have one squared mass difference:

$$\Delta m^2 \equiv \Delta m^2_{21}\,. \tag{3.27}$$

If we can calculate the transition probability using this limit, the only term that survives in Eq. ((3.24)) is the real part because in this approximation the operator U does not have an imaginary part, then we have:

$$\begin{aligned}
P_{\nu_\alpha \to \nu_\beta}(L,E) &= -2\left[\sum_{k>j} \mathrm{Re}(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j})\right]\left[1 - \cos\left(\frac{\Delta m^2_{kj} L}{2E}\right)\right], \\
&= -2\,(U^*_{12} U_{22} U_{11} U^*_{21})\left[1 - \cos\left(\frac{\Delta m^2_{21} L}{2E}\right)\right], \\
&= 2\sin^2\theta\cos^2\theta\left[1 - \cos\left(\frac{\Delta m^2_{21} L}{2E}\right)\right] \\
&= \frac{1}{2}\sin^2(2\theta)\left[1 - \cos\left(\frac{\Delta m^2_{21} L}{2E}\right)\right], \\
&= \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right).
\end{aligned} \tag{3.28}$$

The survival probability is

$$P_{\nu_\alpha \to \nu_\alpha}(L,E) = 1 - P_{\nu_\alpha \to \nu_\beta}(L,E) = 1 - \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right). \tag{3.29}$$

When compared with an experimental result, this theoretical probability must take into account that the energy resolution of the detector is finite. A first approximation to consider the energy resolution is the average oscillation probability:

$$\langle P_{\nu_\alpha \to \nu_\beta}(L,E)\rangle = \frac{1}{2}\sin^2\left(\frac{\theta}{2}\right)\left[1-\left\langle\cos\left(\frac{\Delta m^2_{21}L}{2E}\right)\right\rangle\right], \qquad (3.30)$$

where

$$\left\langle \cos\left(\frac{\Delta m^2_{21} L}{2E}\right)\right\rangle = \cos\left(\left\langle \frac{\Delta m^2_{21} L}{2E}\right\rangle\right) \exp\left[-\frac{1}{2}\left(\frac{\Delta m^2_{21}}{2}\sigma_{L/E}\right)^2\right]. \tag{3.31}$$

Where $\sigma_{L/E}$ is the standard deviation and is proportiona to $< L/E >$. In general we could write as:

$$\sigma_{L/E} = 0.2 < L/E > . \tag{3.32}$$

And we will have the probability:

$$\langle P_{\nu_\alpha \to \nu_\beta}(L,E)\rangle = \frac{1}{2}\sin^2 2\theta \left[1 - \cos\left(\left\langle \frac{\Delta m^2_{21} L}{2E}\right\rangle\right) \exp\left[-\frac{1}{2}\left(\frac{\Delta m^2_{21}}{2}\sigma_{L/E}\right)^2\right]\right]. \tag{3.33}$$

These results are important because the oscillation probability of the sterile neutrino can approximate the two neutrino mixing. As we said above this is an effective model because only uses two flavors, the Standard Model has three flavors of neutrinos that have been observed. One of the most common parametrizations of the oscillation matrix is the Pontecorvo–Maki–Nakagawa–Sakata matrix (PMNS matrix):

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}s_{13} \end{pmatrix}. \tag{3.34}$$

Where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ and δ_{CP} is the CP-violating phase. Although the PMNS matrix is the one that best agrees with the experimental evidence,in many extensions of the standard model, sterile neutrinos are added, and the PMNS might not be the end of the story for the description of the neutrino oscillation.

3.2 Types of neutrino oscillation experiments

We can classify the oscillation experiments into two types. One of them measures transition probabilities between different neutrino flavors. This kind of experiments is called appearance experiments. The second type is called the disappearance experiments and measures the survival probabilities. If the distance between the neutrino source and the detector is too small with respect to the neutrino energy,

$$\frac{\Delta m^2 L}{2E} \gg 1. \quad (3.35)$$

We will not have any oscillation. As it is known, the Δm^2 is a Lorentz invariant and it is fixed. We can design an experiment sensitive to Δm^2 with the right choice of values for the ratio L/E. We say that an experiment is sensitive to Δm^2 when

$$\frac{\Delta m^2 L}{2E} \sim 1. \tag{3.36}$$

Because of this, we can classify the experiments by its L/E average value. For example:

1. Short BaseLine experiments (SBL):
   there are two different short baseLine experiments: Reactor SBL and Accelerator SBL

   (a) Reactor SBL:
   In this type of experiment, that use reactor anti-neutrinos, the range of the ratio L/E and the sensitivity to Δm^2 are:

   $$\frac{L}{E} \leq 10\ m/MeV \Longrightarrow \Delta m^2 \geq 0.1 eV^2. \tag{3.37}$$

   Commonly, the distance between the source and detector is around $10m$.

   (b) Accelerator SBL:
   In these experiments, with beams of neutrinos produced by the decay of kaons, pions and muons, the range of the ratio L/E and the sensitivity to Δm^2 are:

   $$\frac{L}{E} \leq 1km/GeV \Longrightarrow \Delta m^2 \geq 1eV^2. \tag{3.38}$$

   This ratio of L/E can be obtained either with neutrino energies of the order of 1 GeV and baselines $L \approx 1km$ or with the pion decay at rest, with $L \approx 10m$.

2. Long-Baseline experiments (LBL):
   In these experiments, the order of magnitude of L is bigger than the short baseline. Again, we have reactor and accelerator experiments:

   (a) Reactor LBL:
   In this case $L \approx 10^3 m$, the range of the ratio L/E and the sensitivity to Δm^2 are:

   $$\frac{L}{E} \leq 10^3 m/MeV \Longrightarrow \Delta m^2 \geq 10^{-3} eV^2. \tag{3.39}$$

(b) Accelerator LBL:
These accelerators produce muon neutrino from the decay in flight of pions and kaons. In this case, $L \approx 10^2 - 10^3 km$ and the range of the ratio L/E and the sensitivity to Δm^2 are:

$$\frac{L}{E} \leq 10^3 km/GeV \Longrightarrow \Delta m^2 \geq 10^{-3} eV^2. \tag{3.40}$$

3. Very Long-Baseline experiments (VLB)
In this kind of experiments, the distance is larger than the LBL and SBL.

(a) Reactor VLB
In this case, $L \approx 100km$ and the range of the ratio L/E and the sensitivity to Δm^2 are:

$$\frac{L}{E} \leq 10^5 m/MeV \Longrightarrow \Delta m^2 \geq 10^{-5} eV^2. \tag{3.41}$$

(b) Accelerator VLB:

The order of magnitude is thousands of kilometers, the range of the ratio L/E and the sensitivity to Δm^2 are:

$$\frac{L}{E} \leq 10^4 km/GeV \Longrightarrow \Delta m^2 \geq 10^{-4} eV^2. \tag{3.42}$$

There are several neutrino oscillation experiments like MiniBooNE, Icarus, JSNS, etc. For this reason, it is important that we have a way to classify these experiments. This classification helps us to understand the properties of the experiment and gives us important information, such as sensitivity. In the next chapters, we will work with a neutrino oscillation experiment with $L = 1km$ (Long baseline) with a sensitivity around $\Delta m^2 > 0.1$.

Chapter 4

The non-unitary neutrino mixing matrix

Currently, only three neutrino flavors have been observed. However, many theories beyond the standard model postulate the existence of heavy gauge singlet neutrinos, which ones can not participate in the weak interaction. If heavy neutrinos exist, they will modify the oscillation probability because the mixing matrix is not only the 3×3 matrix. We have new states with which the oscillation matrix is non-unitary. The new structure of this matrix would be evidence of new physics beyond the Standard Model. Another scenario is that of light sterile neutrinos, which leads to different phenomenological consequences.

4.1 New neutrino candidates

Before discussing the non-unitary formalism, it is relevant to mention the possible candidates of new neutrinos. Historically, Pontecorvo was the first to propose neutrinos that do not contribute to the weak interaction [3]. It was the first attempt to describe a new type of neutrino. Neutrino masses can emerge from a lepton number violation dimension-five operator O_5. We know that in the Standard Model, the neutrino is a massless particle. Still, when we consider that it can be massive, we can add a Dirac or Majorana mass term in all the possible extensions of the Standard Model. In general, all the massive fermions in the Standard Model have Dirac mass terms and only neutral fermions can have Majorana mass terms. For this description, the neutrino can have both Dirac and Majorana mass terms. We can express the Dirac mass couple left and right-handed fields as follows [9]:

$$m_D \bar{\psi}_L \psi_R + h.c. \tag{4.1}$$

Where m_D is the Dirac mass, ψ_L and ψ_R are left and right-handed Weyl spinor fields. And the Majorana mass term is [9]

$$m_M \bar{\psi}_L^c \psi_L, \quad \psi^c = C\bar{\psi}^T. \tag{4.2}$$

Where m_M is the Majorana mass and C is the charge conjugation matrix. Another important feature is how large the neutrino mass is. This type of neutrino could help to explain current neutrino oscillation anomalies by taking part in it [6]. Another option on which we focus in this thesis is the neutral heavy leptons. Their possible role as messengers of neutrino mass generation in the seesaw mechanism is the strongest motivation to study this type of neutrinos.

4.2 The formalism

We can describe the above candidates in the next non-unitary formalism. The most general matrix with three light neutrinos and n-3 neutral heavy leptons is [6] [10]:

$$U^{n\times n} = \begin{pmatrix} N & S \\ V & T \end{pmatrix}. \tag{4.3}$$

Where N is the 3×3 matrix in the light neutrino sector, and S depends on the coupling parameter of the extra isosinglets states. In this matrix, we can factorize the parameter associated with the heavy leptons from those describing oscillation of the light neutrinos. In other words, put the submatrix U in terms of a multiplication of two matrices. It is possible, but not the only way, to write N in terms of:

$$N = N^{NP}U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U. \tag{4.4}$$

Where U is the PMNS matrix, and N^{NP} is the matrix characterizing the unitary violation and the new physics. The definition of the components of the matrix are:

$$\begin{aligned} \alpha_{11} &= c_{1n}c_{1n-1}c_{1n-2}\cdots c_{14} \\ \alpha_{22} &= c_{2n}c_{2n-1}c_{2n-2}\cdots c_{24} \\ \alpha_{33} &= c_{3n}c_{3n-1}c_{3n-2}\cdots c_{34}, \end{aligned} \tag{4.5}$$

where $c_{ij} = \cos\theta_{ij}$. The off-diagonal terms are:

$$
\begin{aligned}
\alpha_{21} &= c_{2n}c_{2n-1}\cdots c_{25}\eta_{24}\bar{\eta}_{14} + c_{2n}\cdots c_{26}\eta_{25}\bar{\eta}_{15}c_{14} + \cdots + \eta_{2n}\bar{\eta}_{1n}c_{1n-1}c_{1n-2}\cdots c_{14},\\
\alpha_{32} &= c_{3n}c_{3n-1}\cdots c_{35}\eta_{34}\bar{\eta}_{24} + c_{3n}\cdots c_{36}\eta_{35}\bar{\eta}_{35}c_{24} + \cdots + \eta_{3n}\bar{\eta}_{2n}c_{2n-1}c_{2n-2}\cdots c_{24},\\
\alpha_{31} &= c_{3n}c_{3n-1}\cdots c_{35}\eta_{34}\bar{\eta}_{14}c_{24} + c_{3n}\cdots c_{36}\eta_{35}c_{25}\bar{\eta}_{15}c_{14} + \cdots\\
&\quad + \eta_{3n}c_{2n}\bar{\eta}_{1n}c_{1n-1}c_{1n-2}\cdots c_{14}.
\end{aligned}
\tag{4.6}
$$

Where $\eta_{ij} = e^{-i\varphi_{ij}}\sin\theta_{ij}$ this term contains the CP violation phase. Then we realize that the diagonal terms are bigger than the off-diagonal terms. We need the conjugate transpose of N:

$$
N^{NP\dagger} = \begin{pmatrix} \alpha_{11} & \alpha_{21}^* & \alpha_{31}^* \\ 0 & \alpha_{22} & \alpha_{32}^* \\ 0 & 0 & \alpha_{33} \end{pmatrix}. \tag{4.7}
$$

And we can obtain the following equation:

$$
\begin{aligned}
NN^\dagger &= N^{NP}UU^\dagger N^{NP\dagger}\\
&= \begin{pmatrix} \alpha_{11}^2 & \alpha_{11}\alpha_{21}^* & \alpha_{11}\alpha_{31}^* \\ \alpha_{11}\alpha_{21} & \alpha_{22}^2 + |\alpha_{21}|^2 & \alpha_{22}\alpha_{32}^* + \alpha_{21}\alpha_{31}^* \\ \alpha_{11}\alpha_{31} & \alpha_{22}\alpha_{32} + \alpha_{31}\alpha_{21}^* & \alpha_{33}^2 + |\alpha_{31}|^2 + |\alpha_{32}|^2 \end{pmatrix}.
\end{aligned}
\tag{4.8}
$$

It is observed that the unitary condition (3.19) for the oscillation matrix is different in the scheme of this formalism (4.8). Therefore the oscillation probability is not the equation (3.24). The new probability in this formalism is [21]:

$$
\begin{aligned}
P_{\alpha\beta} &= \sum_{i,j}^{3} N_{\alpha i}^* N_{\beta i} N_{\alpha j} N_{\beta j}^* - 4\sum_{j>i}^{3} \mathrm{Re}\left[N_{\alpha j}^* N_{\beta j} N_{\alpha i} N_{\beta i}^*\right] \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E_\nu}\right)\\
&\quad + 2\sum_{j>i}^{3} \mathrm{Im}\left[N_{\alpha j}^* N_{\beta j} N_{\alpha i} N_{\beta i}^*\right] \sin\left(\frac{\Delta m_{ij}^2 L}{2E_\nu}\right).
\end{aligned}
\tag{4.9}
$$

This equation is similar to equation (3.24) but in this case, the probability depends on the matrix N and not on U. An important case is when $L = 0$ known as a zero distance effect:

$$P_{\alpha\beta} = \sum_{i,j}^{3} N_{\alpha i}^{*} N_{\beta i} N_{\alpha j} N_{\beta j}^{*}. \tag{4.10}$$

We can calculate the zero-distance probabilities of all oscillation using equations (4.4) and (4.7)

$$P_{\mu e} = \sum_{i,j}^{3} N^*_{\mu i} N_{ei} N_{\mu j} N^*_{ej} = (N^*_{\mu 1} N_{e1} + N^*_{\mu 2} N_{e2} + N^*_{\mu 3} N_{e3}) \times (N_{\mu 1} N^*_{e1} + N_{\mu 2} N^*_{e2} + N_{\mu 3} N^*_{e3})$$
$$= \alpha^*_{21}\alpha_{11} \times \alpha_{21}\alpha_{11} = \alpha^2_{11}|\alpha_{21}|^2 \qquad (4.11)$$

$$P_{ee} = \sum_{i,j}^{3} N_e{}^*{}_i N_{ei} N_{ej} N_e{}^*{}_j = (N_e{}^*{}_1 N_{e1} + N_e{}^*{}_2 N_{e2} + N_e{}^*{}_3 N_{e3})^2$$
$$= (\alpha^2_{11})^2 = \alpha^4_{11} \qquad (4.12)$$

$$P_{\mu\mu} = \sum_{i,j}^{3} N^*_{\mu i} N_{\mu i} N_{\mu j} N^*_{\mu j} = (N^*_{\mu 1} N_{\mu 1} + N^*_{\mu 2} N_{\mu 2} + N^*_{\mu 3} N_{\mu 3})^2$$
$$= (|\alpha_{21}|^2 + \alpha^2_{22})^2 \qquad (4.13)$$

$$P_{e\tau} = \sum_{i,j}^{3} N^*_{ei} N_{\tau i} N_{ej} N^*_{\tau j} = (N^*_{e1} N_{\tau 1} + N^*_{e2} N_{\tau 2} + N^*_{e3} N_{\tau 3}) \times (N_{e1} N^*_{\tau 1} + N_{e2} N^*_{\tau 2} + N_{e3} N^*_{\tau 3})$$
$$= \alpha_{11}\alpha_{31} \times \alpha_{11}\alpha^*_{31} = \alpha^2_{11}|\alpha_{31}|^2 \qquad (4.14)$$

$$P_{\mu\tau} = \sum_{i,j}^{3} N^*_{\mu i} N_{\tau i} N_{\mu j} N^*_{\tau j} = (N^*_{\mu 1} N_{\tau 1} + N^*_{\mu 2} N_{\tau 2} + N^*_{\mu 3} N_{\tau 3}) \times (N_{\mu 1} N^*_{\tau 1} + N_{\mu 2} N^*_{\tau 2} + N_{\mu 3} N^*_{\tau 3})$$
$$= (\alpha^*_{21}\alpha_{31} + \alpha_{22}\alpha_{32}) \times (\alpha_{21}\alpha^*_{31} + \alpha_{22}\alpha^*_{32})$$
$$= |\alpha_{21}|^2|\alpha_{31}|^2 + \alpha^2_{22}|\alpha_{32}|^2 + \alpha_{22}\alpha_{32}\alpha_{21}\alpha^*_{31} + \alpha_{22}\alpha^*_{32}\alpha^*_{21}\alpha_{31}$$
$$\approx \alpha^2_{22}|\alpha_{32}|^2. \qquad (4.15)$$

In the last line, we ignore the cubic terms. The complete calculation is in appendix A. As we see, in the approximation of zero-distance (short baseline), the probabilities are more straightforward than the general case. In the context of the short baseline approximation, it is helpful to put the oscillation probabilities in terms of the reported experimental parameters to estimate, approximately, the oscillation angle. For this reason, we will use the light-sterile neutrino in the following limit:

$$\frac{\Delta m_{ij} L}{4E} >> 1 \qquad (4.16)$$

$$\langle \sin^2 \left(\frac{\Delta m_{ij} L}{4E} \right) \rangle = \frac{1}{2}. \qquad (4.17)$$

Then, the oscillation probability is [22]:

$$P_{ee} = 2 - \frac{1}{2}[\sin^2(2\theta_{ee})]_{eff} \tag{4.18}$$

$$P_{\mu\mu} = 1 - \frac{1}{2}[\sin^2(2\theta_{\mu\mu})]_{eff} \tag{4.19}$$

$$P_{\mu e} = \frac{1}{2}[\sin^2(2\theta_{\mu e})]_{eff}. \tag{4.20}$$

Using the above equation and the probability oscillation in the zero-distance effect, we can relate the light-sterile neutrino limit to the massive limit in a smooth way as follows

$$[\sin^2(2\theta_{ee})]_{eff} = 2(1 - \alpha_{11}^4) \tag{4.21}$$

$$[\sin^2(2\theta_{\mu\mu})]_{eff} = 2\left[1 - (|\alpha_{21}|^2 + \alpha_{22}^2)^2\right] \tag{4.22}$$

$$[\sin^2(2\theta_{\mu e})]_{eff} = 2\alpha_{11}^2|\alpha_{21}|^2. \tag{4.23}$$

As we see, the zero-distance is just an approximation of the oscillation probability in the non-unitary formalism. The most general probability in this formalism is equation (4.9). However, these results (equations (4.19)-(4.22)) give us a lower bound of new physics because, with this approximation, we get values of the components of the NP matrix. With this, we know in which range of values we are sensitive to new physics in these experiments.

4.3 3+1 case

The easiest case to analyze is with only 1 neutral heavy lepton. In this case, the U matrix is:

$$U^{4\times4} = \begin{pmatrix} N_{3\times3} & S_{3\times1} \\ T_{1\times3} & V \end{pmatrix}. \tag{4.24}$$

Other form to write the mixing matrix is [17]:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} \tag{4.25}$$

Then, using equations (4.5) and (4.6), we can find the components N^{NP} matrix

$$\begin{aligned}
\alpha_{11} &= c_{14}, \\
\alpha_{22} &= c_{24}, \\
\alpha_{33} &= c_{34}, \\
\alpha_{21} &= \eta_{24}\bar{\eta}_{14}, \\
\alpha_{32} &= \eta_{34}\bar{\eta}_{24}, \\
\alpha_{31} &= \eta_{34}c_{24}\bar{\eta}_{14}.
\end{aligned}$$

It is important to see that this result is general, in contrast to the zero-distance effect that we saw in the previous section. This simple case gives us a generallook at the framework with new neutrinos and its possible consequences.

Chapter 5

Non-unitary future tests at a new neutrino source

After discussing the neutrino oscillations in the non-unitary formalism, we can now focus on a new proposed experiment, described in the references [5],[16].The experimental setup uses tagged kaons to produce neutrinos. This neutrino source is attractive because it will provide pure flavor ν_e beams. The objectiveof this new idea is to discover signs of new physics like sterile neutrinos or massive neutrinos using the electron neutrino or muon neutrino that comes from the following decays:

$$K^+ \longrightarrow e^+\nu_e\pi^0 \tag{5.1}$$

$$K^+ \longrightarrow \mu^+\nu_\mu \tag{5.2}$$

$$K^+ \longrightarrow \mu^+\nu_\mu\pi^0. \tag{5.3}$$

This experimental proposal uses kaon decay to obtain the neutrino events using the three channels above. Specifically, this experiment focuses on the signal obtained from the survival probability of electron neutrinos ($\nu_e \longrightarrow \nu_e$). Also, the distance between the detector and the source is $L = 1km$ and the detector has one kt of liquid argon. We will use these parameters in the next section to reproduce the results of this proposed experiment. Although we want to replicate the results, we will use some approximations to facilitate the calculations.

5.1 A First approximation

It is challenging to compute the flux and cross-section for all the channels, including the experimental details. To have a first approximation to this problem and make it more accessible, we will focus only on Eq. (5.1). Moreover, we will consider only the two-body decay:

$$\pi^+ \longrightarrow e^+\nu_e. \tag{5.4}$$

We will compare the events rate of Eq. (5.2) with the one reported in reference [5] and conclude that there is reasonable agreement. The most relevant parts of computing the neutrino events are the neutrino flux, the differential cross-section, and the oscillation probability.

5.1.1 Neutrino flux

The pion decay process is due to incident protons on a target. Therefore, the neutrino flux depends on proton and the pion energy as follows:

$$\lambda(E_\nu) \equiv \frac{d^2N}{dE_\nu\, d\cos\theta} \propto (E_p - E_\pi)^5 \frac{E_\pi E_\nu}{\cos\theta^*}, \tag{5.5}$$

in accordance with refs. [19],[15]. Where E_p is the energy of the incident proton, E_ν is the neutrino energy, E_π is the energy of the pion, and $\cos\theta^*$ is the relative angle of pion and the neutrino in the rest frame of the pion. We can put the energy of the pion and $\cos\theta^*$ in terms of variables that we know well.

$$E_\pi \approx \frac{m_\pi E_\nu}{E^*(1+\cos\theta^*)} \tag{5.6}$$

$$\cos\theta^* \approx \sqrt{1 - \frac{E_\nu^2}{E_\nu^{*2}}\left(\frac{1}{\cos^2\theta} - 1\right)}, \tag{5.7}$$

where m_π is the pion mass and E_ν^* is neutrino energy in the rest frame of the pion. The neutrino energy in the rest frame of pion is easy to compute with the relativistic kinematic of a process $1 \longrightarrow 2$ (appendix B), the result is $E_\nu^* \approx 29.8 MeV$. In this analysis the detector will be aligned with the neutrino flux, in other words, $\theta = 0$. We substitute eqs. (5.6)and (5.7) in Eq. (5.8), we get:

$$\lambda(E_\nu) \propto E_\nu - \frac{m_\pi E_\nu}{2E_\nu^*}^{5} \frac{m_\pi E_\nu^2}{2E_\nu^*}. \tag{5.8}$$

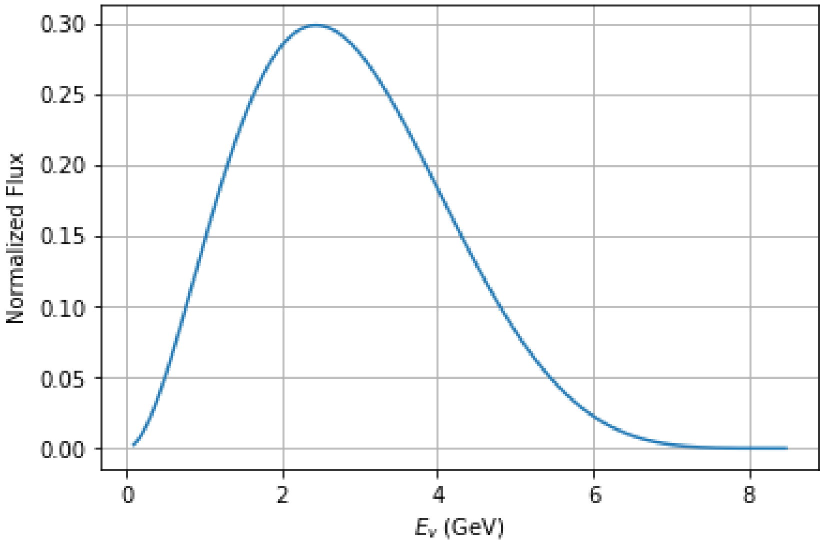

Figure 5.1: The Normalized flux using equation (5.8).

We can see the flux shape in Fig. (5.1). For the part of the oscillation probability, we will use the approximation of two neutrinos Eq. (3.29) and we are interested in the survival probability.

5.1.2 Cross-section

As we already mentioned, to get the neutrino events rate, we also need to compute the corresponding cross-section. In this work, we will use two different processes and, in consequence, two different cross-sections. The first one is the electron neutrino-nucleon scattering, and the second one is the electron neutrino-electron scattering. In both cases, we use the same neutrino flux, given in Eq. (5.8). For the case of neutrino-nucleon, we get the values for its cross-section from refs. [20], [24]. For the case of neutrino-electron scattering, the analytical expression is well-known, in particular, we take the differential cross-section from ref. [22]

$$\frac{d\sigma}{dT} = \frac{2m_e G_F^2}{\pi}\left(g_L^2 + g_R^2\left(1 - \frac{T}{E_\nu}\right)^2 - g_R g_L m_e \frac{T}{E_\nu^2}\right), \tag{5.9}$$

There T is the recoil energy of the electron, g_L, g_R are the coupling constants and G_F is the Fermi constant.

$$g_L = -0.7225$$
$$g_R = -0.2296$$

$$G_F = 1.166 \times 10^{-5} GeV^{-2}.$$

To obtain the total cross-section, we integrate with respect to T, from 0 to T'

$$\sigma = \frac{2m_eG_F^2}{\pi}\left[g_L^2 T' - g_R^2\left(1-\frac{T'}{E_\nu}\right)^3\frac{E_\nu}{3} - g_Rg_Lm_e\frac{T'^2}{2E_\nu} + g_R^2\frac{E_\nu}{3}\right], \tag{5.10}$$

with

$$T' = \frac{2E_\nu^2}{m_e + 2E_\nu}. \tag{5.11}$$

We can see that the Eq. (5.10) only depends on neutrino energy. The differential neutrino events is

$$\frac{dN}{dE_\nu} \propto \sigma\lambda(E_\nu) \tag{5.12}$$

$$N \propto \int \sigma\lambda(E_\nu)dE_\nu. \tag{5.13}$$

We can compute the expected neutrino events rate using the neutrino flux, cross-section, and oscillation probability already discussed here. We should first confirm that the expected number of events in the absence of oscillations coincide with the reported values in the literature. Afterward, we can compute the expected events rate for a given oscillation probability with fixed values for the corresponding parameters, Δm^2, and oscillation angle. In the next section, we will discuss how to compute the events number with and without oscillation and perform a χ^2 analysis to get the expected future sensitivity to this kind of new physics. We should stress that ref. [5] uses a nucleon-neutrino interaction. Here we will also consider the case of a neutrino-electron interaction.

5.2 Computing the neutrino number of events

Using the information described in the previous chapters, we have computed the neutrino events numerically. Afterward, we computed the expected sensitivityto the oscillation mixing angle for high values of Δm^2 values. As a first step, we created a grid with two vectors: The first vector is the $\sin^2 2\theta$ with values ranging from 0.01 to 1 and the second vector is the Δm^2 with values ranging from 0.01 eV^2 to 100 eV^2. In our case, equation (5.8), the neutrino flux, has $E_p = 20GeV$ and has a physical limit in $E_\nu = 8.5405GeV$ for both processes.

We have confirmed that our expected differential number of events, that takes into account neutrino flux and cross-section, is in reasonable agreement with the reported literature. However, since we can not reproduce all the experimental details, we have to fix a total normalization, $Norm(\lambda(E_\nu))$, to the reported expected number of events in the experimental proposal. We can define the differential number of events as:

$$\frac{dN}{dE_\nu} = targets\ \ yr\ \ \sigma\lambda(E_\nu)/Norm(\lambda(E_\nu)) \tag{5.14}$$

$$N = \int targets\ \ yr\ \ \sigma\lambda(E_\nu)/Norm(\lambda(E_\nu))dE_\nu, \tag{5.15}$$

where yr refers to the years that detector would work, one year in this case. We can compute the number of targets considering a detector with one kt of Argon.For the case of neutrino-electron scattering, the targets are the electrons, and for the neutrino-nucleon case, the targets are the argon nucleons

$$Targets_e = 1.206 \times 10^{32} \tag{5.16}$$

$$Targets_{nucleons} = 1.507 \times 10^{31}. \tag{5.17}$$

We compute the expected neutrino events without oscillation probability and normalize the result for the neutrino-nucleon case to $N_{exp} = 1568$ events [5].

For the results on neutrino-electron scattering, we will assume that it would be possible to construct an upgraded version of the detector, with 10 kt of argon. This is necessary in this case because, otherwise, the statistics will not be enoughto obtain a reasonable constraint.

Once we have fixed the normalizations for both neutrino-nucleon and neutrino-electron interactions, we can include the oscillation probability for a given region of Δm^2 and $\sin^2 2\theta$. Finally, we integrate the oscillation probability times the new neutrino events rate. This will allow us to get χ^2 value with the next formula

$$\chi^2 = \frac{(N_{exp} - N_{theo})^2}{N_{exp} + (N_{theo} * \sigma_{exp})^2}, \tag{5.18}$$

where σ_{exp} is the systematic error in percent. Since we are considering an experimental proposal that has not taken events yet, we will assume that in the future it will measure the SM prediction, $N_{exp} = 1568$ events. In this way we can have a forecast for the sensitivity of the future experiment to this kind of new physics. N_{theo} stands for theoretical prediction of the expected number of events for given values of Δm^2 and $\sin^2 2\theta$. After computing the χ^2 function, we can plot the values of Δm^2 and $\sin^2 2\theta$ for which $\Delta\chi^2 < 9.21$ to have the 99% Confidence Level (CL). By showing this curve in a two-dimensional plot, we will see thesensitivity of the experiment in the plane Δm^2 $\sin^2 2\theta$. For this work, we will use two different test values for the systematic error $\sigma_{syst} = 2\%$ and $\sigma_{syst} = 5\%$.

We would expect that our results of neutrino-nucleon interaction will be similar to ref. [5], while the results of neutrino-electron interaction will be less restrictive since the expected number of events will be lower.

5.3 Expected sensitivity for the non-unitary case

After obtaining an expression for the χ^2 function, we can compute the expected sensitivity for the non-unitary test in this experimental proposal. We will first present the expected sensitivity to the light sterile neutrino case, both for the neutrino-nucleon and for the neutrino-electron interaction. The light sterile results will be useful to compare them with those already reported in the literature [5]. Moreover, using equation (4.21) and the sensitivity to the light sterile neutrino case, we can obtain the expected sensitivity for the non-unitary case. That is, we will get the results for the term α_{11} of the N^{NP} matrix.

5.3.1 Light sterile neutrino results

With the computations that we have described up to now, we can compute the values of the χ^2 function, eq. (5.18) for the case of light sterile neutrinos. We show in Figs. (5.2) and (5.3) the sensitivity to the parameters for the case of neutrino-nucleon interaction for the case of a future systematic error of 2% and 5 %. For neutrino-electron interaction case, we see the corresponding sensitivity in the Figs. (5.4) and (5.5). Again for a systematic error of 2 % and 5 %, respectively. We can notice that for the case of neutrino-nucleon scattering there is a reasonable agreement with the previous result reported in literature [5]

Since we are interested in applying these results to the non-unitary case, we pay special attention to the region where $\Delta m^2 > 20eV^2$. In this region, the sensitivity to $\sin^2 2\theta$ is basically constant. We can get the constraints on light sterile neutrinos from these plots. In the case of neutrino-nucleon interaction the constraints for each systematic error (2% and 5 %) are:

$$[\sin^2(2\theta_{ee})] \leq 0.16 \tag{5.19}$$

$$[\sin^2(2\theta_{ee})] \leq 0.29. \tag{5.20}$$

At 99 % CL for each systematic error, respectively. The constraints that we get from ref [5] for each systematic error are: $[\sin^2(2\theta_{ee})] \leq 0.11$ and $[\sin^2(2\theta_{ee})] \leq 0.26$, respectively. Therefore we see that our results are in reasonable accordance with them.

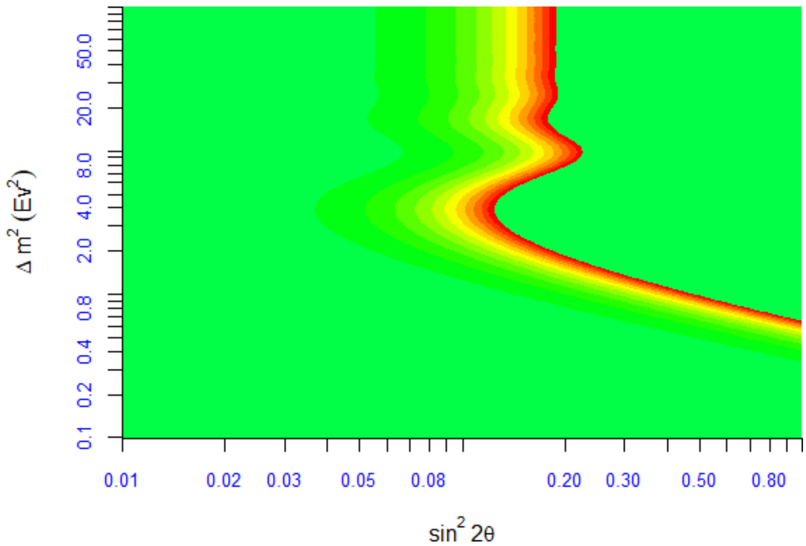

Figure 5.2: The sensitivity in Δm^2-$\sin^2 2\theta$ plane with 2% of systematic error in the nucleon-neutrino interaction.

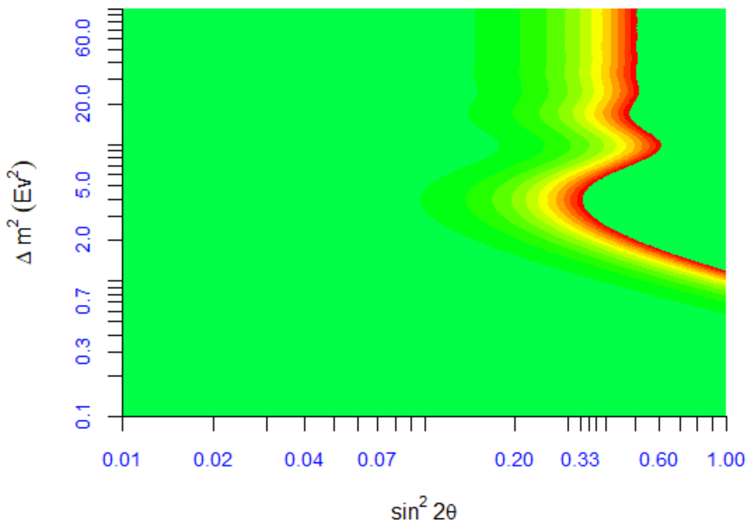

Figure 5.4: The sensitivity in Δm^2-$\sin^2 2\theta$ plane with 2% of systematic error in the neutrino-electron interaction.

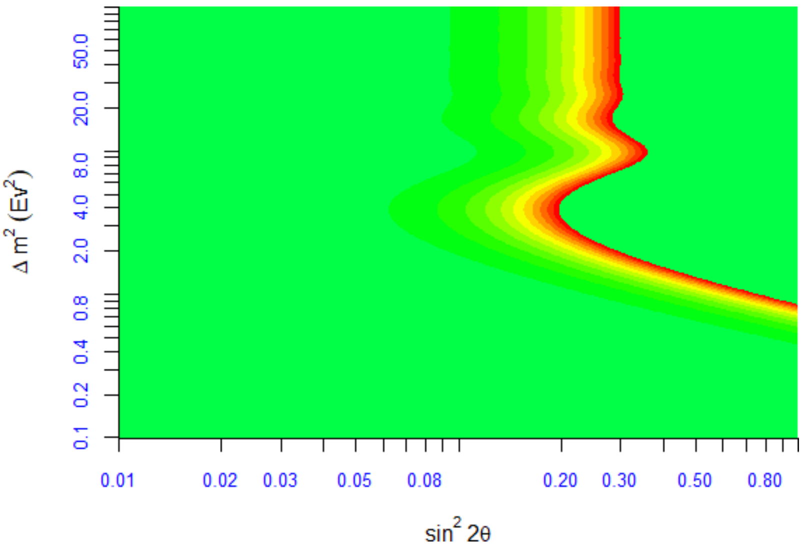

Figure 5.3: The sensitivity in Δm^2-$\sin^2 2\theta$ plane with 5% of systematic error in the nucleon-neutrino interaction.

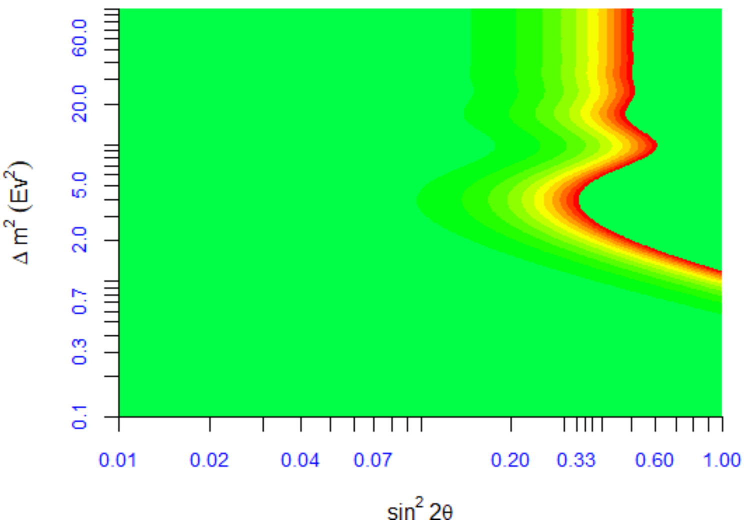

Figure 5.5: The sensitivity in Δm^2-$\sin^2 2\theta$ plane with 5% of systematic error in the neutrino interaction.

The results in the case of neutrino-electron interaction are:

$$[\sin^2(2\theta_{ee})] \leq 0.5 \tag{5.21}$$

$$[\sin^2(2\theta_{ee})] \leq 0.53. \tag{5.22}$$

Again at 99 % CL for each systematic error, respectively. As we expected, the neutrino-electron interaction will be less accurate since the cross-section is smaller in comparison with the case of neutrino-nucleon interaction.

5.3.2 Non-unitary expected sensitivity

We link the results on light sterile neutrinos with the non-unitary case using the eqs. (4.21), as we observe, we only have information of the electron survival probability (e e). For ~~thi~~s reason, we only get the constraints for α_{11} of the N^{NP} matrix in both cases. In the case of neutrino-nucleon interaction, we havethe constraints are:

$$\alpha_{11}^2 \geq 0.959 \tag{5.23}$$

$$\alpha_{11}^2 \geq 0.925, \tag{5.24}$$

at 99% CL for each systematic error, respectively. In the case of neutrino-electron interaction, the constraints of α_{11} are:

$$\alpha_{11}^2 \geq 0.866 \tag{5.25}$$

$$\alpha_{11}^2 \geq 0.857, \tag{5.26}$$

again, at 99% CL. We can compare these constraints with those reported for α_{11} in ref. [6], using data from NOMAD experiments. Their constrain is $\alpha_{11}^2 \geq 0.989$ at 90 % CL. The NOMAD experiments are based on neutrino-nucleon interaction and their constraints are similar to the ones expected from the experiment thatwe are studying here. Therefore, this experimental proposal will be also useful to study the type of new physics that considers the effects of neutral heavy leptonson neutrino oscillation.

Chapter 6

Conclusions

The neutrino oscillation data suggest that it is necessary to have an extension of SM. One of the explanations for the neutrino mass is the neutral heavy lepton. These neutral heavy leptons could act as a messenger to give mass to the active neutrinos. Therefore, they could be the answer for the nature of the neutrino mass. We hope that using the non-unitary oscillation formalism provides a theory that predicts anomalies in the oscillation probability that could be measured inthe future experiments.

For this reason, we introduced the oscillation probability using the non-unitary matrix Eq. (4.10). We described one of the most important effects using the zero-distance case (4.21). In this picture, the zero-distance effect predicts that the survival probability is different from one if $L = 0$. This effect is a consequence of the neutral heavy lepton with light neutrino states. We put this formalism in the context of a new proposed experiment [16]. We studied the sensitivity to $\sin^2 2\theta$ in this experiment, focusing on high values of Δm^2 (more than 20 eV^2). To replicate the result of ref. [5] we used some approximations described in chapter 5 to get a similar result. Afterward, we add a new result taking into account the neutrino-electron scattering, this last interaction is one of the original results in the work.

The sensitivity to $\sin^2 2\theta$ and Δm^2 was useful to obtain the expected sensitivities for the diagonal non-unitary parameter α_{11}. We introduce the expected sensitivity to this component of the NP matrix in eqs. (5.23 -5.26). Our forecast for α_{11} is in reasonable accordance with the literature, for instance, ref. [6].

As a perspective of this work, there is a lot of room for improvements that will lead to a more robust result. The first point that we can consider is to use a more accurate neutrino flux for the kaon decay [2]. Due to lack of time, this information is not added to this work. We have obtained the sensitivity for α_{11} component of NP matrix, but what happens with the remaining components? We need to compute the other channels that could be measured in the proposed experimental array. For example, α_{22} and α_{21} are related to the survival probability of the muon neutrino. In this case, we need to estimate the sensitivity to $\sin^2 2\theta_{\mu\mu}$ and Δm^2 in consequence, we need more computations to get the sensitivity to more elements of the NP matrix. As we observe, there are many issues that we can do in future to improve our work, and we need to wait for future experiments to get more data and have enough evidence to conclude if these extensions are the next step in the search for new physics.

Appendix A

Neutrino probabilities in the non-unitary case for the zero-distance effect

In the case of very short distances, the oscillation probabilities in the non-unitary case will be constant and given by Eq. (4.10). To compute explicitly these probabilities, we start from the components of matrix N using the following equation

$$N_{\alpha\beta} = \sum_{\kappa} N^{NP}_{\alpha\kappa} U_{\kappa\beta}. \tag{A.1}$$

The explicit form for every component is:

$$\begin{aligned}
\mathrm{N}_{ej} &= \alpha_{11} U_{1j} \\
\mathrm{N}^*_{ej} &= \alpha_{11} U^*_{1j} \\
\mathrm{N}_{\mu j} &= \alpha_{21} U_{1j} + \alpha_{22} U_{2j} \\
\mathrm{N}^*_{\mu j} &= \alpha^*_{21} U^*_{1j} + \alpha_{22} U^*_{2j} \\
\mathrm{N}_{\tau i} &= \alpha_{31} U_{1i} + \alpha_{32} U_{2i} + \alpha_{33} U_{3i} \\
\mathrm{N}^*_{\tau i} &= \alpha^*_{31} U^*_{1i} + \alpha^*_{32} U^*_{2i} + \alpha_{33} U^*_{3i}.
\end{aligned} \tag{A.2}$$

Using these expression and the unitary condition (Eq. (3.19)) for the U matrix, we will obtain the corresponding probabilities:

$$\begin{aligned}
P_{ee} &= \sum_{i,j}^{3} N^*_{ei} N_{ei} N_{ej} N^*_{ej} = \sum_{i}^{3} (\alpha_{11} U^*_{1i} \times U_{1i} U^*_{1i}) \sum_{j}^{3} (\alpha_{11} U_{1j} \times \alpha_{11} U^*_{1j}) \\
&= \alpha^4_{11} \sum_{i}^{3} (U_{1i} U^*_{1i}) \sum_{j}^{3} (U_{1j} U^*_{1j}) \\
&= \alpha^4_{11}
\end{aligned}$$

$$P_{\mu e} = \sum_{i,j}^{3} N^*_{\mu i} N_{ei} N_{\mu j} N^*_{ej} = \sum_{i}^{3} [\alpha_{11} U^*_{1i}][\alpha_{21} U_{1i} + \alpha_{22} U_{2i}] \sum_{j}^{3} [\alpha_{11} U_{1j}][\alpha^*_{21} U^*_{1j} + \alpha_{22} U^*_{2j}]$$
$$= \sum_{i}^{3} [\alpha_{11} U^*_{1i} \alpha_{21} U_{1i}] \sum_{j}^{3} [\alpha_{11} U_{1j} \alpha^*_{21} U^*_{1j}]$$
$$= (\alpha_{11}\alpha_{21})(\alpha_{11}\alpha^*_{21}) = \alpha^2_{11} |\alpha_{21}|^2$$

$$P_{\mu\mu} = \sum_{i,j}^{3} N^*_{\mu i} N_{\mu i} N_{\mu j} N^*_{\mu j}$$
$$= \sum_{i}^{3} [\alpha^*_{21} U^*_{1i} + \alpha_{22} U^*_{2i}][\alpha_{21} U_{1i} + \alpha_{22} U_{2i}] \sum_{j}^{3} [\alpha_{21} U_{1j} + \alpha_{22} U_{2j}][\alpha^*_{21} U^*_{1j} + \alpha_{22} U^*_{2j}]$$
$$= \left[|\alpha_{21}|^2 \sum_{i}^{3} U^*_{1i} U_{1i} + \alpha^2_{22} \sum_{i}^{3} U^*_{2i} U_{2i} \right] \left[|\alpha_{21}|^2 \sum_{j}^{3} U_{1j} U^*_{1j} + \alpha^2_{22} \sum_{j}^{3} U_{2j} U^*_{2j} \right]$$
$$= (|\alpha_{21}|^2 + \alpha^2_{22})(|\alpha_{21}|^2 + \alpha^2_{22}) = (|\alpha_{21}|^2 + \alpha^2_{22})^2$$

$$P_{e\tau} = \sum_{i,j}^{3} N^*_{ei} N_{\tau i} N_{ej} N^*_{\tau j}$$
$$= \sum_{i}^{3} [\alpha_{11} U^*_{1i}][\alpha_{31} U_{1i} + \alpha_{32} U_{2i} + \alpha_{33} U_{3i}] \sum_{j}^{3} [\alpha_{11} U_{1i}][\alpha^*_{31} U^*_{1j} + \alpha^*_{32} U^*_{2j} + \alpha_{33} U^*_{3j}]$$
$$= \alpha_{11}\alpha_{32} \sum_{i}^{3} U^*_{1i} U_{1i} \times \alpha_{11}\alpha^*_{32} \sum_{j}^{3} U_{1j} U^*_{1j} = \alpha^2_{11} |\alpha_{32}|^2$$

$$P_{\mu\tau} = \sum_{i,j}^{3} N^*_{\mu i} N_{\tau i} N_{\mu j} N^*_{\tau j}$$
$$= \sum_{i}^{3} [\alpha^*_{21} U^*_{1i} + \alpha_{22} U^*_{2i}][\alpha_{31} U_{1i} + \alpha_{32} U_{2i} + \alpha_{33} U_{3i}] \sum_{j}^{3} [\alpha_{21} U_{1j} + \alpha_{22} U_{2j}]$$
$$\times [\alpha^*_{31} U^*_{1i} + \alpha^*_{32} U^*_{2i} + \alpha_{33} U^*_{3i}]$$
$$= [\alpha^*_{21}\alpha_{31} \sum_{i}^{3} U^*_{1i} U_{1i} + \alpha_{22}\alpha_{32} \sum_{i}^{3} U^*_{2i} U_{2i}][\alpha_{21}\alpha^*_{31} \sum_{j}^{3} U_{1j} U^*_{1j} + \alpha_{22}\alpha^*_{32} \sum_{j}^{3} U_{2j} U^*_{2i}]$$
$$= (\alpha^*_{21}\alpha_{31} + \alpha_{22}\alpha_{32}) \times (\alpha_{21}\alpha^*_{31} + \alpha_{22}\alpha^*_{32})$$
$$= |\alpha_{21}|^2 |\alpha_{31}|^2 + \alpha^2_{22} |\alpha_{32}|^2 + \alpha_{22}\alpha_{32}\alpha_{21}\alpha^*_{31} + \alpha_{22}\alpha^*_{32}\alpha^*_{21}\alpha_{31}$$

$$\approx \alpha_{22}^2 |\alpha_{32}|^2$$

Appendix B

Kinematics of $1 \longrightarrow 2$ process

In this appendix, we briefly explain the kinematics involved in describing a process of type $1 \longrightarrow 2$ in the rest frame. The process that we want to describe is

$$\pi^+ \longrightarrow \mu^+ \nu_\mu. \tag{B.1}$$

The first step is to define the 4-momentum of all the particles:

$$P^\mu_\pi = (m_\pi, 0) \tag{B.2}$$

From 3-momentum conservation, the sum of the 3-momentum of muon and muon neutrino must be zero. The 4-momentum of muon is

$$P^\mu_{muon} = \left(E_\mu, \vec{p}\right) \tag{B.3}$$

and the 4-momentum of the muon neutrino is

$$P^\mu_\nu = (E_\nu, -\vec{p}) \,. \tag{B.4}$$

We remember the mandelstam variable, t, is

$$t = \left(P^\mu_i - P^\mu_{2f}\right)^2 = \left(P^\mu_{1f}\right)^2 \tag{B.5}$$

where subscript i refers to the initial state and f is the final state. in our case this becomes

$$\left(P^\mu_{muon}\right)^2 = \left(P^\mu_\pi - P^\mu_\nu\right)^2$$

$$\left(P^\mu_{muon}\right)^2 = (P^\mu_\pi)^2 + (P^\mu_\nu)^2 - 2P^\mu_\pi P^{\mu\nu}. \tag{B.6}$$

We compute the squared 4-momentum as follows

$$
\begin{aligned}
\left(P_{\pi}^{\mu}\right)^2 &= m_{\pi}^2 \\
(P_{muon}^{\mu})^2 &= E_{muon}^2 - |\vec{p}|^2 = m^2{}_{muon} \\
P_{\pi}^{\mu} P_{\mu\nu} &= m_{\pi} E_{\nu} \\
(P_{\nu}^{\mu})^2 &= E_{\nu}^2 - |\vec{p}|^2 = m^2{}_{\nu} = 0.
\end{aligned}
$$

As we can see we neglected the neutrino mass. By replacing the above results in Eq. (B.6) we have

$$
\begin{aligned}
m_{muon}^2 &= m_{\pi}^2 - 2 m_{\pi} E_{\nu} \\
E_{\nu} &= \frac{m_{\pi}^2 - m_{muon}^2}{2 m_{\pi}} = 29.8 MeV
\end{aligned}
\tag{B.7}
$$

where $m_{\pi} = 134.9766 MeV$ and $m_{muon} = 105.658 MeV$

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