Centro de Investigación y de Estudios Avanzados del I.P.N.
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Regulación de salida para una clase de sistemas conmutados

Tesis que presenta:

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Output regulation for a class of Switched System

Thesis presented by:
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to fulfill the requirements to obtain the degree of:
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Resumen

En la actualidad tanto en la industria como en el hogar, encontramos máquinas modernas cuyo comportamiento depende de ciclos programados; las cuales se encuentran sujetas a diversos procesos. Existen también otros tipos de máquinas que cambian su comportamiento a consecuencia de factores internos o externos a ellas, cambios que se encuentran condicionados a niveles de tolerancia preestablecidos, es decir, son máquinas que cambian su estructura interna constantemente de manera instantánea y repentina. Un ejemplo es el sistema de tracción de un automóvil, el cual cambia su comportamiento en el instante en el cual es realizado un cambio de velocidad; siendo éste un factor externo si el cambio es realizado por una persona ó interno si es de trasmisión automática. Si este sistema fuese modelado, cada cambio de velocidad estaría relacionado a un sistema de ecuaciones diferenciales; sin embargo físicamente sigue siendo el mismo sistema.

Otro ejemplo frecuente lo encontramos en los procesos industriales, en los cuales un solo equipo puede estar sometido a varios procesos; por lo tanto cada proceso se modelaría por un grupo de ecuaciones. En la industria moderna la mayoría de los sistemas se encuentran automatizados, por lo tanto los cambios de los procesos se encuentran condicionados a niveles en los valores de presión, calor, humedad, alcalinidad, etc.

Los sistemas que presentan estas características son llamados sistemas híbridos. En la actualidad este tipo de sistemas ha tomado una gran importancia como campo de investigación y de manera más particular se han estudiado activamente una clase de sistemas híbridos: los sistemas conmutados. Éstos básicamente se componen de dos dinámicas, una de eventos discretos modelados comúnmente por autómatas híbridos, redes de Petri, etc., y la cual describe el comportamiento de los cambios del sistema y controladores; y una segunda dinámica de tipo continua la cual describe cada subsistema para cada evento discreto.

El problema principal en este tipo de sistemas es que algunas propiedades de interés, tales como la estabilidad, se mantengan respecto al cambio de la dinámica del sistema híbrido. En particular, un resultado conocido es que la estabilidad individual del conjunto de dinámicas continuas no es una condición suficiente para garantizar la estabilidad total conmutado. Además al conmutar estas dinámicas, el controlador cambia respecto al estado actualizado, produciéndose en cada conmutación una dinámica

transitoria que generalmente afecta al error de seguimiento o de estabilización.

Por otro lado, para resolver el problema de seguimiento asintótico de trayectorias de referencia, entre los diversos esquemas disponibles en la lectura, el problema de la regulación ha proveído una solución relativamente atractiva, ya que se basa en encontrar una región de espacio de estado donde el error de seguimiento de la trayectoria es cero, y que se fuerza a ser atractiva con una selección adecuada de la señal de control. En términos generales, la solución al problema de la regulación se encuentra basada en la solución de un conjunto de ecuaciones diferenciales parciales conocidas como ecuaciones FIB (Francis-Isidory-Byrnes).

Por este motivo, debido a la importancia de la teoría de la regulación y la facilidad con la cual las máquinas o los dispositivos pueden ser modelados por los sistemas conmutados; la presente tesis se enfoca en la solución del problema de la regulación para los sistemas conmutados. Específicamente se analizarán dos tipos de estructuras: la conmutación exacta y la conmutación con acotación del error.

En la conmutación exacta, una vez que el error es cero y por lo tanto el sistema alcanza su referencia; el error debe mantenerse en cero aún entre las conmutaciones. La solución propuesta para este problema es conmutar en la intersección de las regiones invariantes de cada sistema, aprovechando el hecho de que los estados no pueden escapar de estas regiones cuando se encuentran dentro de ellas.

Por otro lado, la segunda estructura propuesta es la conmutación con acotación del error. En aplicaciones reales es difícil ó imposible encontrar la solución de la condición para realizar la conmutación exacta; por este motivo es necesario relajar las condiciones de conmutación permitiéndose por diseño un error de seguimiento permitido. Para la solución de este problema, se proponen condiciones para encontrar una región de conmutación bajo la cual el error permanece acotado.

En sistemas conmutados se sabe que si las dinámicas continuas modeladas son estables y si la conmutación es suficientemente lenta tal que los efectos del transitorio son disipados, entonces el sistema conmutado es estable. Esta situación da lugar al concepto de "dwell average", y será determinante para encontrar condiciones que permitan mantener la estabilidad para resolver el problema de la regulación para sistemas conmutados.

Abstract

Nowadays, either in the industry, at home, or everywhere; we can find modern machines whose behavior frequently depend on programmed cycles, executing diverse activities in order to obtain a final objective. These machines may also need to change their behavior when some pre-specified events occur. The models describing such systems represent a special class of dynamical models, namely, those systems whose overall performance change by the time.

An automobile traction system can be taken as an example of such systems. Here, its internal structure changes when a gear change or speed ratio takes effect. The model of this system is thus formed by a set of differential equations; each of these set related to a speed ratio, even though physically is the same traction system.

In general, these systems can be divided in two subsystems; one, generally a logic module containing a discrete or discrete event behavior, and another that can be governed by a set of differential/difference equations, whose contribution to the overall performance depends on the rules prescribed by the logic subsystem, based either on preset values on their states, or commuting at some pre-specified set of time instants. Recently, the study of this kind of systems, called *hybrid systems*, has earned a great importance. A special class of hybrid systems is that of switched systems. These systems are composed by two dynamics. The first one is a discrete events dynamic which is commonly modeled by hybrid automatons or Petri nets, governing the behavior of the so-called warning variables which define when the switched system changes; the second is a continuous dynamic modeled by differential equations for each discrete event.

Switching systems are generally controlled by individual controllers designed for each subsystem. The problem arising with this choice is how to guarantee the stability of the overall system and at the same time to reduce the transient behavior at the switching instants.

On the other hand, the output regulation theory has been of great importance, namely, those systems that need to reach a prescribed trajectory. For instance, a motor needs to be controlled in order to reach a desired speed. The output regulation allows achieving an asymptotic tracking of prescribed trajectories and/or an asymptotic rejection of undesired disturbances. The solution is based on solving a set of partial differential equations, called FIB equations (Francis-Isidori-Byrnes).

Due to the importance of the output regulation theory and the easy form on modeling diverse devices by switched systems, the present work are devoted to study the switched systems by means of the output regulation theory; namely, the cases of exact switching and bounded error switching.

In the case of exact switching, when a system tracks the reference, the tracking error goes to zero, and it never must increase at switching instants. The proposed solution on solving this problem is to switch the subsystems between the intersections of the invariant manifolds of each continuous dynamics, which are found by solving the FIB equations.

However, in real applications is hard or simply impossible to find the solution in the case of exact switching, for this reason, we need to relax the switching conditions. Therefore, a scheme allowing bounded tracking error is considered here. The proposed solution for solving this problem is to find a switching region near the origin for which a bounded error is guaranteed.

It is well known that a switched system is stable if all individual subsystems are stable and the switching is sufficiently slow, so as to allow the transient effect to be dissipated after each switch. The time constant fulfilling the previous properties is called dwell time, and by means of this, the stability in the output regulation problem in switched systems is proved.

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Chapter 1

Introduction

Many systems encountered in practice involve a coupling between continuous dynamics and discrete events. Systems in which these two kinds of dynamics coexist and interact are usually called hybrid. These systems have attracted considerable attention in recent years driven by rapid advances in modern technology of digital controllers. Hybrid systems are usually represented by hybrid automatons [20], [8] and [22]. Such automatons are an easy form to describe the systems. An application related with this kind of systems is found in [7], here, the authors modeled electromagnetic valves for camless engines, and controlled them by means of the regulation theory [3]. Another work is found in [27], here the authors characterize the minimal set of extra output information to be provided by continuous signals in order to satisfy observability conditions.

A special class of hybrid systems is that of switched systems. A switched system may be obtained from a hybrid system by neglecting the details of the discrete behavior and instead considering all possible switching patterns from a certain class [12]. Recently, the research field of switched systems has attracted the attention of many people with diverse backgrounds; hence, several publications have been diffused in switched systems theory. For instance, in [11] and [12] the author describes the most recent and important results on stability and control in switched systems. An special characteristic of switched systems is the one in which stability for each individual subsystem is not an enough condition for ensuring stability, for this reason, most results in switched systems are essentially developed for the stability theory, for example in [21], [26], [29], and [16].

A sketch of this kind of systems is illustrated in figure 1.1, where a system (plant) and n dynamical controllers are steered by a logical controller (supervisor).

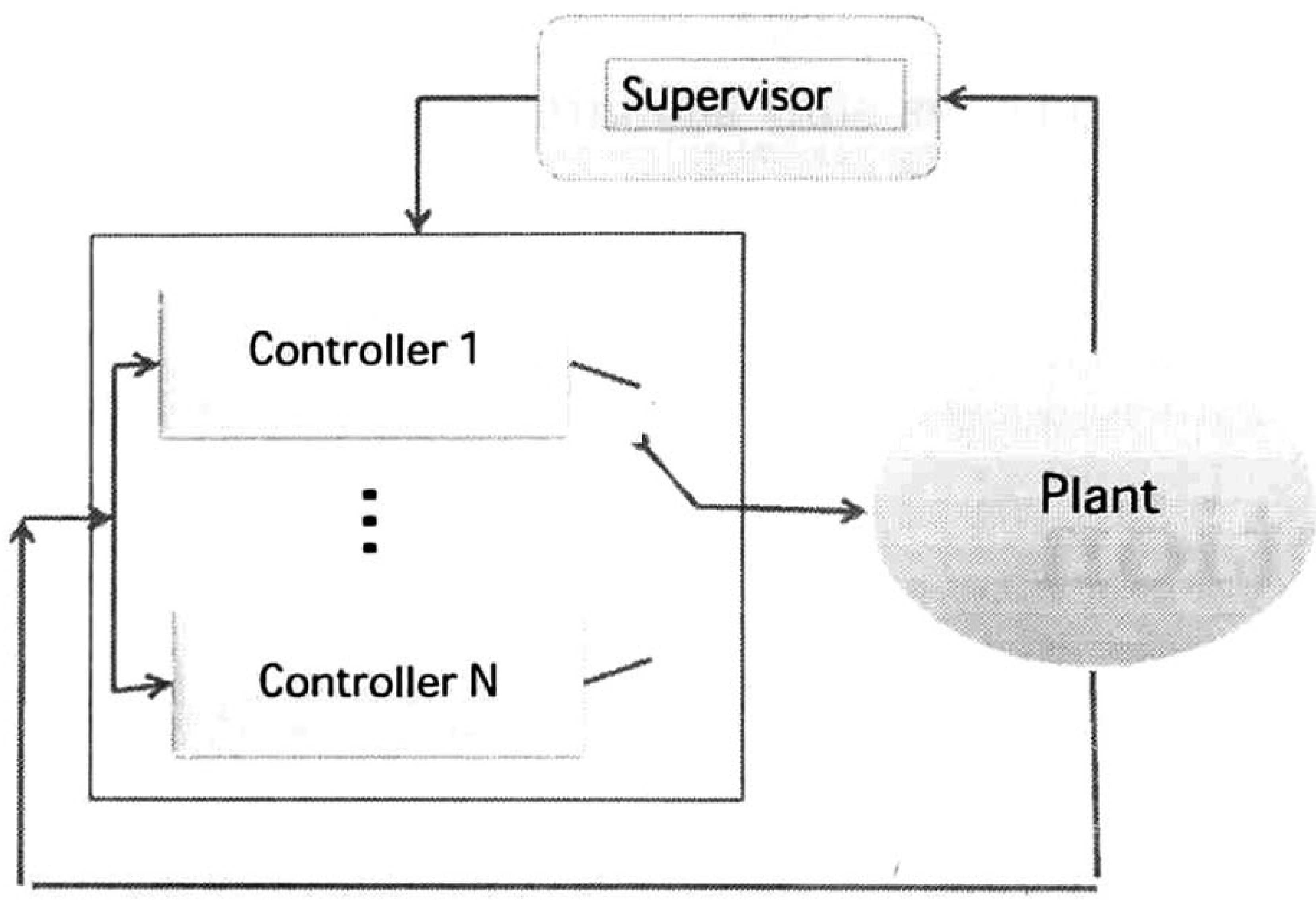

Figure 1.1: A switched system structure

1.1 Preliminaries

In control theory there are many approaches to study the stability. For example, stability in the sense of Lyapunov is based in the system energy, where a stable system tends to an equilibrium state when the system energy is dissipated and leads to infinity with infinity energy. A good exposition of many results on stability can be found on [17]. Other approaches on stability are the concept of input to state stability [14] and input output stability. In the case of discrete dynamical systems, some results can be found in [23], [15], and [9].

However, most of these approaches are hard to be applied to the case of switched systems, because it is difficult to find in many cases a common Lyapunov function. Furthermore, it is known that switched systems can be unstable even if their individual subsystems are stable, the opposite result is also true. This fact is illustrated in figure 1.2, where for two individual stable subsystems and a bad switching policy, the switched system might be unstable. Hence, there are several results on stability in switched systems, for example [26], [29], and [13], however, most of them depend of proper subsystems [16].

Another stability approach is the so-called dwell time τ, the one which with a τ sufficiently large, allows to assure asymptotic stability of switched system providing stable subsystems. This must fulfill the property that all switching times $t_1, t_2, \ldots$ satisfy the inequality $t_{i+1} - t_i \geq \tau$ for all i. The disadvantage of this approach is always to wait for this time.

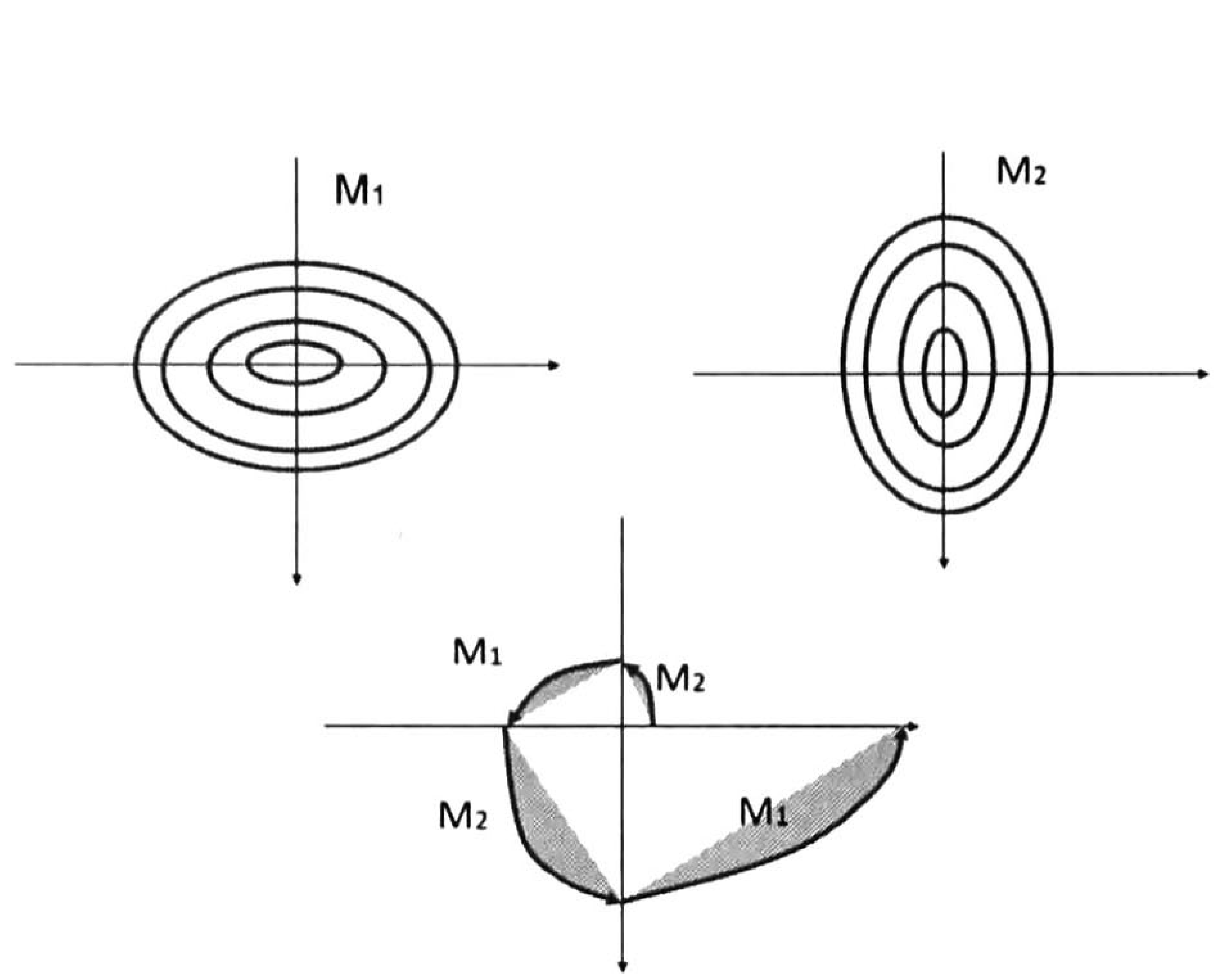

Figure 1.2: Fase portrait of an unstable switched system

For this reason, in 1999 the approach of Average dwell time τ_a was introduced by Hespanha and Morse [21]. Here, a switched system can switch a subsystem before the time τ_a, and later compensate it, along the interval of time T. Works and developments in this topic can be found in [13] and [28] for the continuous-time case, and in [25] in the case of discrete systems. Recently, the average dwell time has been frequently used to prove stability in switched systems. For example in [24] the input to state stability of switched systems was proved. In [12] and [14] the most current and important results are summarized.

On the other hand, the output regulation theory was mainly developed by the work of Francis, Isidory and Byrnes. They introduced the center manifold theory in the output regulation problem, and found that it was possible to use a set of mixed nonlinear partial differential and algebraic equations, called regulator equations, in order to characterize the steady state response of the nonlinear system. These important results are described in [3], where the problem of full information, error feedback, and structurally stable nonlinear output regulation are solved. The regulator equations are also named FIB equations (Francis-Isidory-Byrnes). Another important theoretical and application works can be found in [1, 2, 4, 5, 10, 19]

Another important work is found in references [18] and [6]; here, a control scheme for robust regulation of a discretized nonlinear system is presented to ensure a ripple-free behavior in the intersampling time. This approach was motivated because a discretized control cannot guarantee robust regulation between sampling times. The proposed solution is based in an exponential holder, which shapes the steady state response between sampling times, achieving robust regulation.

Despite the many results in the case of both continuous and discrete non hybrid systems, for the case of switched systems, there are few results concerning the regulation problem. In [31], Liu and Zhao study the output regulation problem of a class of linear systems with disturbances. In this work, the switched linear system has the same error equation, and the authors give conditions to guarantee the output regulation. Another recent work was made by V. Gazi in [30]. Here, the same linear system is maintained along the commutations, and only the exosystems are switched in order to generate more complex trajectories of that generated under the classical regulation theory, assuring the zero error in the switching instants.

1.2 Motivation

At present, modern machines are being designed in order to perform more complex tasks, whose processes in most cases are frequently steered by digital controllers. For this reason, the hybrid systems theory has had an increasing relevance within control theory in the last years.

As mentioned before, switched systems are a special case of hybrid systems. Here, even though many results have been developed, there is no a systematic qualitative theory yet. For this reason, the motives

- The study of Switched Systems by means of the Output Regulation Theory
- Feasibility to model extensive kinds of devices
- Starting a new research line for this topic

1.3 Objectives

There are many results on stability of switched systems; however, more structures of control are needed to be developed. On the other hand, the output regulation has been an important structure of control within the classical control theory. Therefore, the main objective whereby this work is developed is

- To propose a structure of control by means of the regulation theory in switched systems.

In particular, this work is concerned with the following goals

- To establish switching conditions to ensure a zero error.

- To find a switching zone to ensure a maximum bounded error.

To achieve the previous goals, it is necessary

1. To study the output regulation theory in the cases of full information and error feedback output regulation.
2. To review stability in sense of Lyapunov.
3. To learn the basic theory of switched systems.
4. To check the current research on stability of switched systems.
5. In general to check current research of switched systems.

1.4 Thesis structure

- **Chapter one**.- This chapter provides to the reader a brief summary on the main results in switched systems and the output regulation theory. This chapter also presents the motivation whereby this work is developed, and the goals to be reached.
- **Chapter two**.- Important results in switched systems and the output regulation theory are presented in this chapter. In the first part, concepts and theorems on stability in sense of Lyapunov are provided. The second part deals with the switched system theory, where basic theorems and concepts are presented. Finally, the last section presents some important results on regulation theory in the cases of full information and error feedback output regulation.
- **Chapter three**.- This chapter is concerned to find conditions for switched systems controlled by means of the regulation theory in order to maintain a zero error, assuring stability of the system. This result is given for the cases of full information and error feedback output regulation, in both continuous and discrete dynamics.
- **Chapter four**.- This chapter is concerned about finding a switching region that allows to remain the tracking error within a preset region, assuring stability. These conditions are given in the cases of discrete and continuous systems.
- **Chapter five**.- Conclusions and final comments are given, as well as some suggestions for future research.

Chapter 2

Mathematical background

This chapter presents basic results to be used through this work. In the first section, important concepts and theorems on stability of dynamical systems are given, based mainly in [17, 15]. Namely, stability in sense of Lyapunov for nonlinear systems is presented in both cases continuous and discrete systems.

In the second section, theorems and definitions for switched systems are provided. First, definitions and characteristics about switched systems are exhibited, and then switching event definition and its classifications are exposed. Finally, it is introduced an important concept on stability the so-called average dwell time [12], which allows to ensure asymptotic stability in switched systems.

The last section contains the principal results of the classical regulation theory [3]. First, the output regulation problem is established, providing important concepts and definitions. In the last part is solved the output regulation problem in the cases of full information and error feedback output regulation.

2.1 Dynamical systems

In most cases, the evolution of physical systems can be approximately modeled by a set of differential equations

$$\dot{x} = f(t, x, u) \tag{2.1}$$

where $f : D \to \Re^n$ is a locally Lipschitz map from a domain $D \subset \Re^n$ into $\Re^n$ Choosing a control input $u = g(x(t), t)$, the closed loop system of (2.1) can be written as

$$\dot{x} = f(t, x). \tag{2.2}$$

where a special case of (2.2) is when the function f does not depend explicitly on time t, in other words

$$\dot{x} = f(x) \tag{2.3}$$

In this case the system is said to be autonomous or time-invariant.

An important concept in dealing with the state equation is the concept of equilibrium points. For the system (2.3), the equilibrium points are the real roots $\bar{x} \in D$ of the equation

$$f(\bar{x}) = 0.$$

Such points can be stables, unstable, or asymptotically stable. The following definition describes the stability of equilibrium points

Definition 1. The equilibrium point $x = 0$ of (2.3) is

- stable if, for each $\epsilon > 0$, there is $\delta = \delta(\epsilon)$ such that

$$\|x(0)\| < \delta \Rightarrow \|x(t)\| < \epsilon, \quad \forall\ t \geq 0$$

- unstable if not stable

- asymptotically stable if is stable and δ can be chosen such that

$$\|x(0)\| < \delta \Rightarrow \lim_{t \to \infty} x(t) = 0$$

See [17].

2.1.1 Stability

The concept of stability is one of the most important properties of dynamical systems, this plays a central role in the system theory and engineering. The stability of equilibrium points is usually characterized in the sense of Lyapunov, where an equilibrium point is stable if all solutions starting at nearby point stay nearby; otherwise, it is unstable. The following theorem gives sufficient condition to test stability

Theorem 1. Let $x = 0$ be an equilibrium point of (2.3). Let $V : \Re^n \to \Re$ be a continuously differentiable function, such that

$$\begin{aligned} V(0) = 0 \quad V(x) &> 0, \quad \forall x \neq 0 \\ \dot{V}(x) &< 0, \quad \forall x \neq 0 \end{aligned}$$

then $x = 0$ is asymptotically stable.

Proof. See [17]. □

Similar results can be formulated for discrete time systems. Consider the autonomous discrete system

$$x[k+1] = f(x[k]), \qquad f(0) = 0 \tag{2.4}$$

the following result gives conditions to test the stability of the equilibrium point of such system.

Theorem 2. Let $x[k] = 0$ be an equilibrium point of (2.4). Let $V : \Re^n \to \Re$ be a scalar function, such that

$$\begin{aligned} V(0) &= 0 \\ V(x[k]) &\geq 0, \qquad V(x[k]) = 0 \Leftrightarrow x[k] = 0 \\ \Delta V(x[k]) &= V(f(x[k])) - V(x[k]) \leq 0, \quad \Delta V(x[k]) = 0 \Leftrightarrow x[k] = 0 \end{aligned}$$

then $x[k] = 0$ is asymptotically stable.

Proof. See [15]. □

The following theorem is a converse theorem of stability and is concerned about exponential stability, which will be used in the development of the present work

Theorem 3. Let $x = 0$ be an equilibrium point for the nonlinear system

$$\dot{x} = f(t, x) \tag{2.5}$$

where $f : [0, \infty) \times D \to \Re^n$ is continuously differentiable, $D = \{x \in \Re^n | \|x\| \leq r\}$, and the Jacobian matrix $\partial f / \partial x$ is bounded on D, uniformly in t. Let κ, γ and r_0 be positive constants with $r_0 \leq r/k$. Let $D_0 = \{x \in \Re^n | \|x\| \leq r_0\}$. Assume that the trajectories of the system satisfy

$$\|x(t)\| \leq \kappa \|x(t_0)\| e^{-\gamma(t-t_0)}, \quad \forall\, x(t_0) \in D_0, \ \forall t \leq t_0 \leq 0 \tag{2.6}$$

Then, there is a function $V : [0, \infty) \times D_0$ that satisfies the inequalities

$$\|x(t)\| \leq \kappa \|x(t_0)\| e^{-\gamma(t-t_0)}, \quad \forall\, x(t_0) \in D_0, \ \forall t \leq t_0 \leq 0 \tag{2.7}$$

for some positive constants c_1, c_2, c_3 and c_4. Moreover, if $r = \infty$ and the origin is globally exponentially stable, then $V(t, x)$ is defined and satisfies the above inequalities on $\Re^n$ Furthermore, if the system is autonomous, V can be chosen independent of t.

2.2 Switched systems

Traditionally, in the development of classical control either continuous or discrete behaviors has been taken into account separately, however, dynamical systems described by an interaction between both continuous dynamics and discrete events are usually called hybrid systems.

An interesting class of hybrid systems is represented by switched systems. Such systems consist in a collection of dynamical systems for which, the overall behavior is obtained by a suitable switching policy. In these systems, the discrete behavior is replaced by an auto policy. In order to define formally a switched system, discrete event system needs to be defined

Definition 2. A discrete event system is a tuple $D = (Q, Q_0, E, \Psi, \eta)$ such that

1. Q is a finite set of N discrete states
2. $Q_0 \in Q$ is the set of initial conditions
3. $E \subset Q \times Q$ is a collection of edges, each edges $e \in E$ is an ordered pair of discrete states, the first component of them is the source and is denoted by $s(e)$, while the second is the target and is denoted by $t(e)$.
4. Ψ is the finite set of discrete output symbols.
5. $\eta \rightarrow \Psi$ is the output function, that associates to each edge one discrete output symbol.

See [27].

Definition 3. A switched system is a tuple $S = (D, X, X_0, U, Y, \xi)$ such that

1. $D = (Q, Q_0, E, \Psi, \eta)$ is a discrete event system as in Definition (2).
2. $X \subset \Re^n$ is a continuous state space.
3. $X_0 \in X$ is the set of initial continuous conditions.
4. $U \subset \Re^m, Y \subset \Re^p$ are the sets of continuous control input and observable output.
5. $\{\xi_q\}_{q\in Q}$ associates to each discrete states $q \in Q$ the continuous time invariant dynamics

$$\xi_q : \dot{x} = f_q(x, u) \tag{2.8}$$

with output $y = g_q(x)$. The solution of (2.8) exists and is unique under the assumption that f_p is continuous with respect to time and Lipschitz continuous with respect to the dependent variables.

See [27].

2.2.1 Switching events

Switching events specify the policy whereby all subsystems on switched systems are commuted. Switched systems can be categorized due to several kinds of switching events, such events can be classified into

- State dependent versus time dependent
- Autonomous (uncontrolled) versus controlled

where a switched system can have combinations of several types of switching. These will be briefly described.

State dependent switching. A switched system with this type of switching event is specified by

1. The family of switching surfaces and resulting operation regions
2. The family of continuous time subsystems
3. The reset map

These are illustrated in figure 2.1, where the thick curves denote the switching surfaces, the curves with arrows denote the continuous positions of the trajectories and the dashed lines symbolize the jumps [12].

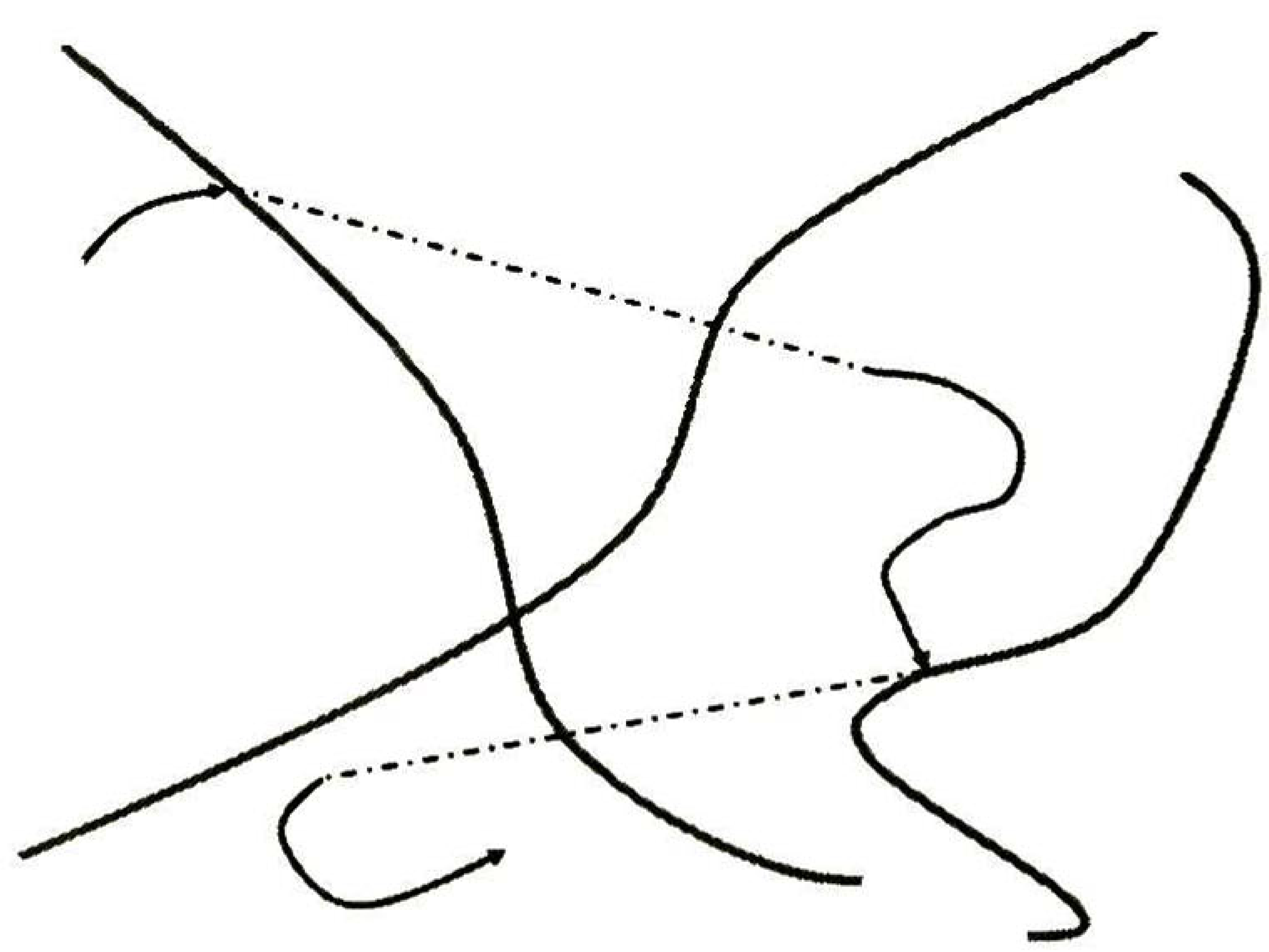

Figure 2.1: State dependent switching

Time dependent switching. Consider a family of systems

$$\dot{x} = f_p(x), \qquad p \in P \tag{2.9}$$

evolving on $\Re^n$ A switching signal is a piecewise constant function $\sigma : [0, \infty) \rightarrow P$. Such a function σ has a finite number of discontinuities called switching times on every bounded time interval and takes a constant value on every interval between two consecutive switching times [12].

This switching signal can be described by means of a hybrid time set, which is defined as follows

Definition 4. A hybrid time set is a sequence of intervals $\tau = \{I_0, I_1, ..., I_N\} = \{I_i\}_{i=0}^{N}$, finite of infinite (i.e. $N = 0$ is allowed) such that

1. $I_i = [\tau_i, \tau_i']$ for all $i < N$;
2. if $N < \infty$ then either $I_N = [\tau_N, \tau_N']$ or $I_N = [\tau_N, \tau_N')$, and
3. $\tau_i < \tau_i' = \tau^{i+1}$ for all i.

See [20].

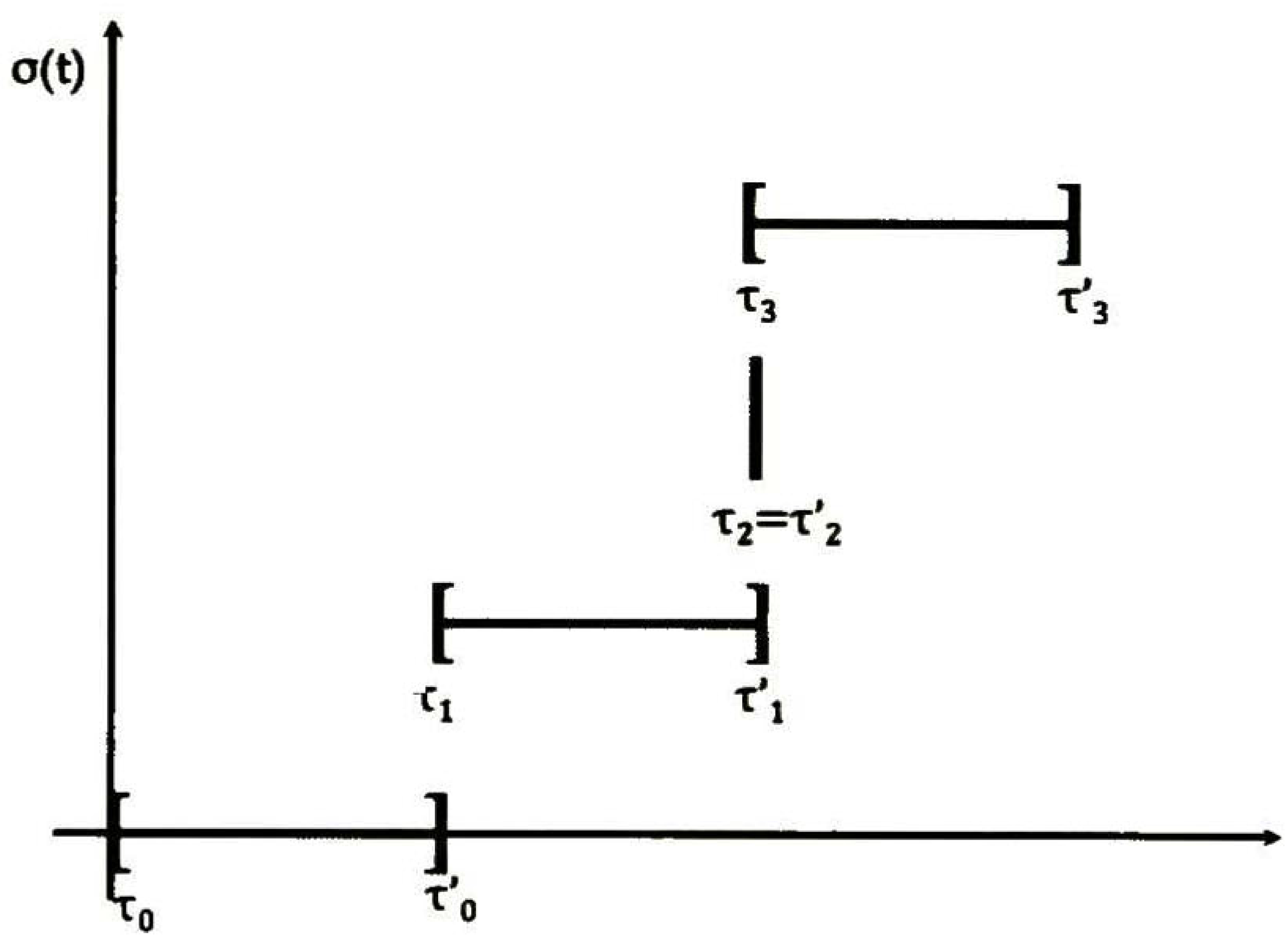

Figure 2.2: Hybrid time set

A switched system with time dependent switching can be described by the equation

$$\dot{x} = f_\sigma(x) \tag{2.10}$$

and this is illustrated in figure 2.3

Autonomous and controlled switching. Autonomous switching event is the one in which the switching mechanism that triggers the discrete events is not controlled

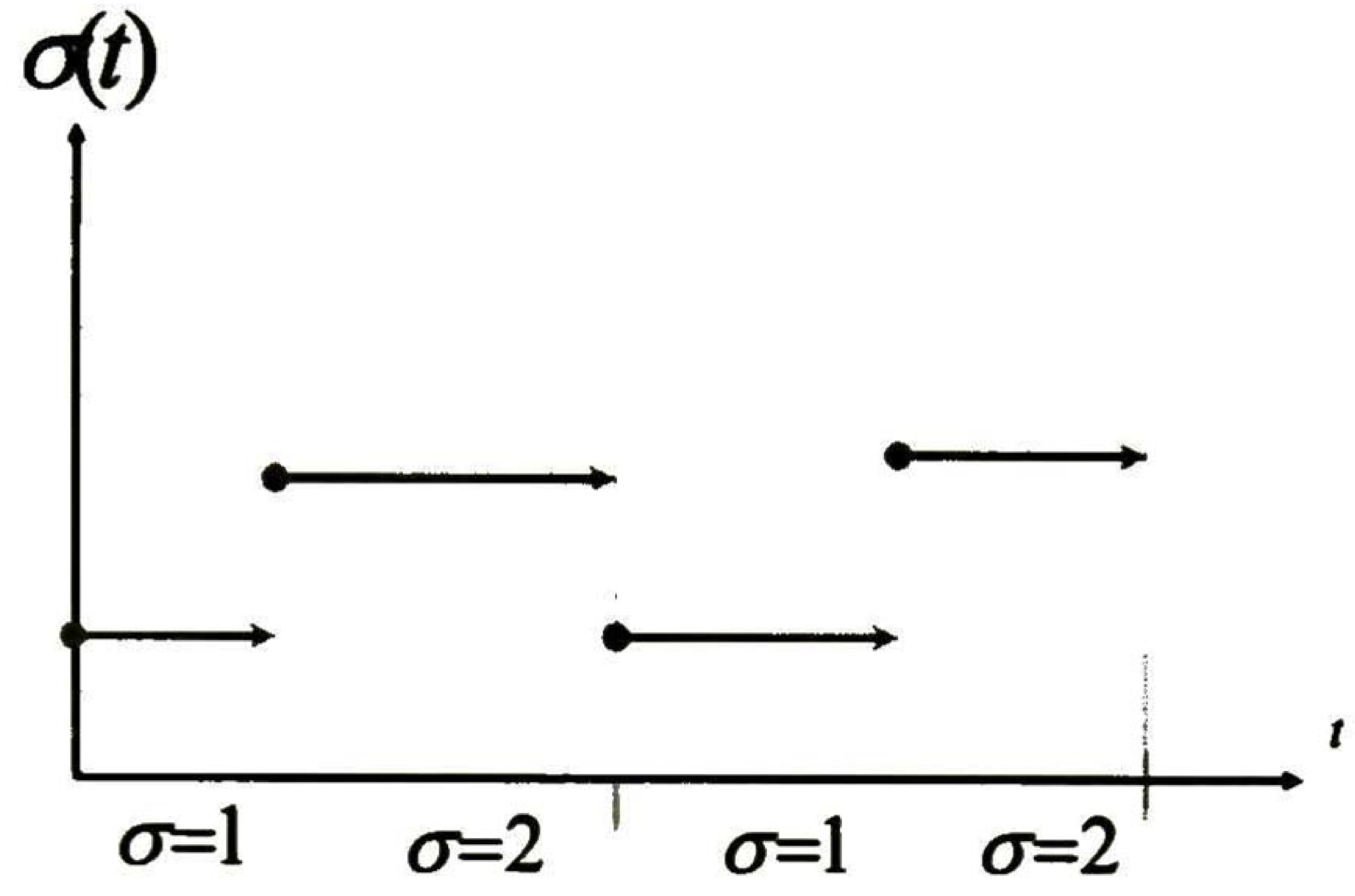

Figure 2.3: Time dependent switching

directly. In this category are included systems with state dependent switching in which the location of the switching surfaces are predetermined as well as systems with time dependent switching when the switching signal is unknown.

Controlled switching event is the one in which the switching mechanism is directly controlled by the designer. The switching is actually imposed by the designer in order to achieve a desired behavior of the system.

2.2.2 Stability of switched systems

There are several results about stability in switched systems. However in most of them, these depend on the structure of systems. In switched systems, it is known that if all subsystems are stable and the switching is sufficiently slow such that the transient effects are dissipated, the switched system become stable.

The simplest way to specify slow switching is to introduce a number $\tau > 0$ and restrict the class of admissible signals to signals with the property that for all switching times $t_1 . t_2, \ldots$ the inequality $t_{i+1} - t_i \geq \tau$ is satisfied. This constant of time between switching is called dwell time τ. The disadvantage of this approach is not to be able to switch before τ.

Thus, Hespanha and Morse introduced the concept of average dwell time, which allows switching faster between subsystems when it is necessary, and then slower to compensate the time.

A switching signal σ has the property of average dwell time if there exist two positive numbers N_0 and τ_a, such that the following inequality holds throughout the interval of time T

$$N_\sigma(T,t) \leq N_0 + \frac{T-t}{\tau_a} \qquad \forall T \geq t \geq 0 \tag{2.11}$$

This property is illustrated by figure 2.4, where $V_\sigma(t)$ with $i \in \sigma$ represent the Lyapunov function for each active subsystem

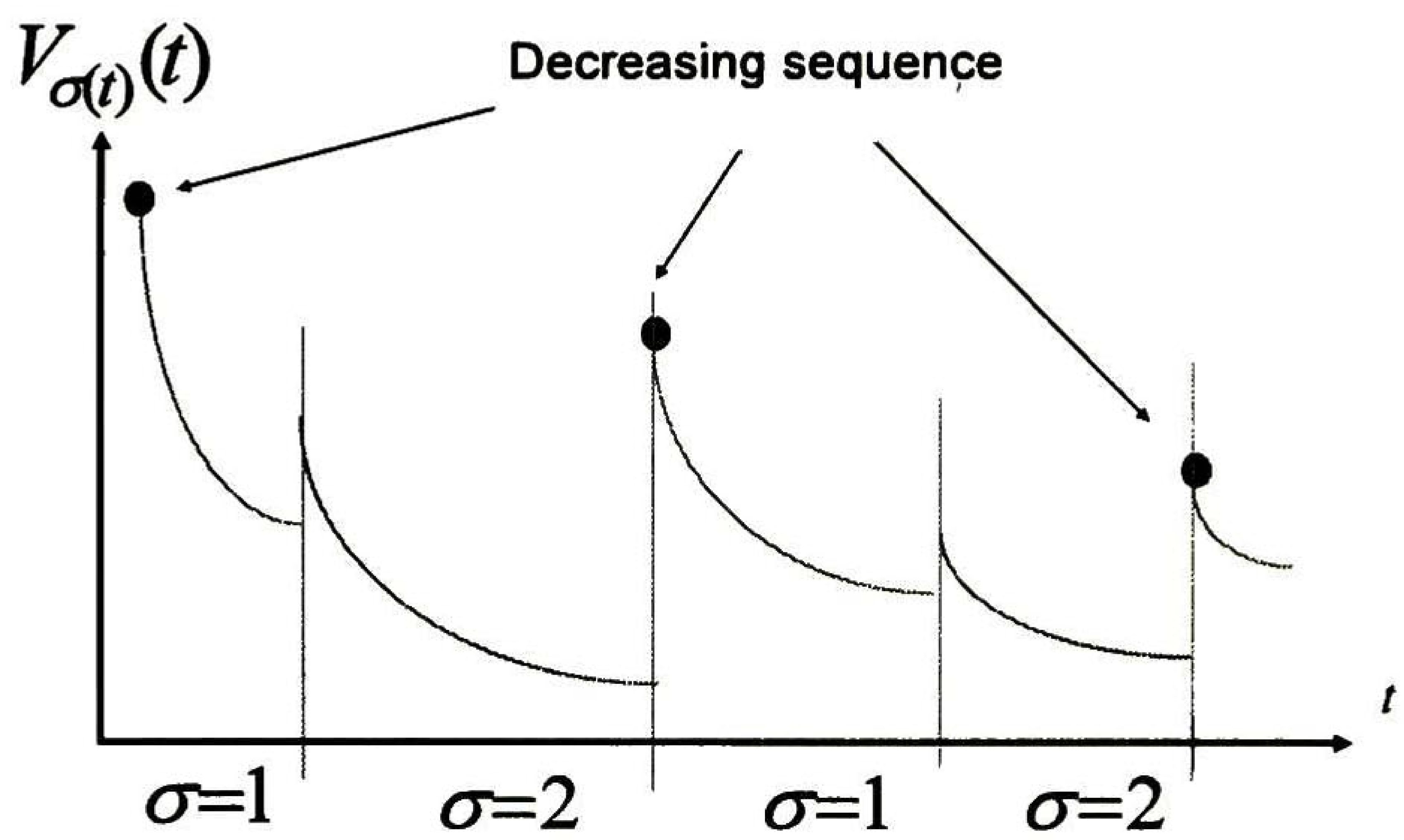

Figure 2.4: A system sequence with average dwell time

Using this property, the next theorem gives conditions to assure asymptotic stability of the system (2.8).

Theorem 4. Consider a family of systems (2.8). Suppose that there exist functions $V_q : \Re^n \rightarrow \Re$, $q \in Q$, two functions α_1 y α_2, and a positive number λ_0 such that we have

$$\alpha_1(|x|) \leq V_q(x) \leq \alpha_2(|x|) \ \ \forall x, \quad \forall q \in Q \tag{2.12}$$

$$\frac{\partial V_q}{\partial x} \leq -2\lambda V_q(x) \ \ \forall x, \quad \forall q \in Q \tag{2.13}$$

suppose also that holds. Then the switched system (2.8) is globally asymptotically stable for every switching signal σ with average dwell time

$$\tau_a > \frac{\ln \mu}{2\lambda_0} \tag{2.14}$$

Proof. See [12]. □

2.3 The Output regulation problem

Dynamical systems usually have two types of responses to control inputs, one called transitory response which disappears when the time goes to infinity, and another called response in steady state and this remains while the system is under the control law.

Analyzing the following system

$$\dot{x} = f(x, u) \tag{2.15}$$

where $x \in \Re^n$ is defined near to the origin, $u \in \Re^m$ is the input control, and assuming that $f(0,0) = 0$; asymptotic stability can be described by the following expression

$$\lim_{t \to \infty} ||f(x_0, u) - f(x^*, u)|| = 0$$

where x_0 is in a neighborhood of x^* Then $f(x^*, u)$ is the steady state response of the system 2.15 and the one in which is specified by f_{ss}.

The response f_{ss} is interesting when the control inputs are persistent, in other words, inputs are periodic and bounded. This fact is more interesting if f_{ss} can be generated by another dynamical system providing an exogenous input, whose dynamics can be modeled by the following equations

$$\begin{aligned} \dot{w} &= s(w) \\ u &= p(w) \end{aligned} \tag{2.16}$$

where w is defined in a neighborhood of $W \in \Re^r$ near the origin and $w(0) = 0$ imply that $p(w) = 0$.

This exogenous system must possess the following features

- $w = 0$ is an equilibrium point,
- there exist an open neighborhood of $w = 0$ for which every point is Poisson stable.

These two features are referred as the property of neutral stability, and this implies that the matrix

$$S = \left[\frac{\partial s}{\partial w}\right]_{w=0} \tag{2.17}$$

with S defined as the first approximation of the vector field $s(w)$ at $w = 0$, has all its eigenvalues on the imaginary axis.

Analyzing the system (2.15) with the control input generated by (2.16), one obtain the following closed loop system

$$f(x, p(w)) \tag{2.18}$$

where using the center manifold theory, the behavior of the system (2.18) has some characteristics, which are formalized in the following proposition

Proposition 1. Assume that (2.16) is neutrally stable. Assume that the equilibrium point $x = 0$ of $\dot{x} = f(x, 0)$ is asymptotically stable in first approximation. Then, exists a map $x = \pi(w)$ defined in a neighborhood $W^o \subset W$ of the origin, with $\pi(0) = 0$, which satisfies

$$\frac{\partial \pi}{\partial w} = f(\pi(w), p(w))$$

for all $w \in W^o$ Moreover, for each $w \in W^o$, the input

$$u^*(t) = p(\phi(w))$$

produces an answer in a defined steady state, which is given for

$$x_{ss} = x(t, \pi(w^*), u^*)$$

2.3.1 Problem setting

Roughly speaking, the output regulation problem is concerned to design a feedback control law that takes the system into a desired steady state response.

Consider the nonlinear systems modeled by the following equations

$$\begin{aligned} \dot{x} &= f(x, w, u) \\ e &= h(x, w) \end{aligned} \tag{2.19}$$

where the first equation describes the dynamics of a plant whose state x is defined in a neighborhood U of the origin in $\Re^n$, with control input $u \in \Re^m$ and subject to a set of exogenous input variables $w \in \Re^r$ which includes disturbances to be rejected y/o references to be tracking. The second equation defines an error variable $e \in \Re^m$ which represents the tracking error, and this is in function of the state x and the exogenous input w.

The family of exogenous inputs w is the set of all time functions, which are solution of a homogeneous differential equation

$$\dot{w} = s(w) \tag{2.20}$$

with initial condition $w(0)$ ranging on some neighborhood W of the origin or $\Re^r$

The output regulation problem is solved if is found a control law fulfilling the following conditions

- The origin is an equilibrium point asymptotically stable in first approximation of the closed loop system.
- There exist a neighborhood of the origin, such that for each initial conditions the error $e(t)$ converges to zero.

Due to these conditions and the center manifold theory, the closed loop system has an invariant manifold mapped by $x = \pi(w)$. Then, the control law is required to be formed by two parts. One part in order to stabilize the system and takes it into the invariant manifold, and another to keep the system inside the manifold.

2.3.2 Output regulation in the case of Full Information

The nonlinear regulation problem for the system defined by

$$\dot{x}(t) = f(x(t), w(t), u(t)) \tag{2.21}$$
$$\dot{w}(t) = s(w(t)) \tag{2.22}$$
$$e(t) = h(x(t), w(t)) \tag{2.23}$$

where the state $x(t) \in U \subset \mathbb{R}^n$ is defined near the origin, with input control $u(t) \in \mathbb{R}^m$ and subject to a set of exogenous input variables $w(t) \in W \subset \mathbb{R}^r$ which includes disturbances to be rejected and reference to be tracked; consists in finding a controller that takes the state of the plant to a desired steady state response.

It is necessary to mention that the equation (2.23) describes the tracking error $e(t) \in \mathbb{R}^m$, which is defined as the difference among the output of the system ($y_i(t)$ with $i = 1, .., m$) and the desired reference. It is also assumed that $f(0,0,0)$,$s(0)$ and $h(0,0)$ are analytic functions, with $s(0) = 0$, $f(0,0,0) = 0$ and $h(0,0) = 0$.

Full information output regulation for nonlinear systems

The output regulation problem for the system (2.21) with all states measured, is to find a control law

$$u(t) = \alpha(x(t), w(t)) \tag{2.24}$$

fulfilling the following conditions

S) With $w = 0$, the equilibrium point $x(t) = 0$ of the closed loop system

$$\dot{x}(t) = f(x(t), 0, \alpha(x(t), 0))$$

is asymptotically stable in first approximation,

R) there exist a neighborhood $V \subset U \times W$ near of $(0,0)$ such that, for all initial condition $(x(0), w(0))$, the output of the closed loop system (2.21), (2.22), (2.23) and (2.24) satisfy

$$\lim_{t \to \infty} e(t) = 0.$$

Defining A, B, P, C, S and Q by

$$\begin{array}{llllll} A & = \frac{\partial f(x,w,u)}{\partial x}|_{(0,0,0)}, & B & = \frac{\partial f(x,w,u)}{\partial u}|_{(0,0,0)}, & P & = \frac{\partial f(x,w,u)}{\partial w}|_{(0,0,0)}, \\ C & = \frac{\partial h(x,w)}{\partial x}|_{(0,0)}, & S & = \frac{\partial s(w)}{\partial w}|_{(0,0)}, & Q & = \frac{\partial h(x,w)}{\partial w}|_{(0,0)}, \end{array}$$

the main result for the full information output regulation problem is given by the following theorem

Theorem 5. If the next conditions are sustained

R1) The system $\dot{w}(t) = Sw(t)$ is neutrally stable,

R2) there exist a matrix K such that $A + BK$ in Hurwitz stable,

R3) there exist a map in steady state $x_{ee}(t) = \pi(w(t))$ and $u_{ee}(t) = c(w(t))$ with $\pi(0) = 0$ and $c(0) = 0$, in a neighborhood $W^\circ \subset W$ near the origin, such that

$$\frac{\partial \pi(w(t))}{\partial w(t)} s(w(t)) = f(\pi(w(t)), w(t), \alpha(\pi(w(t)), w(t))), \tag{2.25}$$

$$0 = h(\pi(w(t)), w(t)), \tag{2.26}$$

for all $w(t) \in W^\circ$;

then the full information output regulation problem is solved by the control law $u(t) = \alpha(x(t), w(t)) = K[x(t) - \pi(w(t))] + c(w(t))$.

Proof. See [3] □

Full information output regulation for linear systems

In the case of the full information output regulation problem for linear systems, the theorem (5) can be reduced in a simpler form given by

Theorem 6. If the following conditions are sustained

R1) The system $\dot{w}(t) = Sw(t)$ is neutrally stable

R2) There exist a matrix K such that $A + BK$ in Hurwitz stable

R3) There exist a map in steady state $x_{ee}(t) = \pi w(t)$ and $u_{ee}(t) = \Gamma w(t)$ with $w(0) = 0$ such that

$$\Pi S = A\Pi + B\Gamma + P, \tag{2.27}$$

$$0 = C\Pi + Q, \tag{2.28}$$

for all $w(t)$;

then the full information output regulation problem in the case of linear systems is solved by the control law $K[x(t) - \pi w(t)] + \Gamma w(t)$.

Proof. See [3]. □

2.3.3 Output regulation in the case of Error Feedback

However, in real situations, many times the state cannot be measured, only the tracking error. For this reason, is important to design an error feedback control law in order to achieve the output regulation.

Error feedback output regulation for nonlinear systems

In this case, the control law can be expressed by the following equations

$$\dot{\xi}(t) = \eta(\xi(t), e(t)), \quad (2.29)$$
$$u(t) = \theta(\xi(t)), \quad (2.30)$$

where $\xi(t) \in \Xi \subset R^{\nu}$

This control require the next conditions

E) with $w = 0$, the equilibrium point $(x(t), \xi(t)) = (0, 0)$ of the closed loop system

$$\begin{aligned} \dot{x}(t) &= f(x(t), 0, \theta(\xi(t))) \\ \dot{\xi}(t) &= \eta(\xi(t), h(x(t), 0)) \end{aligned}$$

is asymptotically stable in first approximation;

R) there exist a neighborhood $V \subset U \times \Xi \times W$ of $(0, 0, 0)$ such that, for all initial conditions $(x(0), \xi(0), w(0))$, the solution of the closed loop system (2.21), (2.22), (2.23), (2.29) and (2.30) the following condition is held

$$\lim_{t \to \infty} e(t) = 0.$$

Defining the matrix F, G y H with

$$F = \frac{\partial \eta(\xi, e)}{\partial \xi} |_{(0,0)}, \quad G = \frac{\partial \eta(\xi, e)}{\partial e} |_{(0,0)}, \quad H = \frac{\partial \theta(\xi)}{\partial \xi} |_{(0)},$$

The main result for error feedback output regulation problem is expressed by the following theorem

Theorem 7. If the followings conditions are sustained

R1) The system $\dot{w}(t) = Sw(t)$ is neutrally stable;

R2) There exist a matrix G such that $\begin{pmatrix} A & BH \\ GC & F \end{pmatrix}$ is Hurwitz stable

R3) There exist maps $x(t) = \pi(w(t))$ and $\xi(t) = \sigma(w(t))$ with $\pi(0) = 0$ and $\xi(0) = 0$, defined in a neighborhood $W^\circ \subset W$ near the origin, and satisfying the next equalities

$$\begin{aligned} \frac{\partial \pi(w(t))}{\partial w(t)} s(w(t)) &= f(\pi(w(t)), w(t), \theta(\sigma(w(t)))), \\ \frac{\partial \sigma(w(t))}{\partial w(t)} s(w(t)) &= \eta(\sigma(w(t)), 0) \\ 0 &= h(\pi(w(t)), w(t)), \end{aligned}$$

for all $w(t) \in W^\circ$;

then the error feedback output regulation is solved by the controller (2.29) and (2.30).

Proof. See [3]. □

Error feedback output regulation for linear systems

For the error feedback output regulation problem in the case of linear systems, theorem 7 can be reduced in a simpler form by the following theorem

Theorem 8. If the followings conditions are sustained

R1) The system $\dot{w}(t) = Sw(t)$ is neutrally stable;

R2) There exist a matrix G such that $\begin{pmatrix} A & BH \\ GC & F \end{pmatrix}$ is Hurwitz stable

R3) There exist a map in steady state $x_{ee}(t) = \pi w(t)$ and $\xi(t) = \Sigma w(t)$ with $w(0) = 0$ such that

$$\begin{aligned} \Pi S &= A\Pi + B\Gamma + P, && (2.31) \\ 0 &= C\Pi + Q, && (2.32) \\ \Sigma S &= F\Sigma, && (2.33) \\ \Gamma &= H\Sigma && (2.34) \end{aligned}$$

for all $w(t)$;

then the error feedback output regulation is solved by the controller

$$\begin{aligned} \dot{\xi}(t) &= F\xi(t) + Ge \\ u &= H\xi(t) \end{aligned} \qquad (2.35)$$

Proof. See [3]. □

Chapter 3

Exact switching conditions

3.1 General scheme

In any real application, to control a system and reducing the energy cost are important requirements for the control designer. When a machine is powered, this needs an extra energy in order to reach a programmed reference. For instance, motors need a lot of energy before to reach a desired speed.

When the subsystems of a switched system are switching, the control law always needs an extra power at switching instants in order to track the next desired reference. This fact is produced by the so-called transient state. In order to eliminate the transient some restrictions need to be imposed to the switching signal in order to maintain the zero error, and hence, to reduce the energy cost.

When a system is switching between subsystems, a discontinuity takes place between present and past states and therefore is produced a tracking error. This forces the controller to reach the reference again.

Therefore, this chapter is concerned to establish conditions among switching instants in order to eliminate the transient response, achieving an exact switching by means of the output regulation theory.

The basic idea to achieve this goal is the one in which for all switching times, the states always are in the invariant manifold of the current subsystem, maintaining the zero tracking error. This idea is illustrated in figure 3.1. Therefore, the problem is to find switching conditions such that

1. The zero error is maintained among switching.

2. Stability of switched systems is assured.

Figure 3.2 illustrates the behavior of the error under switching.

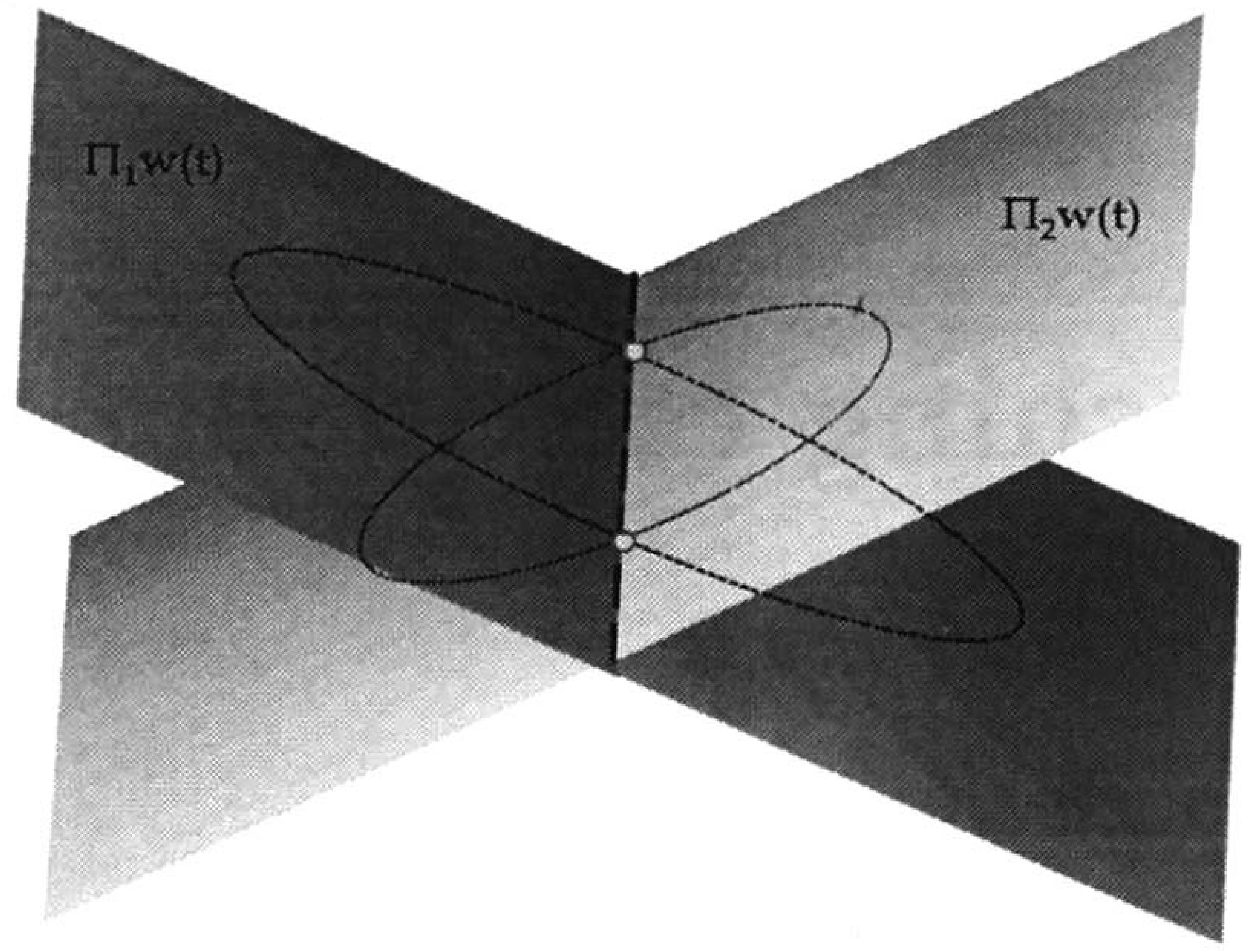

Figure 3.1: Exact switching instant

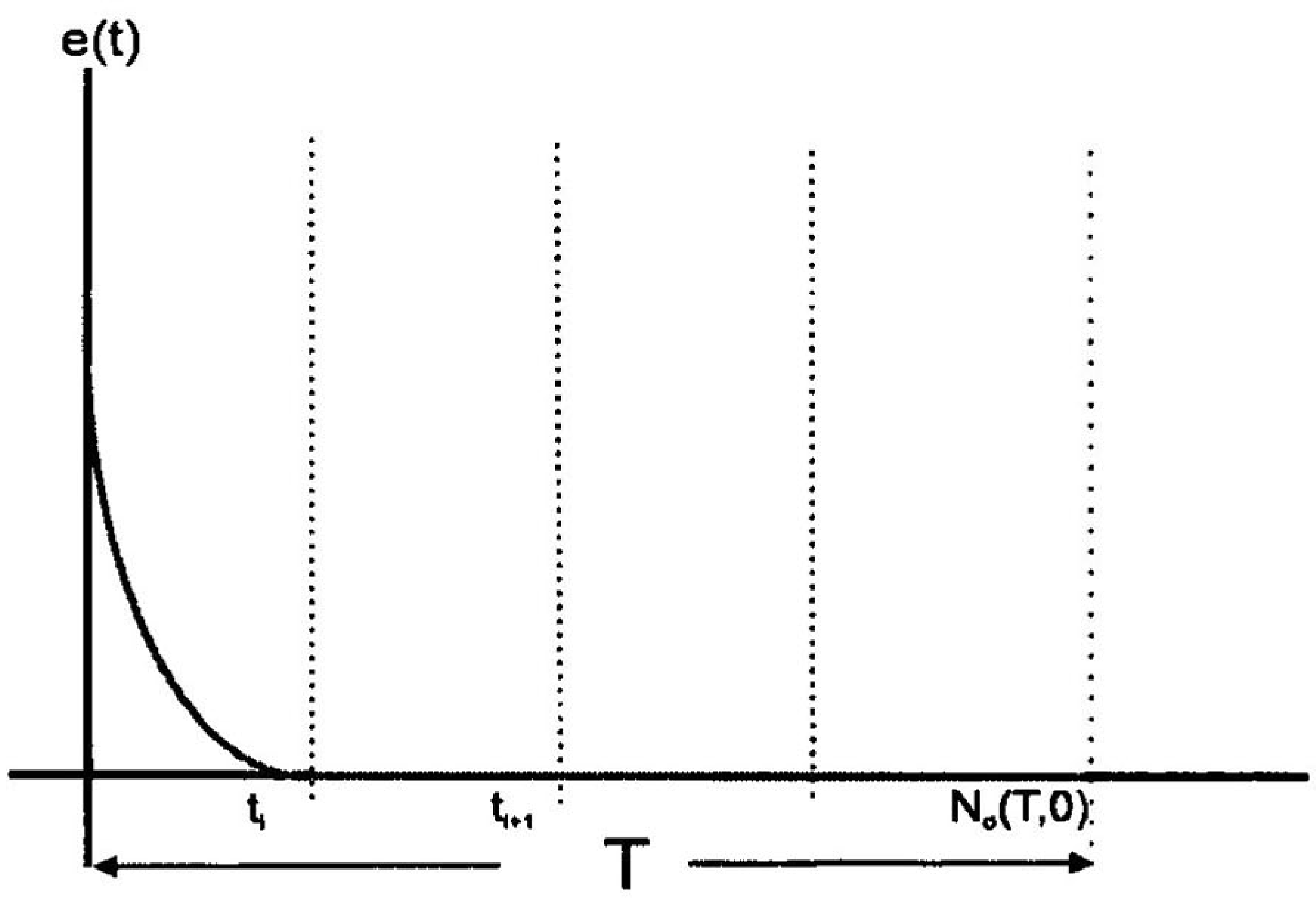

Figure 3.2: Error behavior on exact switching

3.2 Continuous time switched systems

Consider the family of systems

$$\dot{x} = f_q(x, u, w) \quad \forall\, q \in Q \tag{3.1}$$

where the state $x \in \Re^n$ is in a neighborhood of the origin, $u \in \Re^m$ is the control input, $w \in \Re^r$ is an exogenous input containing the disturbances to be rejected and the references to be tracked, and Q represents a set of index $q \in Q$ for each subsystem. The function f_q is locally Lipschitz and the origin is an equilibrium point for all subsystems such that $f_q(0,0,0) = 0 \quad \forall\, q \in Q$.

A continuous system generated by the family of systems (3.1) and a switching signal σ is represented by the following equation

$$\dot{x} = f_i(x, u, w) \quad \forall\, i \in \sigma \tag{3.2}$$

where $\sigma : [0, \infty)$ is a piecewise constant function to segments, continuously from the right, specifying at every time the index of the active subsystem. Moreover, the switched

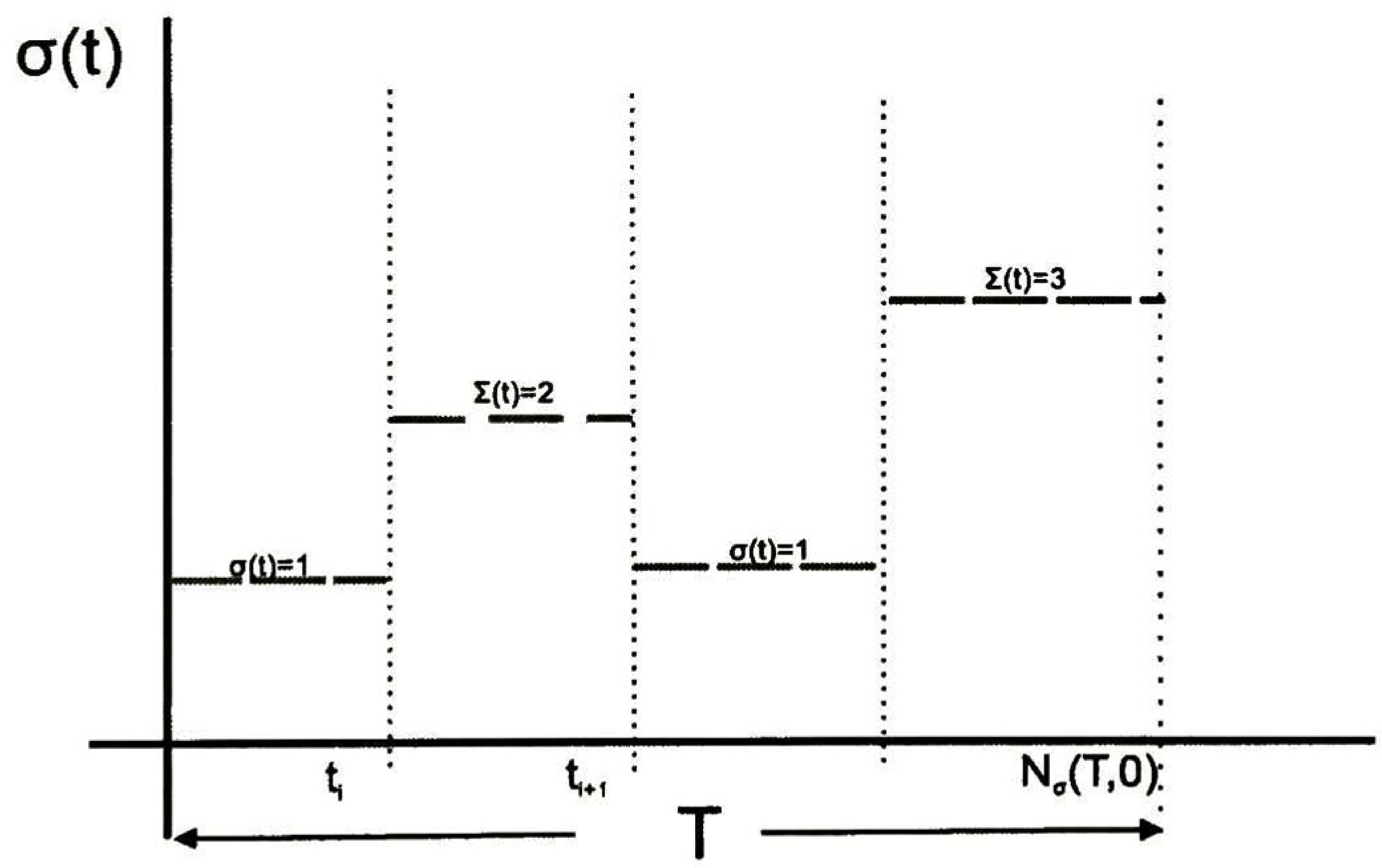

Figure 3.3: Switching signal σ

system to be analyzed through this chapter is given by the following equations

$$\begin{aligned} \dot{x} &= f_i(x, u, w) \\ \dot{w} &= S_i(w) \qquad\qquad \forall\, i \in \sigma \\ e &= h_i(x, w) \end{aligned} \tag{3.3}$$

where the first equation represents the family of all subsystems, the second one represents the family of all exogenous systems, and the last one represents the tracking error of the switched system.

3.2.1 Full information output regulation for switched systems

Before continuing with the main result, some assumptions need to be establish about all subsystems

Assumption 1. For all subsystems with $i \in \sigma$ the output regulation problem is solvable using full information.

If a subsystem is not solvable using the full information output regulation theory this will also lead the switching regulation problem unsolvable. However, the assumption 1 is not a sufficient condition because switching among stable systems not guarantees stability, hence the switched system maybe lead an instability situation.

In order to analyzed the problem, first is found the zero error condition, next is analyzed the stability condition.

Zero error condition

The difference between the actual state and the desired steady state is given by

$$\xi = x - \Pi_i(w_i(t)).$$

such a equation, when $t \longrightarrow \infty$, ξ goes to zero

$$\begin{aligned} 0 &= x - \Pi_i(w_i(t)) \\ x &= \Pi_i(w_i(t)). \end{aligned}$$

With $i+1$, the following active subsystem is represented by

$$\xi = x - \Pi_{i+1}(w_{i+1}(t))$$

hence, the switching instants T_s are represented by the following expression

$$\xi = \Pi_i(w_i(t)) - \Pi_{i+1}(w_{i+1}(t)) \quad \forall t \in T_s.$$

The only way to maintain the zero error is when the following equation is held for all switching instant T_s

$$\Pi_i(w_i(t)) - \Pi_{i+1}(w_{i+1}(t)) = 0 \quad \forall t \in T_s \tag{3.4}$$

Therefore, if there exist a switching time T_s for which (3.4) is satisfied, then the zero error is held and the condition (1) is satisfied.

For linear systems the condition (3.4) is reduced by the following expression

$$(w_{i+1}(t)) \in \ker(\Pi_i - \Pi_{i+1}) \quad \forall t \in T_s \tag{3.5}$$

Stability condition

Stability of the switched system is proved by the average dwell time, for the closed loop system $f_i = (x, u, 0)$ such that the origin is locally asymptotically stable in first approximation.

Furthermore, by means of the regulation theory [3], the nonlinearities vanish at the origin with their first order derivatives. Hence, the system to be analyzed is given by

$$\dot{x} = (A_i + B_i K_i)x \tag{3.6}$$

where changing variables $Acl_i = (A_i + B_i K_i)$

$$\dot{x} = Acl_i x \quad \forall\, i \in \sigma \tag{3.7}$$

Defining continuously differentiable functions $V_i = x^T R_i x$, we find a set of symmetric, positive definite matrices R_i and positive definite matrices Q_i such that

$$Acl_i^T R_i + R_i Acl_i = -Q_i \tag{3.8}$$

with all subsystems fulfilling the next conditions

$$\begin{aligned} a|x|^2 \leq V_i(x) &\leq b|x|^2 \\ \dot{V}_i &\leq -\lambda_0 V_i(x) \\ V_i(x) &\leq \mu V_{i+1}(x) \end{aligned} \tag{3.9}$$

where a, b and μ are given by

$$\begin{aligned} a &= \inf_{i\in\sigma} \lambda_{min}(R_i) \\ b &= \sup_{i\in\sigma} \lambda_{max}(R_i) \\ \mu &= \sup_{i\in\sigma} \frac{\lambda_{max}(R_{i+1})}{\lambda_{min}(R_i)} \end{aligned} \tag{3.10}$$

and the constant λ_0 is such that

$$\dot{V}_i \leq -\lambda_0 V_i(x) \quad \forall i \in \sigma$$

Therefore, with an average dwell time $\tau_C > \frac{\ln \mu}{\lambda_0}$ for a switching signal σ, the system is locally asymptotically stable and the condition (2) is satisfied. The constant τ_C will be computing in the next chapter.

Main result

Theorem 9. Consider the nonlinear output regulation problem of the system (3.3). If assumption (1) holds, then given a piecewise constant switching signal $\sigma\colon [0,\infty)$ the nonlinear output regulation problem in the case of full information is solved in a neighborhood $W \in U \times V$ near the origin by the switched controller

$$u_i = K_i x + (c_i(w) - K_i \Pi_i(w)) \quad \forall \ i \in \sigma \tag{3.11}$$

if exist a finite time $\bar{\mathrm{t}}$ such that at all switching instant $t_i \geq \bar{\mathrm{t}}$ we have

$$\Pi_i(w_{t_i}) = \Pi_{i+1}(w_{t_i}) \tag{3.12}$$

at the switching instants t_i and $\sigma(t) \in S_{ave}[\tau_C, N_0]$ for some positive constants N_0 and τ_C.

3.2.2 Error feedback output regulation for switched systems

Likewise to the full information case, it is necessary to assume that switching occur among a family of systems for which the error feedback output regulation is solvable, hence for the sake of the proof the next assumption is given

Assumption 2. For all subsystems with $i \in \sigma$ the output regulation problem is solvable using error feedback.

In order to solve the error feedback output regulation problem, the concept of the immersion of one system into another will be introduced.

Given two systems defined on two different state spaces, but having the same output space $\Re^m$, a system (X, f, h) is immerse into another $(\tilde{X}, \tilde{f}, \tilde{h})$ if exist a C^k mapping $\tau : X \to \tilde{X}$, with $k \geq 1$, satisfying $\tau(0) = 0$ and

$$h(x) \neq h(z) \Rightarrow \tilde{h}(\tau(x)) \neq \tilde{h}(\tau(z)) \tag{3.13}$$

such that

$$\begin{aligned} \frac{\partial \tau}{\partial x} f(x) &= \tilde{f}(\tau(x)) \\ h(x) &= \tilde{h}(\tau(x)) \end{aligned} \tag{3.14}$$

for all $x \in X$.

Using this concept in the error feedback output regulation problem, we get the immersion of the autonomous systems

$$\begin{aligned} \dot{w} &= s_i(w) \\ u_{ss} &= c_i(w) \end{aligned} \tag{3.15}$$

into the system

$$\dot{\xi}_0 = \varphi_i(\xi_0) \tag{3.16}$$
$$u_{ss} = \gamma_i(\xi_0) \tag{3.17}$$

by the mapping

$$\xi_0 = \tau_i(w) = \begin{pmatrix} c_i(w) \\ L_f c_i(w) \\ \dots \\ L_f^{v-2} c_i(w) \\ L_f^{v-1} c_i(w) \end{pmatrix} \tag{3.18}$$

where v is the new state space dimension of the new system.

Zero error condition

Denoting by E_{obs} the observation error, consider the following dynamic

$$\begin{aligned} \xi &= x - \Pi_i(w_i(t)) \\ E_{obs} &= \xi_0 - \tau_i(w_i(t)) \end{aligned}$$

when $t \to \infty$, ξ and E_{obs} go to zero

$$\begin{aligned} x &= \Pi_i(w_i(t)) \\ \xi_0 &= \tau_i(w_i(t)) \end{aligned}$$

the following subsystem to be switched with $\sigma = i + 1$ is given by

$$\begin{aligned} \xi &= x - \Pi_{i+1}(w_{i+1}(t)) \\ E_{obs} &= \xi_0 - \tau_{i+1}(w_{i+1}(t)) \end{aligned}$$

hence, for all switching instant T_s the next equalities are held

$$\begin{aligned} \xi &= \Pi_i(w_i(t)) - \Pi_{i+1}(w_{i+1}(t)) \quad \forall t \in T_s \\ E_{obs} &= \tau_i(w_i(t)) - \tau_{i+1}(w_{i+1}(t)) \quad \forall t \in T_s \end{aligned}$$

Moreover, the only way to maintain the zero error is finding a switching instant T_s for which the next equations are held

$$\Pi_i(w_i(t)) - \Pi_{i+1}(w_{i+1}(t)) = 0 \quad \forall t \in T_s \tag{3.19}$$
$$\tau_i(w_i(t)) - \tau_{i+1}(w_{i+1}(t)) = 0 \quad \forall t \in T_s \tag{3.20}$$

Therefore, if there exist a switching time Ts for which (3.19) and (3.20) are maintained, then zero error is held.

Stability condition

Stability is proved in the same form of the full information case, taking the following closed loop system $Acl_i = \begin{pmatrix} A_i & B_iH_i \\ G_iC_i & F_i \end{pmatrix}$, where the new system is given by

$$\dot{x} = Acl_i x \quad \forall\, i \in \sigma \tag{3.21}$$

where, taking a constant $\tau_C > \frac{\ln \mu}{\lambda_0}$ and satisfying conditions 3.8, 3.9, then the system is locally asymptotically stable and condition 2 is satisfied.

In the case of linear systems the condition (3.19) and (3.20) can be express by the following expression

$$(w_{i+1}(t)) \in \ker(\Pi_i - \Pi_{i+1}) \quad \forall t \in T_s \tag{3.22}$$
$$(w_{i+1}(t)) \in \ker(\tau_i - \tau_{i+1}) \quad \forall t \in T_s \tag{3.23}$$

Theorem 10. Consider the output regulation problem of the system $\dot{x} = f_i(x, \xi, w)$ using error feedback. If assumption 2 holds, then given a piecewise constant switching signal $\sigma\colon [0, \infty)$ the output regulation problem is solved on a neighborhood $W \in U \times \Xi \times V$ of the origin with $i \in \sigma$ by the switching controller

$$\begin{aligned} \dot{\xi_0} &= \varphi_i(\xi) + N_i e \\ \dot{\xi_1} &= K_i\xi + L_i e \\ u &= \gamma_i(\xi_0) + M_i\xi_1 \end{aligned} \tag{3.24}$$

if there exist a finite time $\bar{t}$ such that at all switching instants $t_i \geq \bar{t}$ the next expressions is held

$$\begin{aligned} \Pi_{i-1}(w_{t_i}) &= \Pi_i(w_{t_i}) \\ \tau_{i-1}(w_{t_i}) &= \tau_i(w_{t_i}) \end{aligned} \tag{3.25}$$

at the switching instants t_i and $\sigma(t) \in S_{ave}[\tau_D, N_0]$ for some positive constants N_0 y τ_C.

3.2.3 EXAMPLE

This section contains a numerical example. The switched system is formed by two nonlinear subsystems, which the first one is given by the plant $f_1(x, u, w)$

$$\begin{aligned} \dot{x_1} &= x_2 + x_1^2 \\ \dot{x_2} &= x_1 + x_2 + u \\ e &= x_1 - w_1 \end{aligned}$$

and the second one is given by $f_2(x, u, w)$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -4x_1 + x_2^2 + u \\ e &= x_1 - w_1 \end{aligned}$$

The reference trajectories are assumed to be generated by two linear exosystems $\dot{w} = S_i w$ where

$$S_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad S_2 = 0$$

and $i \in 1, 2$ is an index set which indicates the active subsystem. In order to understand the problem, fig[3.4] illustrates the problem and the switching conditions. Each circle represents one subsystems, this representation is based in [20] and [8]. Now, consider

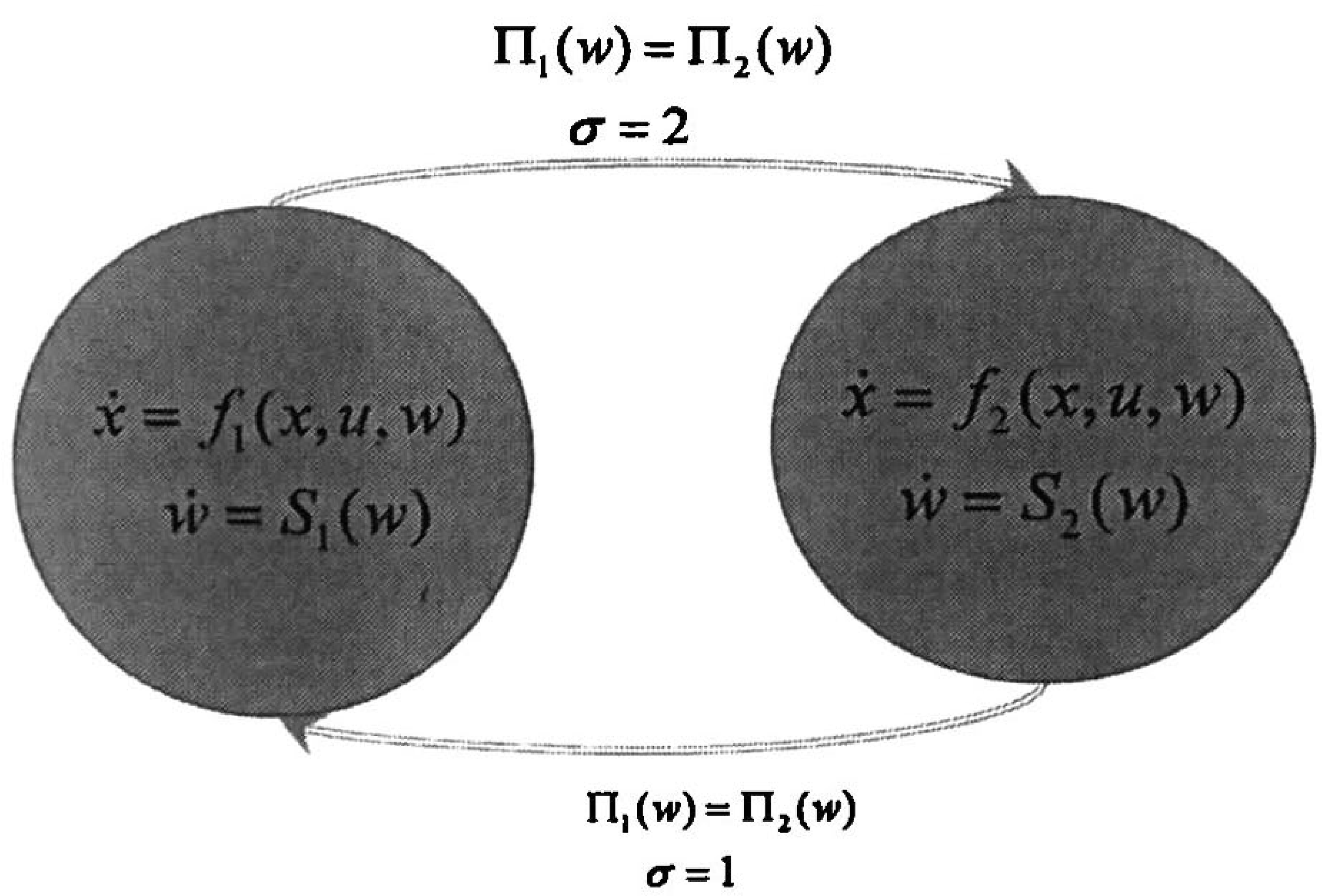

Figure 3.4: Automaton of the continuous time switched system

the full information case. Finding $\Pi_i(w)$ and $c_i(w)$ for each subsystems with $i \in 1, 2$ we get

$$\Pi_1(w) = \begin{pmatrix} w_1 \\ w_2 - w_1^2 \end{pmatrix} \tag{3.26}$$

$$c_1(w) = -2w_1 - w_2 + w_1^2 - 2w_1 w_2$$

the next sub-exosystem will be labeled by w_3

$$\Pi_2(w) = \begin{pmatrix} w_3 \\ 0 \end{pmatrix}$$

$$c_2(w) = 4w_3$$

Last, calculating the gains K_i for each subsystems

$$K_1 = \begin{pmatrix} -9 & -7 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} -4 & -6 \end{pmatrix}$$

the control is given by

$$u_i = K_i x + (c_i(w) - K_i \Pi_i)$$

Switching conditions are obtained by

$$\Pi_1(w) - \Pi_2(w) = 0$$

computing them by

$$\begin{pmatrix} w_1 - w_3 \\ w_2 - w_1^2 \end{pmatrix} = 0$$

For the stability condition, it is needs to compute the average dwell time.

$$\begin{aligned} a &= 0.109769 \\ b &= 1.123563 \\ \mu &= 10.235677 \end{aligned}$$

choosing λ_0 such that

$$\lambda_0 = \sup_{i \in \sigma} \frac{\lambda_{min}(Q_i)}{\lambda_{max}(R_i)}$$

is obtained $\lambda_0 = 0.890028$. Hence, the average dwell time is given by

$$\tau_D = 2.613265$$

The simulation results are illustrated in the following figures. Figure 3.5 shows the reference and the regulated output. This figure illustrate when the output tracks the reference never loss it, even though the reference and subsystem are switching.

Figure 3.6 illustrates the tracking error. When the tracking error reaches the zero steady state, this never leaves it.

In Figure 3.7 is illustrated the phase portrait. The state starts at the origin and reaches the invariant manifold and remains on it.

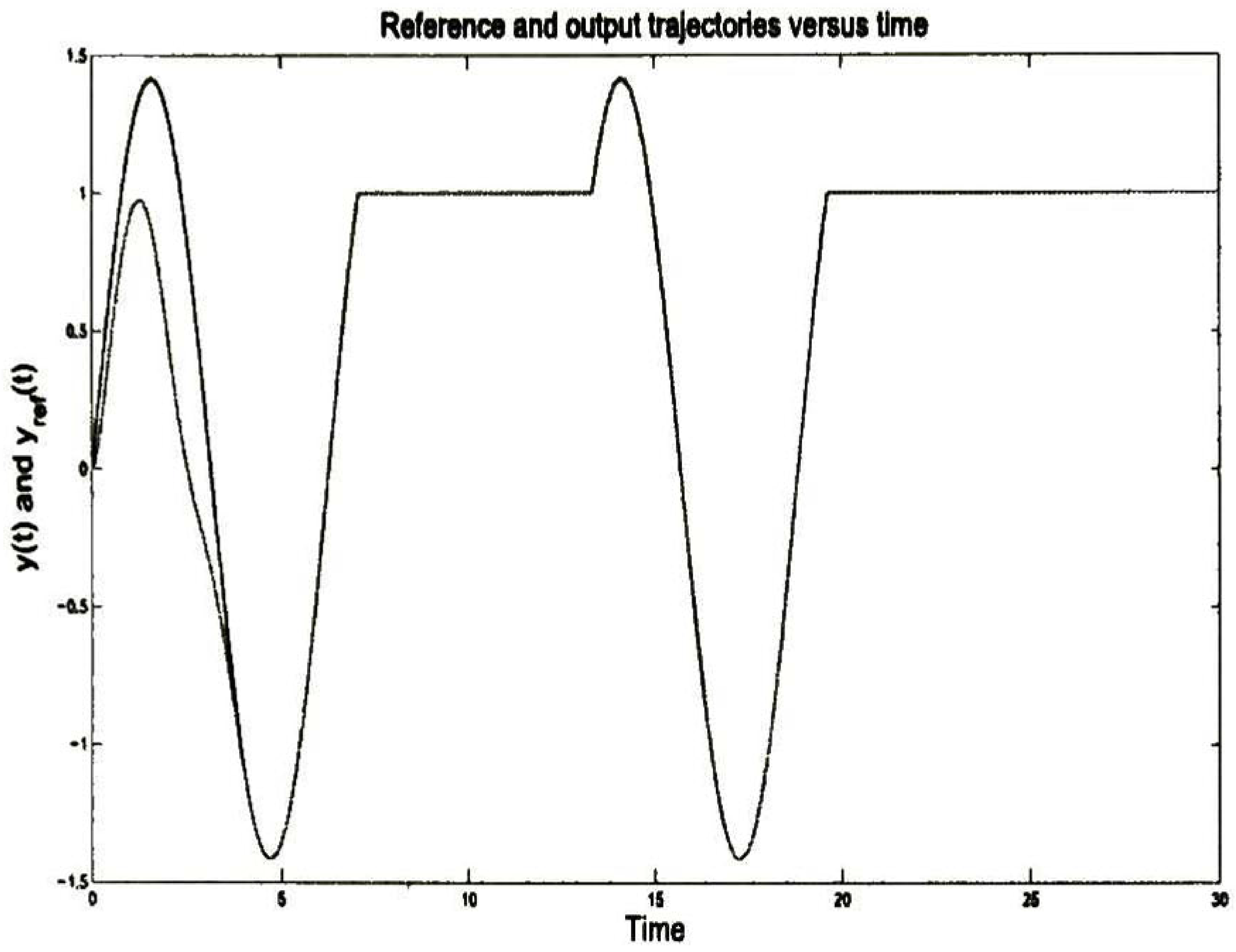

Figure 3.5: The reference and the regulated output of the continuous time switched system using exact conditions

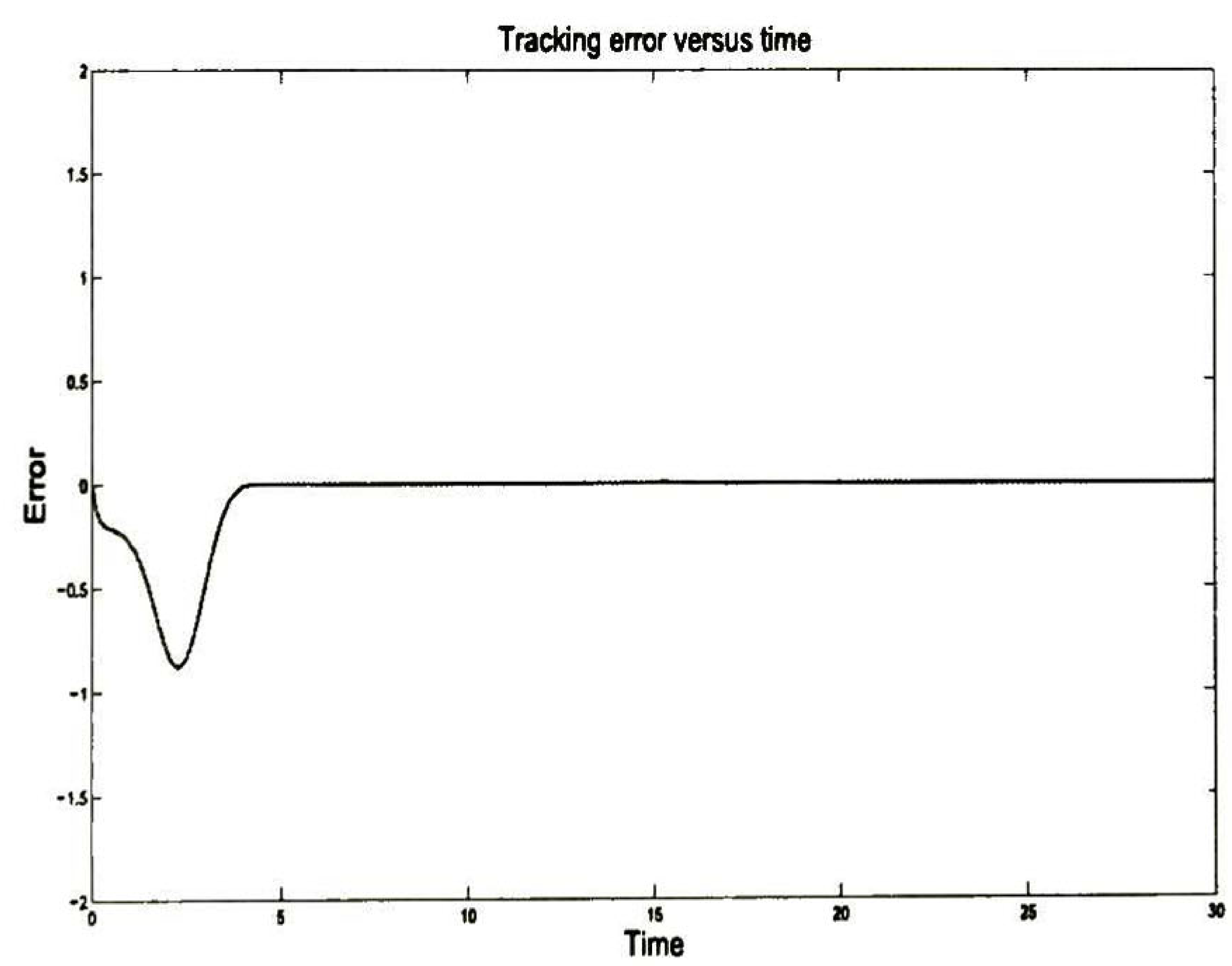

Figure 3.6: The tracking error of the continuous time switched system using exact conditions

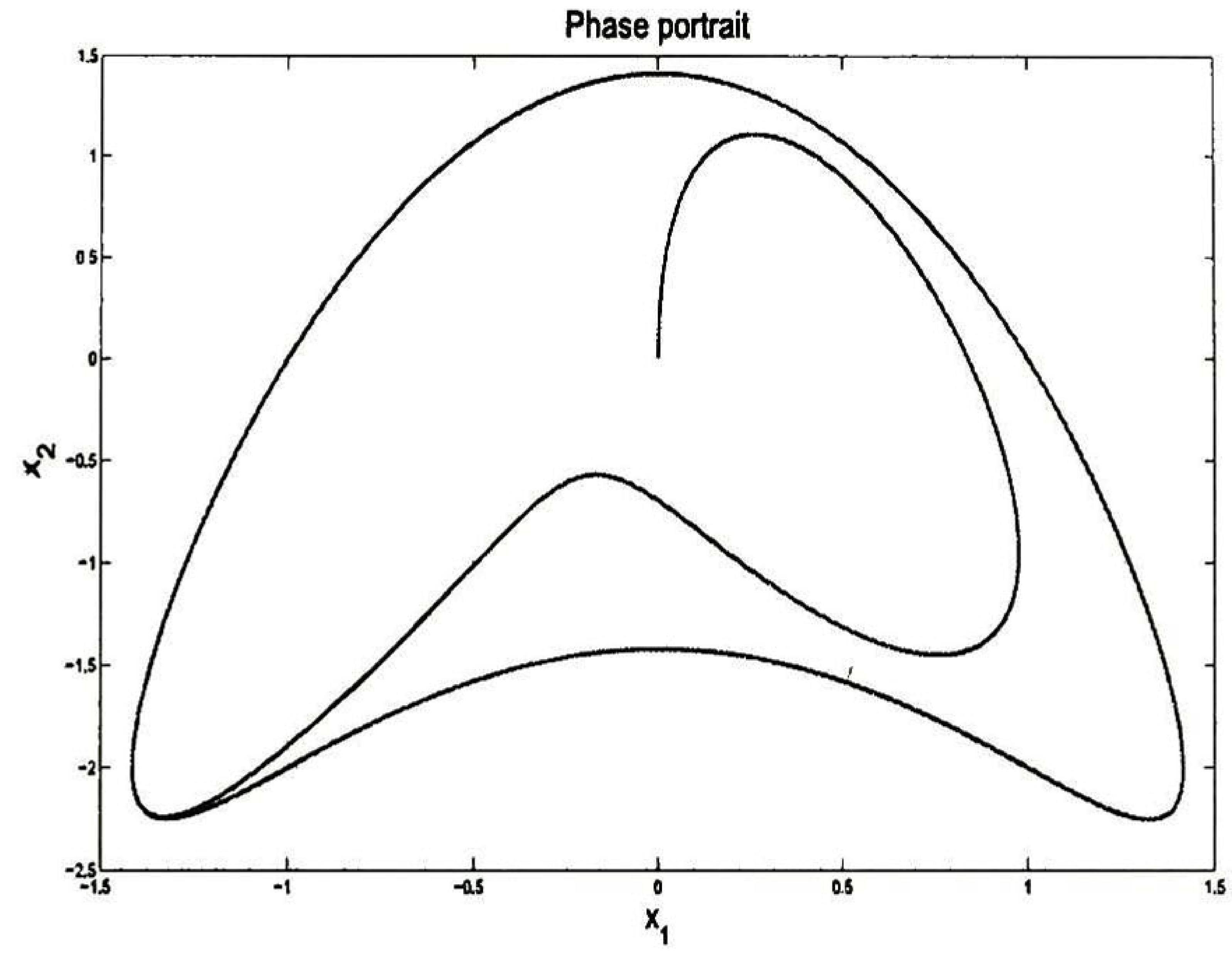

Figure 3.7: The phase portrait of the continuous time switched system using exact conditions

3.3 Discrete time switched systems

This section an extension for the previous one. The difference is the form to compute the average dwell time for discrete systems.

Consider the discrete time switched system given by the following equations

$$\begin{aligned} x[k+1] &= f_i(x[k], \xi[k], w[k]) \\ w[k+1] &= S_i(w[k]) \qquad \forall i \in \sigma \\ e &= h_i(x[k], w[k]) \end{aligned} \tag{3.27}$$

where the first equation represents all subsystem, the second one represents all exosystems and the third one is the tracking error for each $i \in \sigma$ respectively.

3.3.1 Full information output regulation for discrete time switched systems

The average dwell time for discrete time switched systems is given by

$$\tau_D > -\frac{\ln \mu}{\ln(1-\lambda_0)}$$

where τ_D is calculated in the next section in (4.45). Hence, with this constant of time and the assumption 1 held, is satisfied the condition 1. Therefore, The main result is given in the next theorem.

Main result

Theorem 11. Consider the linear discrete time output regulation problem of the system (3.27). If assumption 1 holds, given a piecewise constant switching signal $\sigma\colon [0,\infty)$ the discrete time linear output regulation problem is solved by the switched controller

$$u = K_i x[k] + (\Gamma_i - K_i\Pi_i)w[k] \tag{3.28}$$

if there exist a sampling time $\bar{\mathrm{k}}$ such that at all switching instant $k_i \geq \bar{\mathrm{k}}$ is kept

$$\Pi_{p_{i-1}} w_{k_i} = \Pi_{p_i} w_{k_i} \tag{3.29}$$

at switching instants k_i and $\sigma[k] \in S_{ave}[\tau_D, N_0]$ for some positive constants N_0 y τ_D.

3.3.2 Error feedback output regulation for discrete time switched systems

The switching zero error in the case of error feedback for discrete switched systems is ensured by the next theorem

Main result

Theorem 12. Consider the output regulation problem of the system (3.27) using error feedback. If assumption 2 holds, then given a piecewise constant switching signal $\sigma\colon [0,\infty)$ the output regulation problem is solved on a neighborhood $W \in U \times \Xi \times V$ of the origin with $i \in \sigma$ by the switching controller

$$\begin{aligned} \xi_0[k+1] &= \varphi_i(\xi[k]) + N_i e \\ \xi_1[k+1] &= K_i\xi[k] + L_i e \\ u &= \gamma_i(\xi_0[k]) + M_i\xi_1[k] \end{aligned} \tag{3.30}$$

if there exist a sampling time $\bar{\mathrm{k}}$ such that at all switching instants $k_i \geq \bar{\mathrm{k}}$ we have:

$$\begin{aligned} \Pi_{p_{i-1}}(w_{k_i}) &= \Pi_{p_i}(w_{k_i}) \\ \tau_{p_{i-1}}(w_{k_i}) &= \tau_{p_i}(w_{k_i}) \end{aligned} \tag{3.31}$$

at the switching instants k_i and $\sigma(k) \in S_{ave}[\tau_D, N_0]$ for some positive constants N_0 y τ_D.

3.3.3 EXAMPLE

In this case, the switched system is formed by two discrete linear subsystems. The first one is given by $f_1(x, u, w)$

$$\begin{aligned} x_1[k+1] &= 1.0097x_1[k] - 0.02x_2[k] - 0.0001u \\ x_2[k+1] &= 0.03x_1[k] + 0.9898x_2[k] + 0.0099u \\ e &= x_1[k] - w_1[k] \end{aligned}$$

and the second subsystem is given by $f_2(x, u, w)$

$$\begin{aligned} x_1[k+1] &= 1.02x_1[k] - 0.01x_2[k] + 0.0101u \\ x_2[k+1] &= 0.04x_1[k] + 0.98x_2[k] + 0.0002u \\ e &= x_1[k] - w_1[k] \end{aligned}$$

The reference trajectory are assumed to be generate by two linear exosystems $w[k+1] = S_i w[k]$ where

$$S_1 = \begin{pmatrix} 1.0000 & -0.0100 \\ 0.0100 & 1.0000 \end{pmatrix} \quad S_2 = \begin{pmatrix} 0.9996 & 0.0300 \\ -0.030 & 0.9996 \end{pmatrix}$$

Fig[3.8] illustrates subsystems and switching conditions. Each circle represents one subsystem, and the lines represent the switching instants.

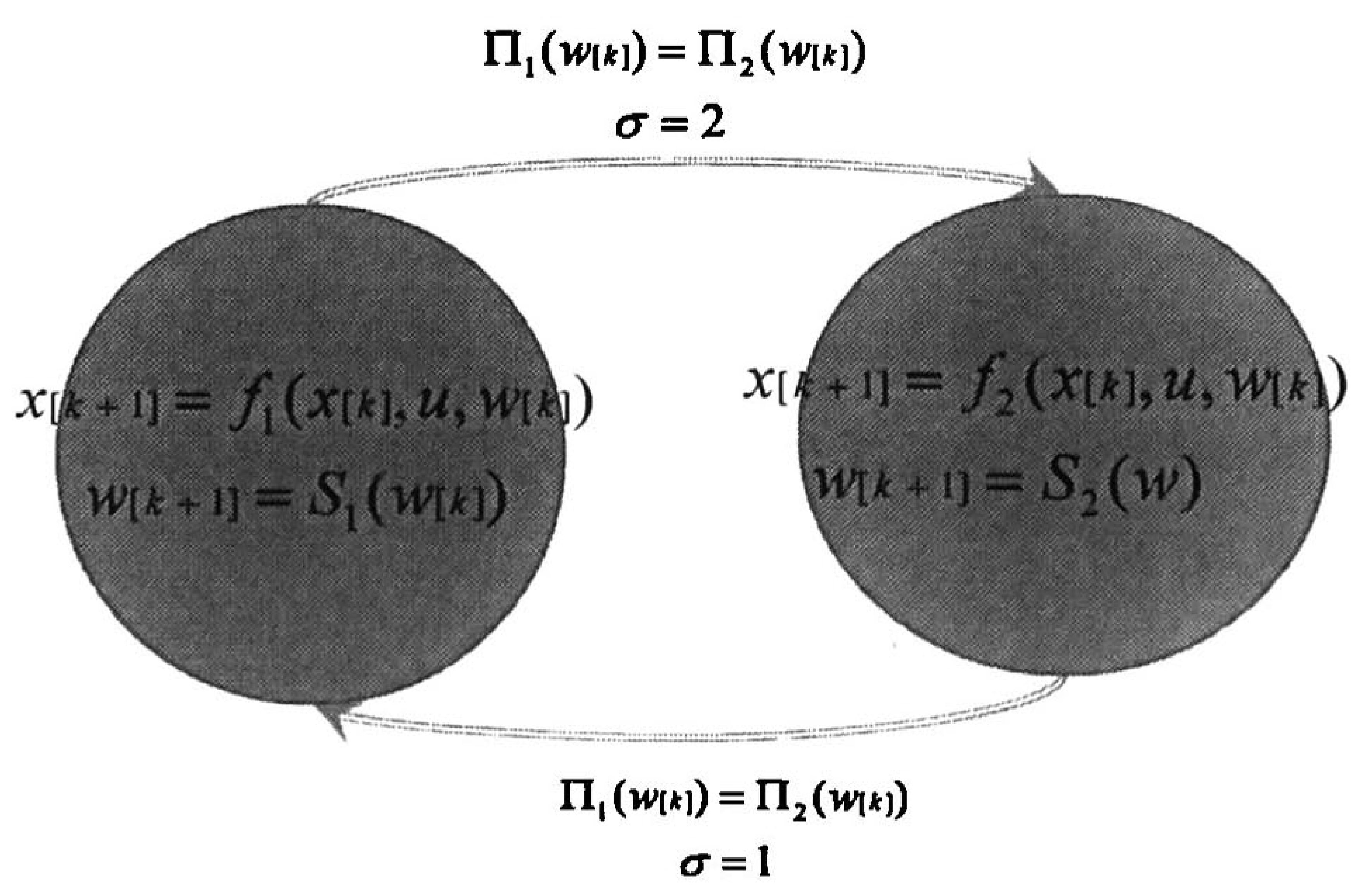

Figure 3.8: Automaton of the discrete time switched system

Considering the full information case and finding $\Pi_i(w[k])$ and $c_i(w[k])$ for each subsystems with $i \in 1,2$, are obtained the following expressions

$$\Pi_1 w[k] = \begin{pmatrix} w_1[k] \\ 0.495076w_1[k] + 0.499925w_2[k] \end{pmatrix}$$

$$\Gamma_1 w_1[k] = -2.015249w_1[k] + 0.014997w_2[k]$$

for the sake of the simulation, the following sub-exosystem will be labeled by w_3

$$\Pi_2 w_2[k] = \begin{pmatrix} w_3[k] \\ 0.500253w_3[k] + 1.499996w_4[k] \end{pmatrix}$$

$$\Gamma_2 w_2[k] = -1.050242w_3[k] + 3.000765w_4[k]$$

Last, calculating K_i for each subsystems is obtained the complete control in the case of full information for both plants

$$K_1 = \begin{pmatrix} 111.9929 & -28.9774 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} -83.8813 & -27.2559 \end{pmatrix}$$

and the control is given by

$$u = K_i x[x] + (c_i w[k] - K_i \Pi_i w[k])$$

The switching condition obtained by the following equality

$$\Pi_1 w_1[k] - \Pi_2 w_2[k] = 0$$

and given by

$$\begin{pmatrix} w_1 - w_3 \\ 0.4950762485w_1 + 0.4999250138w_2 - 2w_3 \end{pmatrix} = 0$$

In order to computing the average dwell time some constant need to be known

$$\begin{aligned} a &= 0.109769 \\ b &= 1.123563 \\ \mu &= 10.235677 \end{aligned}$$

and choosing λ_0 such that

$$\lambda_0 = \sup_{p \in P} \frac{\lambda_{min}(Q_p)}{\lambda_{max}(R_p)}$$

we obtain $\lambda_0 = 0.890028$, hence the average dwell time is given by

$$\tau_D = 2.613265$$

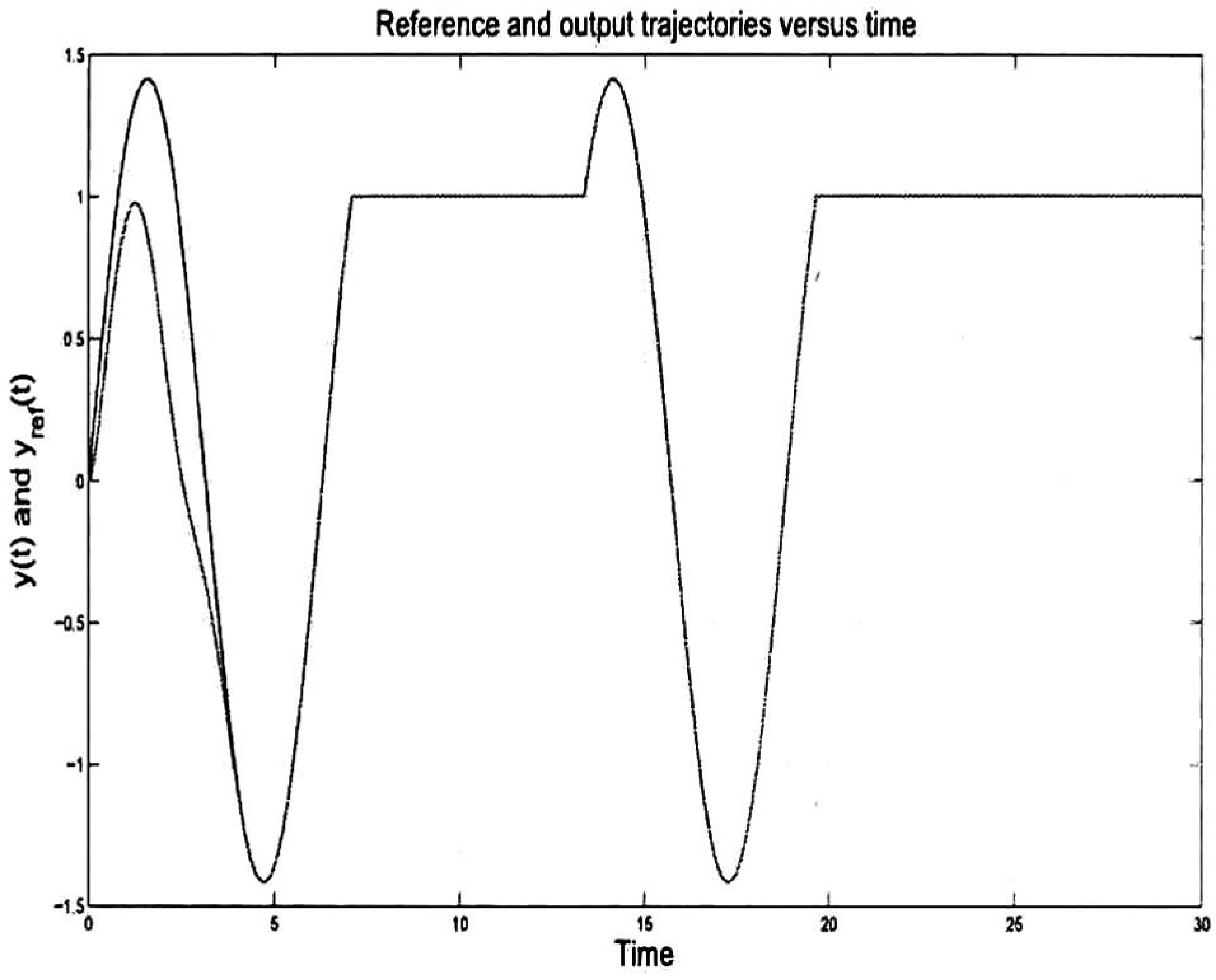

Figure 3.9: The reference and the regulated output of the discrete time switched system

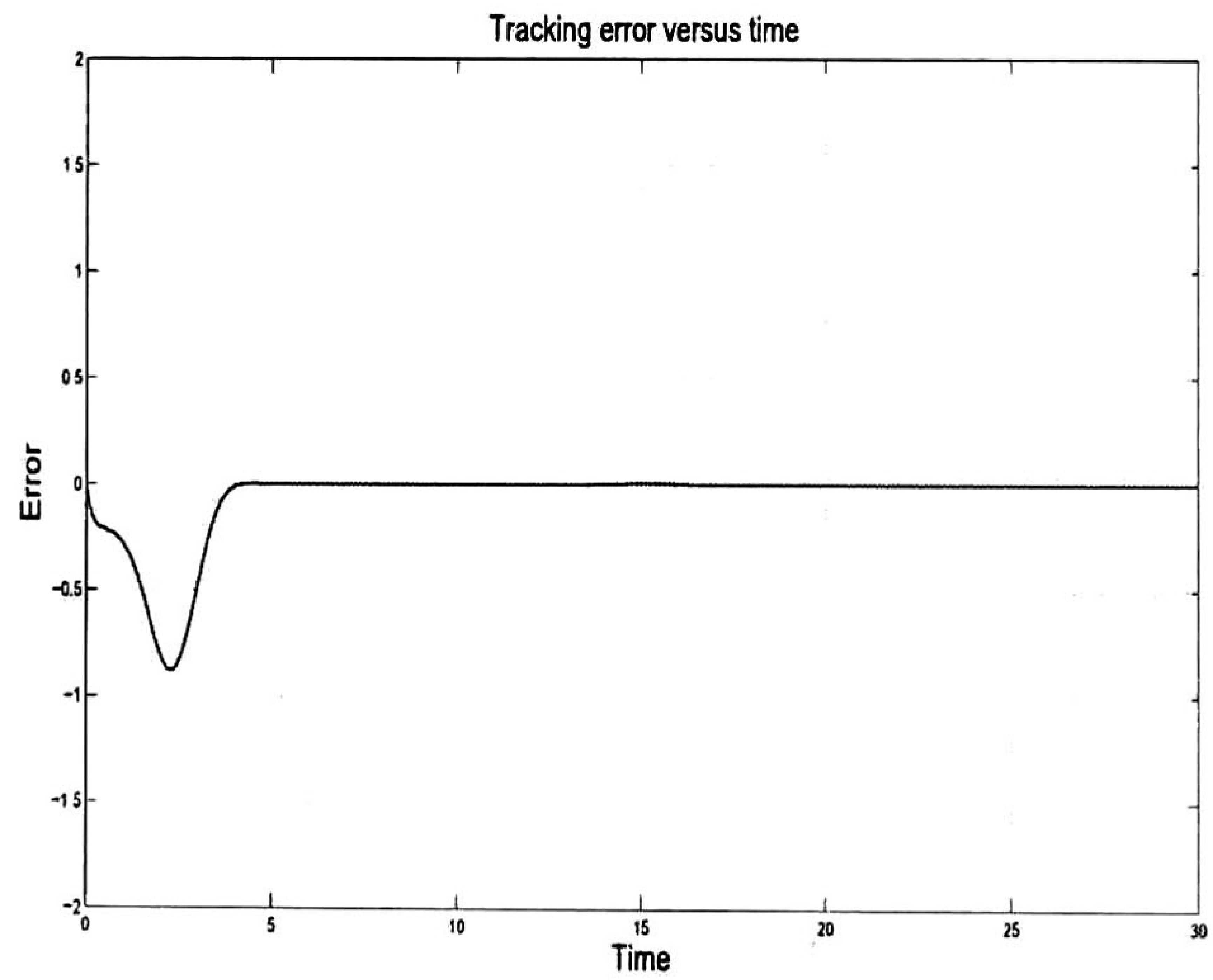

Figure 3.10: The tracking error of the discrete time switched system

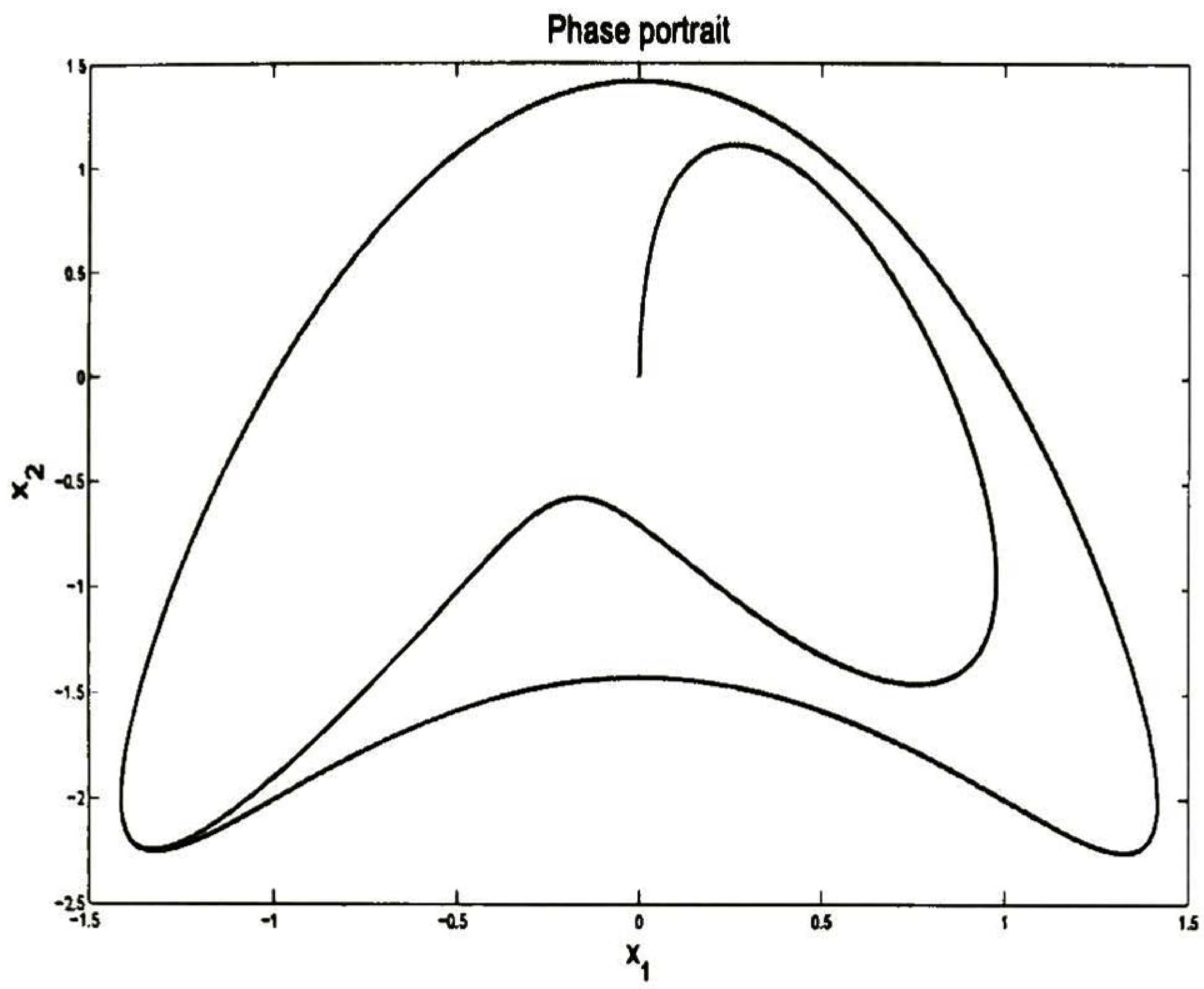

Figure 3.11: The phase portrait of the discrete time switched system

The result of the simulation is illustrated in the next figures. Figure 3.9 shows how the regulated output reaches the reference and never loss it, even though the reference is switching. Figure 3.10 illustrates the tracking error. Figure 3.11 illustrated the phase portrait. This chapter used a nonlinear continuous system and the same system in first approximation for discrete case in both examples. Hence the results seen to be the same.

Chapter 4

Bounded error conditions

4.1 General scheme

In real applications the switching times are not usually known, for this reason the exact switching condition in most cases cannot be fulfilled. On the other hand, there are machines which the switching instants are controlled by measured states not only by time.

Perfect switching is a strong condition, which in most times there is no solution or is difficult to achieve it due to the system design and the reference to be tracked. If a system switch without restrictions, the tracking error will take any value, therefore the controller cannot steer perfectly the dynamic error, in other words, the transients cannot be bounded. For this reason, it is interesting to give conditions for which a maximum error δ is allowed.

This chapter is concerned about this fact, to give switching conditions for which subsystems can commute and maintain within an allowed zone. These conditions are more useful rather than conditions in the previous chapter because are less restrictive and ensure an error bound which is important by control designers.

This chapter is developed for both continuous and discrete switched systems, in the case of nonlinear switched systems and last extending the conditions to linear switched systems. Figure (4.1) illustrates in graphic form the purpose of this chapter.

The main goal is to find a switching zone such that

E) At switching instant an allowed maximum error δ is guaranteed.

S) At switching instant the stability of switched system is assured.

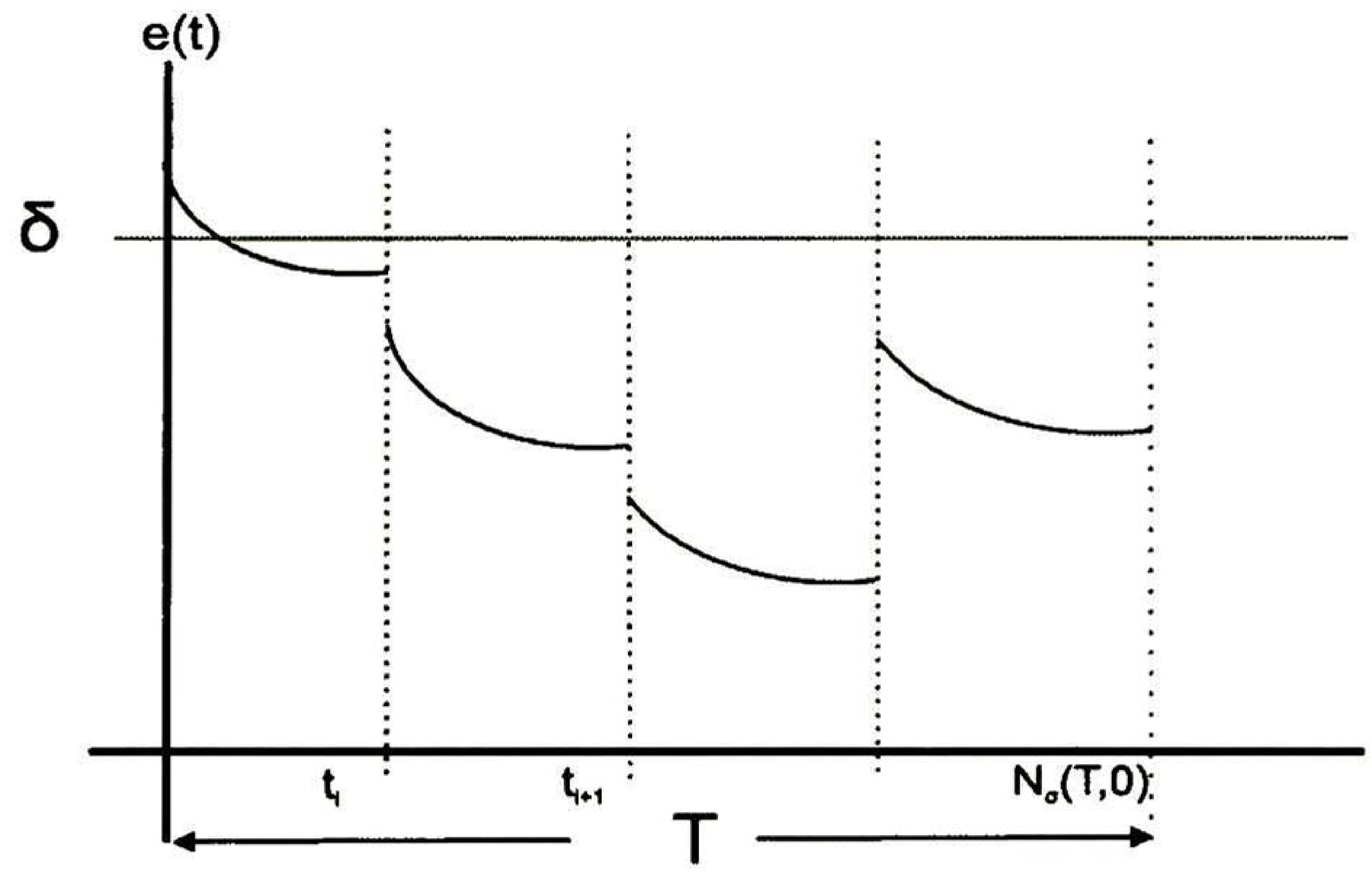

Figure 4.1: The allowed maximum error δ

4.2 Continuous time switched systems

Consider the switched system by the following equations

$$\begin{aligned} \dot{x} &= f_i(x, u, w) \\ \dot{w} &= S_i(w) \qquad\qquad \forall\, i \in \sigma \\ e &= h_i(x, w) \end{aligned} \tag{4.1}$$

where the first equation represents the family of all subsystems, the second represents the family of all exogenous systems which contains the reference to be tracked and the disturbances to be rejected, and the last one represents the family of all tracking error outputs to be regulated.

4.2.1 Output regulation for switched systems

The condition E) is important due to any control designer require remaining the state within an allowed region. The condition S) is about stability on switched systems, it is well known that in switched systems although the subsystems are stable, the switched system can become unstable.

Nonlinear systems

Consider the nonlinear switched system (4.1). The control law fulfilling the full information output regulation problem is given by

$$u_i = c_i(w) + K_i(x - \Pi_i(w)) \tag{4.2}$$

From the system (4.1) and the control law (4.2) is obtained the following closed loop system

$$\begin{aligned} \dot{x} &= (A_i + B_iK_i)x + (B_iL_i + P_i)w + \phi_i(x,w) \quad \forall\, i \in \sigma \\ \dot{w} &= S_iw + \eta_i(w) \end{aligned} \tag{4.3}$$

where

$$L_i = \left[\frac{\partial u_i}{\partial w}\right]_0 \quad \forall\, i \in \sigma \tag{4.4}$$

and $\phi_i(x,w)$ and $\eta_i(w)$ vanish at the origin.

On the other hands, the tracking error is an important equation, by means of this equation the switching zone will be found.

$$e = h_i(x,w)$$

By some manipulations, this equation can be expressed by

$$\begin{aligned} e &= h_i(x,w) \\ &= h_i(x,w) - h_i(\Pi_i(w),w) \quad \forall\, i \in \sigma \\ &= C_i(x - \Pi_i(w)) \\ &= C_i\xi \end{aligned} \tag{4.5}$$

where

$$\xi = x - \Pi_i(w) \tag{4.6}$$

Deriving (4.6) on order to analyze the dynamics, and using the regulation properties the equation is transformed into

$$\begin{aligned} \dot{\xi} &= \dot{x} - \Pi_i\dot{(w)} \\ &= f(x,u_i,w) - f(\Pi_i(w),c_i(w)_i,w) \\ &= (A_i + B_iK_i)\xi + \psi(\xi,w) \end{aligned} \tag{4.7}$$

with $\psi(\xi,w)$ vanishing at the origin and satisfying

$$\|\psi(\xi,w)\| \leq \gamma_i \|\xi_{t_i}\| \tag{4.8}$$

The following analysis is mainly based in the approach of vanishing perturbation [17]. Analyzing the solution of (4.7) throughout the time in a neighborhood of the origin and taking $\psi(\xi, w) = 0$, the solution is given by the following expression

$$\begin{aligned} \xi_{t_i} &= x(t_i) - \Pi_i w(t_i) \\ \xi(t) &= e^{(A_i+B_iK_i)(t-t_i)}\xi_{t_i} \end{aligned} \tag{4.9}$$

Being the origin a locally asymptotically stable equilibrium point for all subsystems (4.1), the following inequalities are held

$$\begin{aligned} \lambda_{\min}(P_i)\left\|\xi\right\|^2 &\leq V_i(\xi) = \xi^T P_i \xi \leq \lambda_{\max}(P_i)\left\|\xi\right\|^2 \\ \dot{V}_i(\xi) &= \xi^T((A_i+B_iK_i)^T P_i + P_i(A_i+B_iK_i))\xi \\ &= -\xi^T Q_i \xi \leq -\lambda_{\min}(Q_i)\left\|\xi\right\|^2 \\ \left\|\frac{\partial V}{\partial \xi}\right\|^2 &\leq 2\lambda_{\max}(P_i)\left\|\xi\right\|^2 \end{aligned}$$

Now, taking $\psi(\xi, w) \neq 0$ and deriving $V_i(\xi)$ along the system trajectories is obtained

$$\dot{V}_i(\xi) \leq -(\lambda_{\min}(Q_i) - 2\lambda_{\max}(P_i)\gamma_i)\left\|\xi\right\|^2 \tag{4.10}$$

where

$$\gamma_i < \frac{\lambda_{\min}(Q_i)}{2\lambda_{\max}(P_i)} \tag{4.11}$$

the one in which is a necessary stability condition. See [17].

With all locally asymptotically stable subsystems, it is possible to find constants a, b and λ_0 near the origin such that the following inequalities are held

$$\begin{gathered} a\left\|\xi\right\|^2 \leq V_i(\xi) \leq b\left\|\xi\right\|^2 \\ \dot{V}_i(\xi) \leq -2\lambda_0 V_i \end{gathered} \tag{4.12}$$

where

$$\begin{aligned} a &= \inf_{i\in i} \lambda_{\min}(P_i) \\ b &= \sup_{i\in i} \lambda_{\min}(P_i) \\ \lambda_0 &= \inf_{i\in i} \frac{\lambda_{\min}(Q_i) - 2\lambda_{\max}(P_i)\gamma_i}{\lambda_{\max}(P_i)} \end{aligned} \tag{4.13}$$

It is important to remember that the output regulation problem is solved in a neighborhood of the origin. For this reason is always possible to find constants M and λ_0 fulfilling the following equation near the origin with ξ as in (4.6) and (4.9)

$$\left\|\xi(t)\right\| \leq \left\|e^{(A_i+B_iK_i)(t-t_i)}\right\| \left\|\xi(t_i)\right\| \leq Me^{-\lambda_0(t-t_i)}\left\|\xi(t_i)\right\| \leq M\left\|\xi(t_i)\right\| \tag{4.14}$$

and M is defined by

$$M = \left[\sup_{i \in \sigma} \frac{\lambda_{max}(P_i)}{\lambda_{min}(P_i)}\right]^{1/2}$$

by the equations 4.5 and 4.14, and the allowed maximum error δ are obtained the following inequalities

$$\begin{aligned} e &= C_i \xi(t) \\ \|e\| &\leq \|C_i\| \, \|\xi(t)\| \leq \|C_i\| \, M \, \|\xi(t_i)\| \leq \delta. \end{aligned}$$

by some algebraic manipulations is obtained

$$\|\xi(t_i)\| \leq \frac{\delta}{M \, \|C_i\|}.$$

Hence, this is the switching zone the one in which ensures a bounded tracking error δ.

Linear systems

In the case of linear systems the nonlinearities are not taken into account

$$\begin{aligned} \phi_i(x, w) &= 0 \\ \eta_i(x, w) &= 0 \\ \psi_i(x, w) &= 0 \\ \gamma_i &= 0 \end{aligned} \tag{4.15}$$

Analyzing the equation (4.5) again

$$\begin{aligned} e &= C_i(x - \Pi_i w) \\ &= C_i \xi(t) \end{aligned} \tag{4.16}$$

the solution of $\xi(t)$ by the time is given by

$$\begin{aligned} \xi &= (x - \Pi_i w) \\ \dot{\xi} &= (A_i + B_i K_i)\xi \\ \xi(t) &= e^{(A_i + B_i K_i)} \xi_{(t_i)} \end{aligned} \tag{4.17}$$

If all subsystems are exponential stable, it is possible to find constants a, b and λ_0 such that the inequalities (4.12) are held

$$\begin{aligned} a &= \inf_{i \in i} \lambda_{\min}(P_i) \\ b &= \sup_{i \in i} \lambda_{\min}(P_i) \\ \lambda_0 &= \inf_{i \in i} \frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)} \end{aligned} \tag{4.18}$$

Then, doing the same procedure, we can realize that the region is the same for both, nonlinear and linear systems. Hence the switching zone is given by

$$\|\xi(t_i)\| \leq \frac{\delta}{M\,\|C_i\|}$$

Stability condition

In order to guarantee stability the concept of Average Dwell Time is used 2.11. If all subsystems are locally asymptotically stable, then we can choose some constants μ and λ_0 such that the following inequalities are held

$$\begin{aligned} V_{i+1}(\xi) &\leq \mu V_i(\xi) \\ \frac{\partial V_i}{\partial \xi} &\leq -2\lambda_0 V_i(\xi) \quad \forall i \in \sigma \end{aligned}$$

For a time period $T > 0$, the initial time is represented by $t_0 = 0$, and the switching times among the interval $(0, T)$ are represented by $t_1, ..., t_{N_i(T,0)}$. Taking the following Lyapunov function

$$W(t) := e^{2\lambda_0 t} V_i(\xi(t))$$

and deriving this function, we obtain the following expression

$$W(t) = 2\lambda_0 W + e^{2\lambda_0} \frac{\partial V_i(\xi(t_i))}{\partial \xi} f_i(\xi)$$

which is defined negative, and therefore it is not growing in the switching instants. Therefore the candidate function fulfills the conditions to be a Lyapunov function.

Analyzing on the switching instant among $(i, i+1)$ we obtain the following expression

$$W(t_{i+1}) := e^{2\lambda_0 t_{i+1}} V_{i(t_{i+1})}(\xi(t_{i+1})) \leq \mu e^{2\lambda_0 t_{i+1}} V_{i(t_i)}(\xi(t_{i+1}))$$

Making this procedure during the whole period in each switching instant is easy to realize that the following inequality is maintained during the period $(0, T)$

$$W(T) \leq \mu W(1) \leq \mu(\mu W(0)) = \mu^2 W(0)$$

and, generalizing, the expression among the whole period is given by

$$W(T^-) \leq \mu^{N_i(T,0)} W(0)$$

using the definition of $W(t)$ again, we substitute in the previous expression and we will get the next inequalities

$$\begin{aligned} e^{2\lambda_0 T} V_{i(T^-)}(\xi(T)) &\leq \mu^{N_i(T,0)} V_{i(0)}(\xi(0)) \\ V_{i(T)}(\xi(T)) &\leq e^{-2\lambda_0 T} \mu^{N_i(T,0)} V_{i(0)}(\xi(0)) \end{aligned}$$

Now, introducing the property of average dwell time in the expression, the expression is reduced to

$$\begin{aligned} V_{i(T^-)}(\xi(T)) &\leq e^{-2\lambda_0 T}\mu^{(N_0+\frac{T}{\tau_C})}V_i(\xi(0)) \\ &\leq e^{-2\lambda_0 T}e^{\ln\mu^{(N_0+\frac{T}{\tau_C})}}V_i(\xi(0)) \\ &\leq e^{(N_0\ln\mu)}e^{(\frac{\ln\mu}{\tau_C}-2\lambda_0)T}V_i(\xi(0)) \end{aligned}$$

Therefore the constant that ensures asymptotic stability during the period T needs to hold the following inequality

$$\frac{\ln\mu}{\tau_C} - 2\lambda_0 < 0$$

finally the constant τ_C is obtained by

$$\tau_C > \frac{\ln\mu}{2\lambda_0}$$

Main result

Assumption 3. For all family of (4.1) for $i \in \sigma$ the output regulation problem is solvable using full information.

All results are stated in the following theorem

Theorem 13. Consider the nonlinear output regulation problem of the system (4.1). If assumption 3 holds, then given a piecewise constant switching signal $\sigma\colon [0,\infty)$ the error bound nonlinear output regulation problem is solved in a neighborhood $W \in U \times V$ near the origin by the switched controller

$$u_i = K_\sigma x + (c_i(w) - K_i\Pi_i(w)) \quad \forall i \in \sigma \tag{4.19}$$

if there exist a switching zone $\mathcal{J}_\mathcal{E}$ such that at all switching instant $t_i \geq \bar{t}$ we have

$$\mathcal{J}_\mathcal{E} = \{\xi_i(t)/\,\|\xi_0\|_i \leq \frac{\delta}{M\,\|C_i\|} \quad \forall i \in \sigma\} \tag{4.20}$$

at the switching instants t_i with $i \in \sigma$ and $\sigma(t) \in S_{ave}[\tau_C, N_0]$ and

$$\tau_C \geq \frac{\ln\mu}{2\lambda_0}$$

where

$$\mu \overset{def}{=} \sup_{i\in i}\frac{\lambda_{max}(P_{i+1})}{\lambda_{min}(P_i)} \tag{4.21}$$

for some positive constant N_0.

4.2.2 EXAMPLE

Consider the same example of the previous chapter. The switching region is the only to be computed. Then for $\delta = 1$ the switching region is given by

$$\mathcal{J}_{\mathcal{E}} = \{\xi_i(t) / \|\xi_0\|_i \leq \frac{1}{M\|C_i\|} \quad \forall i \in \sigma\} \tag{4.22}$$

The result of the simulation is illustrated in the next figures. Figure 4.2 shows the reference and the output. In this figure is observed that the output tracks the reference and has a little error among switching.

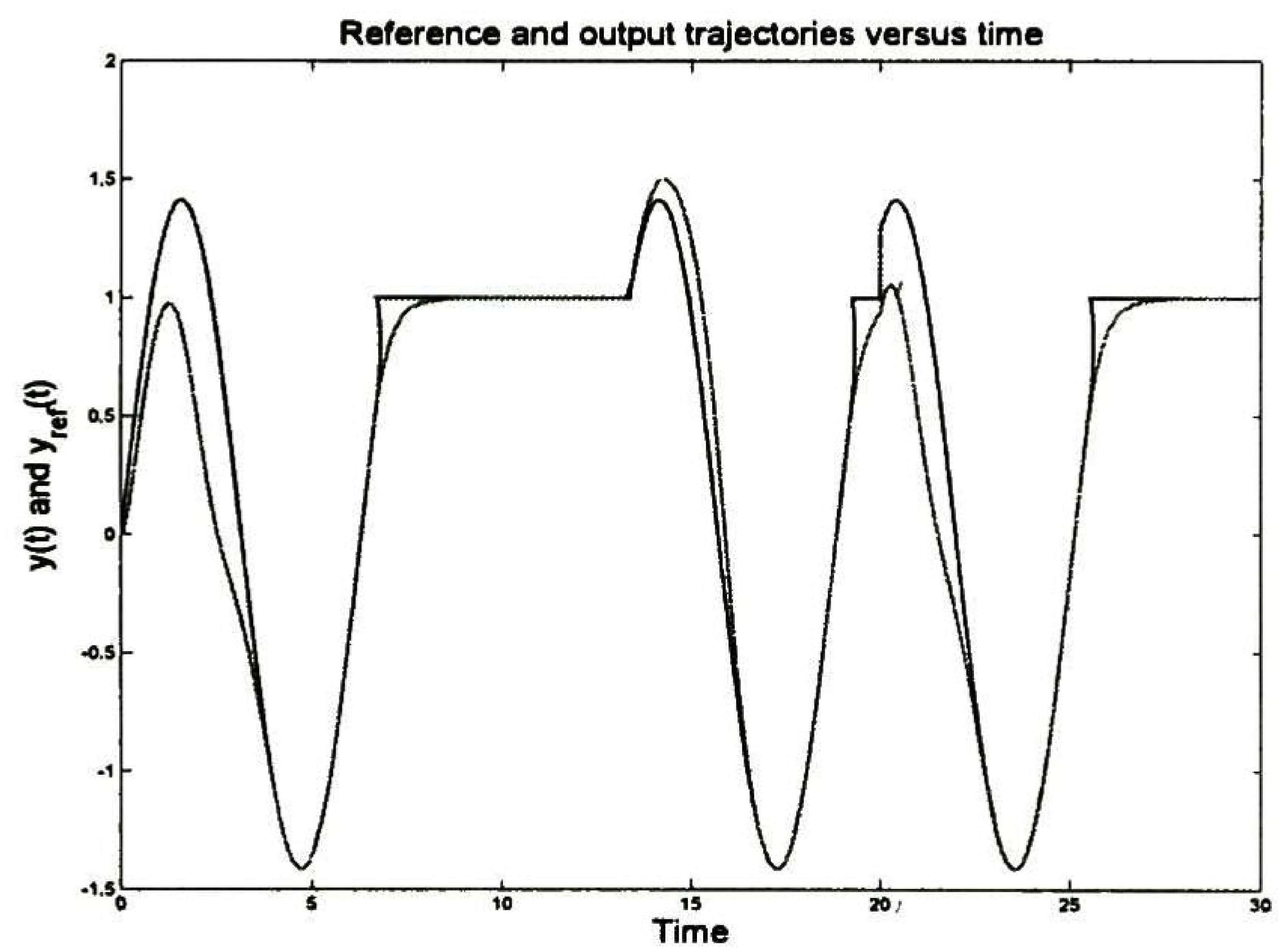

Figure 4.2: The reference and the regulated output of the continuous time switched system using an allowed maximum error.

Figure 4.3 illustrated the tracking error. In this figure is observed that the error has a little chattering in the switching instants but never crosses the switching region.

4.3 Discrete time switched systems

The discrete switched system to be considered in this section is given by the equations

$$\begin{aligned} x[k+1] &= f_i(x[k], u_i, w[k]) \\ w[k+1] &= S_i(w[k]) \qquad\qquad \forall\, i \in \sigma \\ e &= h_i(x[k], w[k]) \end{aligned} \tag{4.23}$$

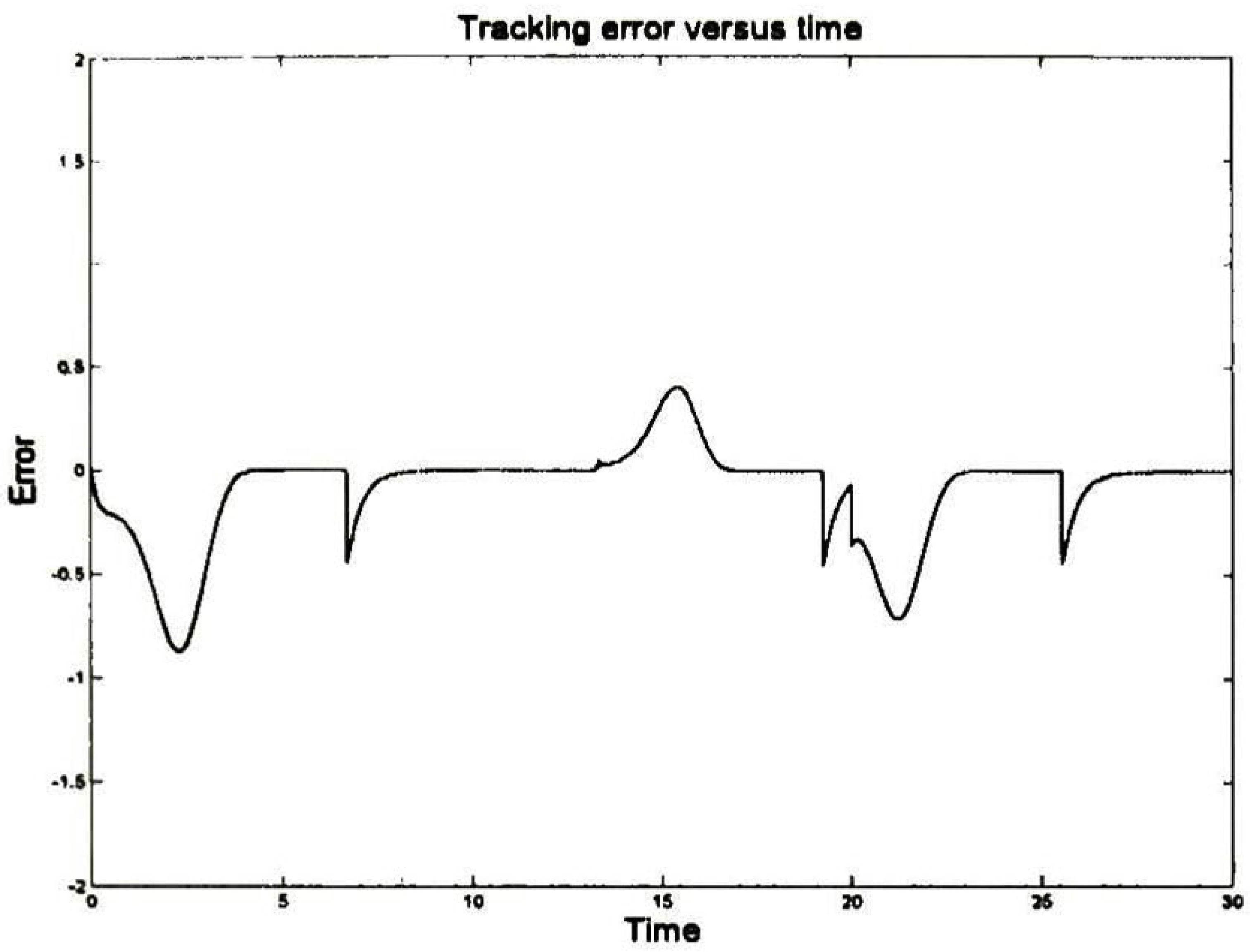

Figure 4.3: The tracking error of the continuous time switched system using an allowed maximum error.

where $x \in \Re^n$ represent the states, $u \in \Re^m$ the control input, $w \in \Re^r$ an exogenous input, and P the group of index $p \in P$ with the subsystem number. The origin is an equilibrium point for all the subsystems, that is $f_p(0,0,0)=0$.

4.3.1 Output regulation switched systems

Nonlinear systems

Consider the discrete switched system given by (4.23), the control law fulfilling the full information output regulation problem is given by

$$u_i = c(w[k])_i + K_i(x[k] - \Pi_i(w[k])) \tag{4.24}$$

Taking the discrete switched system (4.23) and the control law (4.24) the following the closed loop system is obtained

$$\begin{aligned} x[k+1] &= (A_i + B_iK_i)x[k] + (B_iL_i + P_i)w[k] + \phi_i(x[k], w[k]) \;\; \forall\, i \in \sigma \\ w[k+1] &= S_iw[k] + \eta_i(w[k]) \end{aligned}$$

where

$$L_i = c(w[k])_i - K_i\Pi_i(w[k]) \;\; \forall\, i \in \sigma$$

$\phi_i(x, w)$ and $\eta_i(w)$ vanish at the origin.

By some algebraic manipulation the error equation is reduced

$$\begin{aligned} e &= h_i(x[k], w[k]) \\ &= h_i(x[k], w[k]) - h_i(\Pi_i(w[k]), w[k]) \quad \forall\, i \in \sigma \\ &= C_i(x[k] - \Pi_i(w[k])) \\ &= C_i\xi[k] \end{aligned} \tag{4.25}$$

where

$$\xi[k] = x[k] - \Pi_i(w[k]) \tag{4.26}$$

Using the regulation properties the equation (4.26) is reduced in the following form

$$\begin{aligned} \xi[k+1] &= x[k+1] - \Pi_i(w)[k+1] \\ &= f(x[k], u_i[k], w[k]) - f(\Pi_i(w[k]), c_i(w[k])_i, w[k]) \\ &= (A_i + B_iK_i)\xi[k] + \psi(\xi[k], w[k]) \end{aligned} \tag{4.27}$$

with $\psi(\xi, w)$ vanishing at the origin and satisfying

$$\|\psi(\xi, w)\| \leq \gamma_i \|\xi_{t_i}\| \tag{4.28}$$

Hence, the solution of (4.27) with $\psi(\xi, w) = 0$ is given by

$$\xi[k] = (A_i + B_iK_i)^{k-k_i}\xi[k_i] \quad \forall\ k \in \{0, 1, ...\}\ , i \in \sigma \tag{4.29}$$

Being the origin a locally asymptotically stable equilibrium point for all subsystem of (4.1), and taking $\psi(\xi, w) = 0$, this produce the following equations

$$\begin{aligned} \lambda_{\min}(P_i)\|\xi[k]\|^2 &\leq V_i(\xi[k]) = \xi[k]^T P_i \xi[k] \leq \lambda_{\max}(P_i)\|\xi[k]\|^2 \quad \forall\ k \in \{0, 1, ...\}\ , i \in \sigma \\ \Delta V_i(\xi[k+1]) &= \xi[k]^T((A_i + B_iK_i)^T P_i (A_i + B_iK_i) - P_i)\xi[k] \\ &= -\xi[k]^T Q_i \xi[k] \leq -\lambda_{\min}(Q_i)\|\xi[k]\|^2 \end{aligned}$$

these conditions are only held if all the subsystems are stable.

Defining

$$\|A_i\| = a_i$$

taking $\psi(\xi, w) \neq 0$ and analyzing $\Delta V_i(\xi[k])$ along the system trajectories, the following inequality is obtained

$$\Delta V_i(\xi[k+1]) \leq -(\lambda_{\min}(Q_i) - 2\gamma_i\lambda_{\max}(P_i)a_i - \lambda_{\max}(P_i)\gamma_i^2)\|\xi\|^2 \tag{4.30}$$

where

$$\gamma_i \quad < \quad -a_i + \left(a_i^2 + \frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)}\right)^{1/2}$$

is a necessary condition. See Appendix B.

With all subsystems locally asymptotically stable, it is possible to find constants a, b and λ_0 in a neighborhood of the origin such that the following inequalities are held

$$\begin{aligned} a\left\|\xi[k]\right\|^2 \leq V_i(\xi[k]) \leq b\left\|\xi[k]\right\|^2 \\ V_i(\xi[k+1]) \leq -\lambda_0 V_i(\xi[k]) \end{aligned} \tag{4.31}$$

where

$$\begin{aligned} a &= \inf_{i \in i} \lambda_{\min}(P_i) \\ b &= \sup_{i \in i} \lambda_{\min}(P_i) \\ \lambda_0 &= \inf_{i \in i} \frac{\lambda_{\min}(Q_i) - 2\gamma_i \lambda_{\max}(P_i) A_c - \lambda_{\max}(P_i)\gamma_i^2}{\lambda_{\max}(P_i)} \end{aligned} \tag{4.32}$$

Computing a constant M such that

$$\|\xi[k]\| \leq \left\|(A_i + B_i K_i)^{k-k_i}\right\| \|\xi[k_i]\| \leq M J^{k-k_i} \|\xi[k_i]\| \leq M \|\xi[k_i]\| \tag{4.33}$$

where

$$M \quad = \quad \left[\sup_{i \in \sigma} \frac{\lambda_{max}(P_i)}{\lambda_{min}(P_i)}\right]^{1/2}$$

for more information see Appendix A.

Taking the equations (4.25) and (4.33), and by means of an allowed maximum error δ is obtained

$$\begin{aligned} e[k] &= C_i \xi[k] \\ \|e[k]\| &\leq \|C_i\| \|\xi[k]\| \leq \|C_i\| M \|\xi[k]_i\| \leq \delta \end{aligned}$$

where, by some algebraic manipulations

$$\|\xi[k_i]\| \quad \leq \quad \frac{\delta}{M \|C_i\|}$$

Hence, this is the switching zone the one in which ensures a bounded tracking error δ.

Linear systems

For linear systems, the following relations are taken into account

$$\begin{aligned} \phi_i(x, w) &= 0 \\ \eta_i(x, w) &= 0 \\ \psi_i(x, w) &= 0 \\ \gamma_i &= 0. \end{aligned} \tag{4.34}$$

Equation (4.25) can be rewritten as

$$\begin{aligned} e[k] &= C_i(x[k] - \Pi_i w[k]) \\ &= C_i \xi \end{aligned} \tag{4.35}$$

analizing $\xi[k]$ by the time and by regulation properties

$$\begin{aligned} \xi[k] &= x[k] - \Pi_i w[k] \\ \xi[k+1] &= (A_i + B_i K_i)\xi[k] \\ \xi[k] &= (A_i + B_i K_i)^{k-k_i}\xi_i[k_i] \quad \forall\ k \in \{0, 1, ...\} \end{aligned} \tag{4.36}$$

then, if all subsystems are exponential stable, then we can use a Lyapunov function such that

$$\begin{gathered} a\,\|\xi[k]\|^2 \leq V_i(\xi[k]) \leq b\,\|\xi[k]\|^2 \\ \Delta V_i(\xi[k+1]) \leq -\lambda_0 V_i(\xi[k]) \end{gathered} \tag{4.37}$$

where

$$\begin{aligned} a &= \inf_{i \in i} \lambda_{\min}(P_i) \\ b &= \sup_{i \in i} \lambda_{\min}(P_i) \\ \lambda_0 &= \inf_{i \in i} \frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)} \end{aligned} \tag{4.38}$$

Now, doing the same procedure, we can realize that the region is the same for both, discrete nonlinear and linear systems. Hence

$$\|\xi[k_i]\| \leq \frac{\delta}{M\,\|C_i\|}$$

Stability condition

However stability of the system also needs to be assured, for this fact the concept of Average Dwell Time for discrete systems will be used A discrete system has the property of average dwell times (τ_a) if it accomplish the following inequality

$$N(T,k) \leq N_0 + \frac{T-k}{\tau_a} \quad \forall T \geq k \geq 0.$$

If all our subsystems are asymptotically stable, then constants μ and λ_0 can be chosen such that these guarantee the following inequalities

$$V_i(\xi[k]) \leq \mu V_{i-1}(\xi[k]) \tag{4.39}$$
$$\Delta V_i(\xi[k]) = V_i(\xi[k+1]) - V_i(\xi[k]) \leq -\lambda_0 V_i(\xi[k]) \quad \forall i \in \sigma \tag{4.40}$$

Then, for a time period $T > 0$, the initial time will represent it as $k_0 = 0$, and the switching times among an interval $(0,T)$ is given by $k_1, ..., k_{N_i(T,0)}$. For $k \in [k_i, k_{i+1},$ from (4.40) is obtained

$$V_i(\xi[k+1]) \leq (1-\lambda_0)V_i(\xi[k]) \quad \forall i \in \sigma \tag{4.41}$$

solving this system we have

$$V_i(\xi[k]) \leq (1-\lambda_0)^{k-k_i} V_i(\xi[k_i]) \quad \forall i \in \sigma \tag{4.42}$$

then, according to (4.39), we obtain

$$V_i(\xi[k]) \leq (1-\lambda_0)^{k-k_i} \mu V_{i-1}(\xi[k_{i-1}]) \quad \forall i \in \sigma \tag{4.43}$$

Doing the same for all period T, it can be expressed in the next form

$$\begin{aligned} V_i(\xi[k]) &\leq (1-\lambda_0)^{k-k_0} \mu^{N_i(T,0)} V_0(\xi[k_0]) \quad \forall i \in \sigma \\ &\leq ((1-\lambda_0)\mu^{\frac{1}{\tau_D}})^{k-k_0} \mu_0^V(\xi[k_0]) \quad \forall i \in \sigma \end{aligned} \tag{4.44}$$

The constant which ensures asymptotic stability during the period T need to hold the following inequality

$$(1-\lambda_0)\mu^{\frac{1}{\tau_D}} < 1$$

The constant τ_a is given by

$$\tau_D > -\frac{\ln \mu}{\ln(1-\lambda_0)} \tag{4.45}$$

Main result

Assumption 4. For all family of (4.1) for $i \in \sigma$ the output regulation problem is solvable using full information.

The difference between continuous and discrete systems is the form of computing the average dwell time. The following theorem summarize the principal results

Theorem 14. Consider the nonlinear output regulation problem of the system (4.1). If assumption 4 holds, then given a piecewise constant switching signal $\sigma\colon [0, \infty)$ the error bound nonlinear output regulation problem is solved in a neighborhood $W \in U \times V$ near the origin by the switched controller

$$u_i = K_\sigma x + (c_i(w) - K_i \Pi_i(w)) \quad \forall i \in \sigma \tag{4.46}$$

if there exist a switching zone $\mathcal{J}_\mathcal{E}$ such that at all switching instant $t_i \geq \bar{t}$ we have

$$\mathcal{J}_\mathcal{E} = \{\xi_i(t) / \|\xi_0\|_i \leq \frac{\delta}{M\,\|C_i\|} \quad \forall i \in \sigma\} \tag{4.47}$$

at the switching instants t_i with $i \in \sigma$ and $\sigma(t) \in S_{ave}[\tau_D, N_0]$ and

$$\tau_D > -\frac{\ln \mu}{\ln(1 - \lambda_0)}$$

where

$$\mu \stackrel{def}{=} \sup_{i \in i} \frac{\lambda_{max}(P_{i+1})}{\lambda_{min}(P_i)} \tag{4.48}$$

for some positive constant N_0.

Chapter 5

Conclusions

The switched systems theory is an interesting research field that in this moment has acquired a relevant importance. Several modern machines have different kinds of dynamics; hence switched systems allow modeling these machines.

In the present work, two different approaches of switching conditions for switched systems were proposed by means of the output regulation problem in order to guarantee a switching zero error and a bounded switching error. The exact switching is the most ideal switching situation; the applications for this are reduced due to strong conditions.

However, to give conditions in order to bound a required maximum error can relax the switching conditions, increasing the number of applications. Controlling the allowed maximum error, the control input can be bounded during the transient. This fact is important in control theory; in switched systems the control law always needs an extra power in order to reach the steady state response for each discrete event. This is the most important contribution in the present work, to propose conditions to find a switching region in order to reduce this extra power for each switching instant, and to take the control of the tracking error.

The switching conditions in the case of bounded error were developed only for the full information output regulation problem. Therefore, it would be interesting to be able to extend these results to the case of error feedback and robust regulation, for both continuous and discrete dynamics, which are more common cases in applications.

It would be important in future works to expand diverse theories like sliding modes, neuronal networks, and nonlinear control theory to switched systems. Later, to propose new theories and controllers taking advantage of the weaknesses and virtues of this systems. For example in stability theory, systems can be stabilized even if all subsystems are unstable, and the opposite is also true. These characteristics on switched systems expand the analysis on control and stability and represent an important difference for future researches.

Appendix A

Bounded state norm

A.1 Continuous systems

If the system is exponential stable, we can bound the state norm in the following way Consider a Lyapunov function fulfilling the following inequalities

$$a\|x\|^w \le V(t,x) \le b\|x\|^w \tag{A.1}$$

$$V(\dot{t},x) \le -c\|x\|^w \le -\frac{c}{b}V(t,x) \tag{A.2}$$

from equation A.1 is obtained

$$\|x\| \le \left[\frac{V(t,x)}{a}\right]^{\frac{1}{w}} \tag{A.3}$$

Solving $V(\dot{t},x)$ from A.2

$$V(t,x) \le V(t_0,x(t_0))e^{-(\frac{c}{b})(t-t0)} \tag{A.4}$$

from equations A.3 and A.4

$$\begin{aligned} \|x\| &\le \left[\frac{V(t_0,x(t_0))e^{-(\frac{c}{b})(t-t_0)}}{a}\right]^{\frac{1}{w}} \\ &\le \left[\frac{b\|x_0\|^c e^{-(\frac{c}{b})(t-t_0)}}{a}\right]^{\frac{1}{w}} \\ &= \left(\frac{b}{a}\right)^{1/w} \|x_0\| e^{-(c/wb)(t-t_0)} \end{aligned} \tag{A.5}$$

A.2 Discrete systems

Now, doing the same for discrete systems, we have

$$a\left\|x[k]\right\|^{w} \leq V(k, x[k]) \leq b\left\|x[k]\right\|^{w} \tag{A.6}$$
$$\Delta V(k, x[k]) \leq -c\left\|x[k]\right\|^{w} \leq -\frac{c}{b}V(k, x[k]) \tag{A.7}$$

with

$$\Delta V(k, x[k]) = V(k+1, x[k+1]) - V(k, x[k]) \tag{A.8}$$

Hence from A.7 and A.8

$$V(k+1, x[k+1]) - V(k, x[k]) \leq -\frac{c}{b}V(k, x[k])$$
$$V(k+1, x[k+1]) \leq -\frac{c}{b}V(k, x[k]) + V(k, x[k])$$
$$V(k+1, x[k+1]) \leq \left(1-\frac{c}{b}\right)V(k, x[k]) \tag{A.9}$$

solving A.9 and with A.6

$$\begin{aligned} V(k, x[k]) &\leq \left(1-\frac{c}{b}\right)^{k} V(0, x[0]) \quad k \in [0, 1, \ldots) \\ \left\|x[k]\right\|^{w} &\leq \frac{\left(1-\frac{c}{b}\right)^{k} V(0, x[0])}{a} \quad k \in [0, 1, \ldots) \\ \left\|x[k]\right\| &\leq \left(\frac{b}{a}\right)^{1/w}\left(1-\frac{c}{b}\right)^{k/w} x[0] \quad k \in [0, 1, \ldots) \end{aligned} \tag{A.10}$$

Appendix B

Vanishing perturbation for discrete systems

Consider the systems

$$x[k+1] = Ax[k] + g(x[k]) \tag{B.1}$$

The nominal system of (B.1) is given by

$$x[k+1] = Ax[k] \tag{B.2}$$

Now, suppose the perturbation case with $g(0) = 0$ and vanishing at the origin and being a stable equilibrium point of the nominal system (B.2), with $V(x[k]) = x[k]^T P x[k]$ a lyapunov function such that

$$c_1 \|x\|^2 \leq V(x[k]) \leq c_2 \|x\|^2 \tag{B.3}$$

$$\Delta V(x[k]) \leq -c_3 \|x\|^2 \tag{B.4}$$

$$\|A\| = c_4 \tag{B.5}$$

for positive constant c_1, c_2, c_3, c_4 and the perturbation term satisfies the linear growth bound

$$\|g(x)\| = \gamma \|x\| \tag{B.6}$$

Analyzing the stability along the system (B.1), we have

$$\begin{aligned}
\Delta V(x[k]) &= V(x[k+1]) - V(x[k]) \\
&= [Ax[k] + g(x[k])]^T P [Ax[k] + g(x[k])] - x[k]^T P x[k] \\
&= x[k]^T (A^T P A - P) x[k] + (g(x[k]))^T P A x[k])^T \\
&+ (g(x[k]))^T P A x[k]) + g(x[k])^T P g(x[k]) \\
&\leq -c_3 \|x\|^2 + 2 \left\| (g(x[k]))^T P A x[k]) \right\| + \left\| g(x[k])^T P g(x[k]) \right\|
\end{aligned}$$

Now, using the inequality B.3 to B.6 we obtain

$$\begin{aligned} \Delta V(x[k]) &\leq -c_3 \|x\|^2 + 2\gamma c_2 c_4 \|x\|^2 + c_2 \gamma^2 \|x^2\| \\ \Delta V(x[k]) &\leq -(c_3 - 2\gamma c_2 c_4 - c_2 \gamma^2) \|x\|^2 \end{aligned}$$

Then, $\Delta V(x[k])$ is negative definite when

$$c_3 - 2\gamma c_2 c_4 - c_2 \gamma^2 \; > \; 0$$

$$\gamma \; < \; -c_4 + \left(c_4^2 + \frac{c3}{c2}\right)^{1/2}$$

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CENTRO DE INVESTIGACIÓN Y DE ESTUDIOS AVANZADOS DEL I.P.N.
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Regulación de salida para una clase de sistemas conmutados
-Output Regulation for a class of Switched Systems

del (la) C.

JOSE MARIA CORDOBA LAGUNES

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