Centro de Investigación y de Estudios Avanzados
del
Instituto Politécnico Nacional

UNIDAD ZACATENCO
DEPARTAMENTO DE FÍSICA

“Restricciones de No-unitaridad en las mezclas pesado-ligeras en un modelo con dos neutrinos de O(Tev)”

Tesis que presenta

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para obtener el Grado de

Maestro en Ciencias

en la Especialidad de

Física

Director de tesis: **Dr. Pablo Roig Garcés**

Ciudad de México Enero, 2024

CENTRO DE INVESTIGACION Y DE ESTUDIOS AVANZADOS DEL INSTITUTO POLITECNICO NACIONAL

UNIT ZACATENCO
PHYSICS DEPARTMENT

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Acknowledgments

I would like to thank the working people of Mexico, to share me his progress and wealth. I would also like to extend my gratitude to:

- My family for all the collaboration provided;
- The CINVESTAV for hospitality, and research environment;
- My research advisor, Dr. Pablo Roig G. For his patience, motivation and enthusiasm;
- Dr. Gerardo Hernández Tomé for their helpful comments and questions.
- to thank my colleague Luis David Ávalos for his collaboration, teachings and enriching comments.
- Lina Marcela González Paz for her love, understanding and patience.
- CONAHCYT for the postgraduate scholarship that allowed me to carry out my research and academic training.

This thesis is dedicated to my loved ones, who are no longer in this physical world.

Abstract

One of the most popular and best motivated extensions of the standard model of elementary particle physics consists in adding heavy right-handed neutrinos and generating the mass of light neutrinos through the so-called Type-I seesaw mechanism, or some of its lower-energy variants (like the so-called inverse seesaw).

In this work we study the non-unitary constraints arising from the presence of extra states, which may render the violation of the charged lepton flavor measurable, in the context of a model with two sterile Majorana neutrinos, that includes both the type-I seesaw and its inverse realization as limits of interest. In our study, we generate a set of random values for the three independent heavy-light admixtures. These inputs are constrained by the results of a global fit to electroweak precision observables and by data on possible decay and transition processes, where $\mu \to e\gamma$ –that violates lepton flavor– currently dominates. The constraint on the $\mu \to e\gamma$ process is taken in the case where the masses of the two heavy neutrinos are equal and thus form a Dirac field. In this particular case, we conclude that the unitarity constraints are more stringent than those coming from charged lepton flavor violating processes, which deserves further study, both within this simplified model, and more generally, in the context of seesaw scenarios.

Resumen

Una de las extensiones más populares y mejor motivadas del modelo estándar de la física de partículas elementales, consiste en añadir neutrinos derechos pesados, y generar la masa de los neutrinos ligeros a través del llamado mecanismo Seesaw tipo-I, o alguna de sus variantes a energías menores (como el llamado seesaw inverso).

En este trabajo estudiamos las restricciones de no unitariedad provenientes de la presencia de estados adicionales, que pueden causar que la violación de sabor leptónico en el sector cargado sea observable, en el contexto de un modelo con dos neutrinos estériles de Majorana, que incluye el seesaw tipo-I y su realización inversa como límites de interés. En nuestro estudio, generamos un conjunto de valores aleatorios para las tres mezclas pesadas-ligeras independientes. Estos inputs están restringidos por los resultados de un ajuste global a observables de precisión electrodébiles y los datos de diversas transiciones y desintegraciones posibles, donde $\mu \to e\gamma$ –que viola el sabor leptónico– domina actualmente. La restricción del proceso $\mu \to e\gamma$ se toma en el caso en que las masas de los dos neutrinos pesados son iguales y, por tanto forman un campo de Dirac. En este caso particular concluimos que las restricciones de no unitariedad son más fuertes que las de procesos con violación de sabor leptónico en corriente cargada, lo que merece extender este estudio, tanto dentro de este modelo simplificado, como de forma más general, en el contexto de escenarios seesaw.

Notation

In this work we will use the natural units $\hbar = c = 1$. Where $\hbar = h/2\pi$, with h the Planck constant and c the speed of light.

Greek indices μ and ν run over the four spacetime coordinate, usually taken as 0,1,2,3. The Einstein's summation convention is used, i.e. indices that are repeated are summed over.
$\mathcal{L}$ Lagrangian density, frequently called Lagrangian.
$\varepsilon_{\alpha\beta\gamma\delta}$ Levi-Civita symbol.
$g^{\mu\nu}$ components of the Minkowski metric, $\mathrm{diag}\,(1,-1,-1,-1)$.
The transpose of a matrix A is A^T.
The complex conjugate of a matrix A is A^*. The Hermitian adjoint of a matrix A is $A^\dagger = A^{*T}$.
$\left(A^a_{Ad}\right)_c{}^b = if^{abc}$ is the adjoint representation.
$\mathbb{1}$ is the identity matrix, or sometimes called a unit matrix.
$[A,B] = AB - BA$ Commutator.
$\{A,B\} = AB + BA$ Anticommutator
$\not{A} = \gamma^\mu A_\mu$ Feynman slash notation.
Dirac conjugation is expressed by $\bar{\psi} = \psi^\dagger\gamma^0$.
$+h.c$ is the addition of the Hermitian adjoint or complex conjugate.
The Pauli matrices are given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Contents

Chapter 1

Introduction

The Standard Model of Particle Physics (SM) [1–3] is the theory that describes, to the best of our understanding, Nature correctly at the most fundamental level, up to energy scales of several TeVs. In some privileged cases, like the anomalous magnetic moment of leptons, the agreement between experiment and theory is astonishing and reaches many significant figures.

Despite this tremendous success, which rests upon its renormalizability, enabling precise multi-loop calculations to be confronted unambiguously to the extremely accurate measurements, the SM cannot be the final theory in our understanding of the Universe.

A clear and indisputable evidence for this comes from the fact that the SM cannot explain the observed enormous abundance of matter over antimatter (this could have been possible with a lighter Higgs, with mass around 75 GeV instead of the 125 measured). This requires, doubtlessly, physics beyond the SM.

A possible (and elegant) solution goes through the generation of baryon asymmetry from an initial lepton asymmetry [1].

The required CP violation [6] would then be provided by the decays of heavy right-handed neutrinos, which naturally appear in the explanation of the minuscule neutrino masses through the seesaw of type I [7–11]. This theoretically appealing possible origin of our matter universe is linked to the generation of tiny neutrino masses at scales (of the right-handed neutrinos) natural for Grand Unification of the electroweak and strong sectors of the SM.

Unfortunately, this exciting possibility may be untestable [2], which has motivated variants in which the new physics scale is lowered (rendering it even accessible at the LHC), even though the unification of interactions would still need to be achieved (if that is the case). These are generically called low-scale seesaw mechanisms and give some hope for the soon discovery of charged lepton flavor violation, that would surely provide key insights into the neutrino mass generation.

[1]Partial transfer of the latter to the former is possible, complying with the $B-L$ symmetry which is anomaly-free, through non-perturbative effects called sphalerons, that are associated to a second order phase transition above the electroweak scale. This is called baryogenesis through leptogenesis, see e.g. refs. [4,5].

[2]This can happen for normal ordering of neutrino masses, where there is a portion of parameter space yielding unobservably small neutrinoless double beta decay rates, with the lightest neutrino mass $\lesssim 1$ meV [12].

At present, lepton flavor violation has only been found (and measured, with ever-increasing precision) in neutrino oscillations, where searches for physics beyond the three light and active neutrinos paradigm (sometimes called minimally extended SM, in which the three SM neutrinos are endowed with a mass, of unknown origin) are conventionally pursued by searching for non-standard interactions or possible unitarity violations (the former would correspond to new forces and the latter to new states).

Additional motivations for the existence of heavy (mostly sterile) neutrinos come from their possible role as dark matter candidates [13–15] or portals to dark sectors [16–18].

In this thesis we want to investigate the constraints imposed by unitarity tests in neutrino oscillations (that we will take from the updated analysis of ref. [19]) on a particularly simple toy model for neutrino masses [20]. Despite its simplicity, it has the virtue of interpolating smoothly between two cases of interest. On the one hand, it reduces to the type I seesaw (for high new physics scale), while on the other end it inherits the observable signatures (depending on the heavy-light mixings) of the inverse seesaw, possibly the simplest among those at low scale. Besides, it has a parameter whose variation connects continuously the Dirac and Majorana cases for the extra two heavy states.

In ref. [20], the restrictions on the model parameters were worked out using the best limits on the non-observation of charged lepton flavor violating transitions and the model was subsequently employed in the study of lepton flavor-changing Higgs decays in ref. [21]. In particular, this thesis aims to to answer if non-unitarity yields stronger/comparable/weaker constraints than charged lepton flavor violation on this model parameters. In the first case this will motivate the update of the predictions obtained in refs. [20, 21] in light of our improved constraints. In the last instance, a prediction of the class of models represented by our framework would be that non-unitarity effects in neutrino oscillations should not be discovered anytime soon.

This Master Thesis is organized as follows: after this brief introduction, we succinctly review the Standard Model of Elementary Particle Physics in section 2, with a special focus on the mechanism of spontaneous symmetry breaking, that gives rise to all particle masses but (perhaps) those of neutrinos in section 2.2. After that, in section 3, we explain the different spinors that can be used in quantum field theory, as this is fundamental for understanding the possible neutrino mass types which can be generated: Weyl and Dirac spinors are explained in section 3.1 and helicity is reviewed next, in section 3.2, emphasizing that the Lorentz invariant chirality degenerates into it in the massless limit. We end this chapter by recalling the main features of Majorana spinors in section 3.3, which correspond to the particular case where (massive) neutrinos are their own antiparticles. Then, section 4 deals with some of the main different possibilities for describing massive neutrinos. It is well-known that the tiny neutrino masses can be generated in a natural way assuming the existence of heavy right-handed neutrinos with masses around Grand Unification scales ($\sim 10^{15}$ GeV), in what was christened as the type I seesaw mechanism, which is covered in section 4.1. Unfortunately, as elegant as this mechanism is, it may be untestable in current or forthcoming experiments, which motivates its low-scale variants that can be probed nowadays (although typically need ad hoc assumptions and are less grounded formally). The simplest of these, the inverse seesaw, is introduced in section 4.2. The model which will be worked out in this thesis, that is described in section 4.4, can be seen as a simplified toy description which in a limiting case (when the new physics scale if of few TeVs) falls into the inverse seesaw and that goes asymptotically to the type I seesaw. We will specifically study if non-unitarity constraints (section 4.3) restrict the heavy-light mixings of this model beyond the constrictions coming from the non-observation of charged lepton flavor violation, which was worked out, for this model, in ref. [20]. This amounts to compare the general constraints on models with a couple of heavy sterile neutrinos coming from non-unitarity with the findings of ref. [20], which is the topic of chapter 5. Our conclusions are summarized in chapter 6, where perspectives for pending future work that we will undertake are also given. Several useful appendices complement the core material of this thesis. Appendix A reviews the main aspects of neutrino oscillations in vacuum, appendix B relates our model to the effective field theory point of view, appendix C explains how the contraints on non-unitarity of the three light active neutrinos are set, and appendix D illustrates the way in which charged lepton flavor violating processes are used to restrict the heavy-light mixings of our model.

Chapter 2

Standard model

The standard model of elementary particle physics, SM, describes strong, weak, and electromagnetic interactions. This theory is built on the work of Glashow [1], Weinberg [2], and Salam [3]. The SM is a gauge quantum field theory, in order to be phenomenologically viable, it undergoes spontaneous symmetry breaking. The theory is invariant under the Lie symmetry group of local transformations $\mathcal{G}_{SM} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. This group determines the interactions and the vector bosons mediators of the interactions, that is to say

- Group $\mathrm{SU}\,(3)_C$, the C index refers to the fact that it is the color group. It has eight gauge bosons, G_μ (called gluons), which are the mediators of the strong interactions.
- Group $\mathrm{SU}\,(2)_L \times \mathrm{U}\,(1)_Y$ called electroweak group. The L and Y indices refer to left isospin and hypercharge (both of weak type), respectively. It has four gauge bosons, and is spontaneously broken to $\mathrm{U}\,(1)_{em}$. After spontaneous symmetry breaking, three massive bosons ($W_\mu^\pm$ and Z_μ^0) and one massless boson (the photon, A_μ) are produced.

The matter content of the theory is composed of fermions (quarks and leptons) and is presented in three generations or also called flavors. The transformation properties of these particles, under the group $\mathcal{G}_{SM}$ are presented in Table 2.1 and the flavors[1] in Table 2.2.

So for example

- $\ell_L = \begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L \sim \left(1, 2, -\frac{1}{2}\right)$ (where $\ell = e, \mu, \tau$), transforms as singlet under $\mathrm{SU}\,(3)_C$, doublet under $\mathrm{SU}\,(2)_L$ and has hypercharge $-\frac{1}{2}$.
- $q_L = \begin{pmatrix} u_j \\ d_j \end{pmatrix}_L \sim \left(3, 2, \frac{1}{6}\right)$ (where $j = 1, 2, 3$), transforms as triplet under $\mathrm{SU}\,(3)_C$, doublet under $\mathrm{SU}\,(2)_L$ and has hypercharge $\frac{1}{6}$.

In this theory, the left-handed and right-handed components transform in a different way, so the SM is a chiral theory. Also, the selection of the hypercharge Y is arbitrary and its value is fixed to obtain the electrical charge observed experimentally.

[1] Originally the SM was conceived with a null mass for neutrinos, which is not correct due to neutrino oscillation experiments [22, 23].

Matter	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
ℓ_L	1	2	-1/2
e_R	1	1	-1
q_L	3	2	1/6
u_R	3	1	2/3
d_R	3	1	-1/3

Table 2.1: Leptons and quarks in the SM and its representations. The subscripts L and R distinguish between left and right handed fields.

Leptons						
Q	Flavor	Mass (MeV)	Flavor	Mass (MeV)	Flavor	Mass (MeV)
-1	Electron e	0.511	Muon μ	105.7	Tau τ	1777
0	ν_e	0	ν_μ	0	ν_τ	0
Quarks						
+2/3	Up u	2.2	Charm c	1.28×10^3	Top t	173.1×10^3
-1/3	Down d	4.7	Strange s	96	Bottom b	4.18×10^3

Table 2.2: Leptons ℓ (Antileptons $\bar{\ell}$), with spin=1/2. The ν_j $(j = e, \mu, \tau)$ are the corresponding neutrino flavors. Quarks q (Antiquarks $\bar{q}$), with spin=1/2. Taken from [24].

The spontaneous electroweak symmetry breaking, called SEWSB, is forced by a scalar sector. After the SEWSB it is desired to obtain electromagnetism and that the color be preserved. So the scalar spontaneously breaks $SU(2)_L \times U(1)_Y$ into $U(1)_{em}$, so there are three broken generators.

At least three real scalars are needed to play the role of the Nambu-Goldstone bosons, NGB. The smallest representation containing three scalars is $H \sim (1, 2, Y_H)$, this must be charged under $SU(2)_L$ because otherwise it cannot break the symmetry. The hypercharge is fixed at $Y_H = +\frac{1}{2}$.

The scalar field H, with representation $H \sim \left(1, 2, \frac{1}{2}\right)$ under $\mathcal{G}_{SM}$, is known as the Higgs field. This field contains four real scalars, of which three will be NGB and the other the so-called Higgs boson.

With the given representations of the matter fields, the corresponding covariant derivatives are written. For example, considering left quarks $q_l \sim \left(3, 2, \frac{1}{6}\right)$, we have

$$D_\mu q_L = \left(\partial_\mu - i g_s G_\mu^b T_b - i g W_\mu^a \frac{\sigma_a}{2} - i g' B_\mu \frac{\mathbb{1}}{6}\right) q_L. \tag{2.1}$$

2.1 Dynamics of SM

Once we have described the matter content in the SM, the dynamics of the model is given by the following invariant Lagrangian under $\mathcal{G}_{SM}$,

$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{fermions} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yuk}. \tag{2.2}$$

The $\mathcal{L}_{Gauge}$ is associated with the dynamics of gauge bosons that correspond to each group of the $\mathcal{G}_{SM}$. This consists of Yang-Mills type terms, given by

$$\mathcal{L}_{Gauge} = -\frac{1}{4}G_b^{\mu\nu}G_{\mu\nu}^b - \frac{1}{4}W_a^{\mu\nu}W_{\mu\nu}^a - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}, \tag{2.3}$$

where index a runs from 1 to 3 and index b from 1 to 8, and

$$\begin{aligned} G_{\mu\nu}^b &= \partial_\mu G_\nu^b - \partial_\nu G_\mu^b + g_s f^{bac} G_\mu^a G_\nu^c, \\ W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g f^{abc} W_\mu^b W_\nu^c, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \tag{2.4}$$

in these terms g_s and g are the gauge coupling constants of groups SU(3)$_C$ and SU(2)$_L$ respectively.

The structure constants f^{abc} $(a, b, c = 1, \cdots, N^2-1)$ are defined through the generators of the groups[2] as

$$\left[T^a, T^b\right] = i f^{abc} T^c. \tag{2.5}$$

The term $\mathcal{L}_{fermions}$ contains the coupling of fermionic matter fields with gauge fields. The coupling is made through covariant derivation. Because they are fermions, these terms are of Dirac type, but without mass term, $\mathcal{L}_D = \bar{\psi} i \not{D} \psi$, so

$$\mathcal{L}_{fermions} = i \sum_{k=1}^{3} \left(\bar{q}_L^k \not{D}^k q_L^k + \bar{\ell}_L^k \not{D}^k \ell_L^k + \bar{e}_R^k \not{D}^k e_R^k + \bar{u}_R^k \not{D}^k u_R^k + \bar{d}_R^k \not{D}^k d_R^k \right), \tag{2.6}$$

the index k runs over each of the flavors of the fermions.

As it has been said, the Higgs sector is responsible for the SEWSB. The Lagrangian of this scalar field H is

$$\mathcal{L}_{Higgs} = \left(D_\mu H\right)^\dagger \left(D^\mu H\right) - V(H), \quad V(H) = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2, \tag{2.7}$$

where constants μ^2 and λ are > 0, so that the potential $V(H)$ has a non-trivial and stable minimum. The Higgs field is a doublet under SU(2)$_L$, composed of real scalar fields, which can be written as

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} H_1 + iH_2 \\ H_3 + iH_4 \end{pmatrix}. \tag{2.8}$$

The scalar-vector interaction is described through the covariant derivative of the Higgs field

$$D_\mu H = \left(\partial_\mu - igW_\mu^a \frac{\sigma_a}{2} - ig' B_\mu \frac{\mathbb{1}}{2} \right) H. \tag{2.9}$$

The interactions of the scalar Higgs field with fermions is described through $\mathcal{L}_{Yuk}$, which has the form

$$-\mathcal{L}_{Yuk} = \sum_{k,j=1}^{3} \left(Y_e^{kj} \bar{\ell}_L^k H e_R^j + Y_d^{kj} \bar{q}_L^k H d_R^j + Y_u^{kj} \bar{q}_L^k \tilde{H} u_R^j \right) + h.c. \tag{2.10}$$

where $\tilde{H} = i\sigma_2 H^*$ is the conjugate of the doublet H, and Y_f^{kj} are the coupling constants between the fermions and the Higgs field.

[2]The groups SU(N), with $N \geq 2$ are non-abelian.

Putting all the sectors together, we have the Lagrangian of the SM before the spontaneous breaking of the symmetry, given by

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}G_b^{\mu\nu}G^b_{\mu\nu} - \frac{1}{4}W_a^{\mu\nu}W^a_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ &+ i\sum_{k=1}^{3}\left(\bar{q}_L^k \not{D}^k q_L^k + \bar{\ell}_L^k \not{D}^k \ell_L^k + \bar{e}_R^k \not{D}^k e_R^k + \bar{u}_R^k \not{D}^k u_R^k + \bar{d}_R^k \not{D}^k d_R^k\right) \\ &+ (D_\mu H)^\dagger (D^\mu H) + \mu^2 H^\dagger H - \lambda \left(H^\dagger H\right)^2 \\ &- \sum_{k,j=1}^{3}\left(Y_e^{kj}\bar{\ell}_L^k H e_R^j + Y_d^{kj}\bar{q}_L^k H d_R^j + Y_u^{kj}\bar{q}_L^k \tilde{H} u_R^j\right) + h.c.\end{aligned} \tag{2.11}$$

2.2 Spontaneous Symmetry breaking in the SM

The spontaneous symmetry breaking, SSB, occurs when the Lagrangian is invariant under a symmetry group $\mathcal{G}$, but the physical states (particularly the vacuum) are not. When the symmetry transformation is local, we have the so-called Higgs mechanism [25–27], where the gauge bosons associated with the spontaneously broken generators acquire mass. The symmetry $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is spontaneously broken, when the Higgs field acquires a vacuum expected value (vev). The vev is induced by the potential $V(H)$

$$\left.\frac{dV}{d|H|^2}\right|_{\min} = 0 = \left(-\mu^2 + 2\lambda|H|^2\right)|_{\min} \Rightarrow |\langle H\rangle|^2 = \frac{v^2}{2} = \frac{\mu^2}{2\lambda}. \tag{2.12}$$

The degeneracy of the vacuum is infinite, and all are equivalent. Conventionally, we choose

$$\langle H_1\rangle = \langle H_2\rangle = \langle H_4\rangle = 0, \quad \langle H_3\rangle = \frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{2\lambda}}. \tag{2.13}$$

So when the Higgs field acquires a vev, we obtain

$$\langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}. \tag{2.14}$$

When the SEWSB occurs, we have a combination of generators that leave the vacuum invariant, given by

$$Q = T_3 + Y = \frac{\sigma_3}{2} + Y. \tag{2.15}$$

Since we have the breakdown pattern $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{em}$, where the subgroup $\mathrm{U}(1)_{em}$ corresponds to the electromagnetic interaction; then the generator Q must be the electric charge. Once the Higgs field acquires a vev, from the kinetic term $|D_\mu H|^2$, we obtain

$$\mathcal{L}_{mass} = \frac{g^2v^2}{4}W^+_\mu W^\mu_- + \frac{v^2}{8}\left(g^2 + g'^2\right) Z_\mu Z^\mu. \tag{2.16}$$

The charged fields mediators of weak interactions are expressed by $W^\pm_\mu$, and have been defined as

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp iW^2_\mu\right). \tag{2.17}$$

The weak neutral interactions are mediated by the Z^0_μ boson, which is a combination of the B_μ and W_3^μ bosons. From eq. (2.16) we have that, after the SEWSB, the bosons Z^0_μ and $W^\pm_\mu$ have acquired a mass given by

$$M_W = \frac{gv}{2}, \quad M_Z = \frac{v}{2}\sqrt{g^2 + g'^2}. \tag{2.18}$$

The combination orthogonal to Z^0_μ is the boson A_μ, which does not acquire mass in the SEWSB. Such a boson is identified with the photon.

In the Yukawa sector, after the SEWSB, we obtain Dirac-type mass terms for the charged leptons and quarks, given by

$$\mathcal{L}_{Yuk} = -\frac{v}{\sqrt{2}}\left(\bar{\boldsymbol{e}}_L Y_e \boldsymbol{e}_R + \bar{\boldsymbol{d}}_L Y_d \boldsymbol{d}_R + \bar{\boldsymbol{u}}_L Y_u \boldsymbol{u}_R\right) + h.c. \tag{2.19}$$

The following two events have taken place in this sector

- Neutrinos have not acquired mass.
- The matrices $Y_{e,d,u}$, are not diagonal in the flavor space, so we cannot yet associate a mass to fermions.

The diagonalization of the matrices $Y_{e,d,u}$ is done through a bi-unitary transformation, of the form

$$Y = U\hat{Y}V^\dagger, \tag{2.20}$$

where $\hat{Y}$ is a diagonal matrix with positive definite entries. With this diagonalization, we obtain

$$\mathcal{L}_{Yuk} = -\frac{v}{\sqrt{2}}\left(\bar{\boldsymbol{e}}_L U_{e_L}\hat{Y}_e U^\dagger_{e_R}\boldsymbol{e}_R + \bar{\boldsymbol{d}}_L U_{d_L}\hat{Y}_d U^\dagger_{d_R}\boldsymbol{d}_R + \bar{\boldsymbol{u}}_L U_{u_L}\hat{Y}_u U^\dagger_{u_R}\boldsymbol{u}_R\right) + h.c. \tag{2.21}$$

Now, since physics is invariant under redefinitions of the fields, we perform a unitary transformation of the fields in the flavor space as follows

$$\boldsymbol{f}_{L,R} \to U_{f_{L,R}}\boldsymbol{f}_{L,R}. \tag{2.22}$$

For example, $\boldsymbol{e}_R \to U_{e_R}\boldsymbol{e}_R$ and, $\bar{\boldsymbol{e}}_L \to \bar{\boldsymbol{e}}_L U^\dagger_{e_L}$. In this way for the Yukawa sector, we obtain

$$\mathcal{L}_{Yuk} = -\frac{v}{\sqrt{2}}\left(\bar{\boldsymbol{e}}_L \hat{Y}_e \boldsymbol{e}_R + \bar{\boldsymbol{d}}_L \hat{Y}_d \boldsymbol{d}_R + \bar{\boldsymbol{u}}_L \hat{Y}_u \boldsymbol{u}_R\right) + h.c. \tag{2.23}$$

Thus when SEWSB occurs, quarks and charged leptons acquire a well-defined mass given by

$$M_f = \frac{v}{\sqrt{2}}\hat{Y}_f, \quad \text{where} \quad f = e, u, d. \tag{2.24}$$

Thus we have that the transformations leading to the fermion mass basis at the doublet level are

$$\begin{aligned} \ell &= \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \to U_{e_L}\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \\ q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} \to U_{u_L}\begin{pmatrix} u_L \\ U^\dagger_{u_L}U_{d_L}d_L \end{pmatrix} = U_{u_L}\begin{pmatrix} u_L \\ V_{\text{CKM}}d_L \end{pmatrix}. \end{aligned} \tag{2.25}$$

Where we have defined the Cabibbo-Kobayashi-Maskawa matrix, V_{CKM}, which being a product of unitary matrices is itself a unitary matrix.

Such unitary transformations in the flavor space only have an effect on the vector-fermion interactions of the SM.

In the lepton sector, the neutral interactions, which involve bosons Z^0_μ and A_μ, are of the form $\bar{f}_{L,R}\gamma^\mu f_{L,R}$, that is, they involve the same type of fermions, so the unitary transformations will not modify the neutral interactions. In charged interactions, if we consider that neutrinos are nonmassive, they can be transformed as we wish without changing the physics, so charged interactions will not be modified either. In other words, there are no leptonic flavor changes at tree level and for massless neutrinos.

In the quark sector, the neutral interactions are of the same structure as in the leptonic case. Therefore, transformations in the flavor space have no effect. Because the $V_{\rm CKM}$ is not diagonal, charged interactions change the flavor. That is, there are processes mediated by $W_\mu^\pm$ bosons that violate the flavor in the quark sector.

Since it is essential for this thesis, we will in the following give the explicit form that charged and neutral current interactions involving neutrinos have in the SM, after the SSEWSB. This is of central importance in this work, since the constraints on non-unitarity will be given with respect to the SM couplings of neutrinos (among themselves and with charged leptons) mediated by the weak Z^0 and $W^\pm$ gauge bosons –in the neutral and charged currents, respectively–, which are

$$\begin{aligned}\mathcal{L}_{W^\pm} &= -\frac{g}{\sqrt{2}}W_\mu^- \sum_{i,j=1}^{3} U_{ij}^\nu \bar{\ell}_i \gamma_\mu P_L \nu_j + h.c.\,,\\ \mathcal{L}_Z &= -\frac{g}{4c_W} Z_\mu \sum_{i=1}^{3} \bar{\nu}_i \gamma^\mu P_L \nu_i\,,\\ \mathcal{L}_{G^\pm} &= -\frac{g}{\sqrt{2}M_W} G^- \sum_{i,j=1}^{3} U_{ij}^\nu \bar{\ell}_i m_{\ell_i} P_L \nu_j \,+\, h.c. \end{aligned} \tag{2.26}$$

In eqs. (2.26), the 3×3 matrix U_{ij}^ν is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [28, 29], encoding the charged current mixing of the three light (massive) and active neutrinos in the SM with their corresponding charged leptons. Mathematically, this matrix appears in a completely equivalent way as the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the quark sector [30,31], although the numerical values of their respective entries are rather different [24], which is not understood.

2.3 Tree-level Higgs boson physics

To eliminate the Goldstone bosons from the theory (which in the SEWSB process are "eaten" by the three gauge bosons to acquire their mass) we make use of the unitary gauge[3], in which the Higgs doublet is given by

$$H\left(x\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h+v \end{pmatrix}. \tag{2.27}$$

We see that in this gauge $H(x)$ has only one degree of freedom and the physical implications will be clearer to see. The scalar field h is a radial excitation (in the vev direction). This is the physical state called the Higgs boson. The physics of the Higgs boson h, refers

[3] Due to the local symmetry, we can always perform a gauge transformation to completely absorb the dependence on the Nambu Goldstone bosons

$$\varphi = e^{i\xi_a \chi_a}\tilde{\varphi} \rightarrow e^{-i\xi_a \chi_a}\varphi = \tilde{\varphi}.$$

The Nambu Goldstone bosons disappear from the Lagrangian, it is said that with this transformation $\mathcal{L}$ is in the unitary gauge.

to the couplings and the mass of the h field.

The Higgs sector of the SM in the unitary gauge is

$$\mathcal{L}_{Higgs} = \frac{1}{2}\partial_\mu h \partial^\mu h + M_W^2 \left(1 + \frac{h}{v}\right)^2 W_\mu^+ W_-^\mu + \frac{M_Z^2}{2}\left(1 + \frac{h}{v}\right)^2 Z_\mu Z^\mu - V(H), \tag{2.28}$$

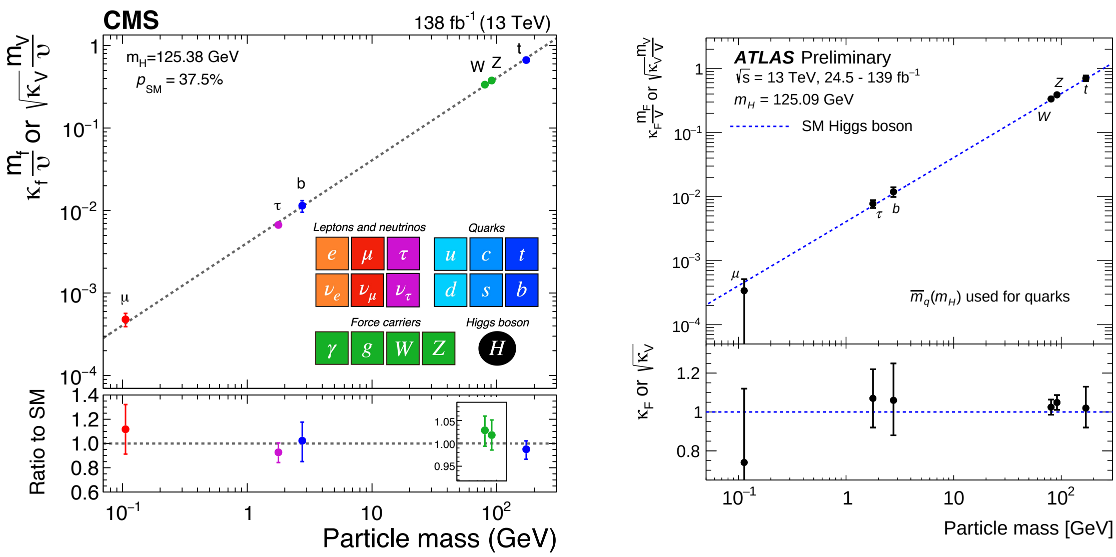
and the Yukawa sector

$$-\mathcal{L}_{Yuk} = \sum_f m_f \left(1 + \frac{h}{v}\right) \bar{f}_L f_R + h.c. \tag{2.29}$$

From equations (2.28) and (2.29) we have the following observations:

- All Higgs boson couplings are proportional to the mass of the particles, with divisions between v or v^2 to have the correct dimensions.
- The couplings of the Higgs boson with the massive gauge bosons are of linear and bilinear type.
- The couplings of the Higgs boson with fermions are of linear type.

The above observations have been experimentally tested and we show the most recent experimental results in Figure 2.1. Similar agreement with SM and data has been found for the Higgs boson couplings to fermions and bosons (not shown in the figure). Thus, at the required precision, the Higgs mechanism of SSEWSB appears to be the one realized in Nature. It is yet unknown if it contributes to neutrino masses, however. In this case, mass generation related to the Weinberg operator (app. B), like the one considered in this thesis, may be essential to understand their tiny values.



(a) CMS (b) ATLAS

Figure 2.1: Couplings of the Higgs boson versus the mass of the SM particles. The Higgs boson couples with a strength proportional to mass (charged fermions) or mass squared (gauge bosons). We observe that the more massive a particle, the stronger the coupling is. Taken from [32] and [33] links.

But these couplings are only at tree level. Phenomenologically, the most important ones arise at one loop and are $h - A_\mu - A^\mu$ (photon-photon) for its decay ($h \to \gamma\gamma$ was one of the two Higgs discovery modes, together with $H \to 4\ell$, coming from one on-shell and one off-shell intermediate Z^0 bosons) and $h - G^a_\mu - G^{\mu\, a}$ (gluon-gluon) for its main production channel at the LHC.

We can read the mass of the Higgs boson from the potential[4] with the substitution $\mu^2 = \lambda v^2$:

$$\begin{aligned} V(H) &= -\frac{\mu^2}{2} v^2 \left(1 + \frac{h}{v}\right)^2 + \frac{\lambda}{4} v^4 \left(1 + \frac{h}{v}\right)^4 \\ &= \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 - \frac{\lambda}{4} v^4. \end{aligned} \tag{2.31}$$

Then, the Higgs boson mass term is $m_h^2 = 2\lambda v^2$, plus trilinear and quartic self-couplings. There are two parameters in the Higgs sector, the mass parameter μ and the quartic coupling parameter λ. One combination of them gives the expected vacuum expectation value, which is measured via the Fermi constant in the muon decay $\mu = \lambda v^2$. The other combination gives the mass of the Higgs boson measured in 2012 at CERN $\lambda = m_h^2/2v^2$. With the above we can express the Higgs self-couplings, only in terms of observables v^2 and m_h^2, that is to say

$$V(H) = \frac{1}{2} m_h^2 h^2 + \frac{1}{2v} m_h^2 h^3 + \frac{1}{8v^2} m_h^2 h^4 - \frac{1}{8} m_h^2 v^2. \tag{2.32}$$

Despite the predictions of the standard model, the tree-level values are very far from the experimental values (some almost 40σ off). Only when we consider the loop corrections do we have a formidable accuracy between theory and experiment. Table 2.3 shows some experimental values together with the predictions of the standard model.

Quantity	**Exp. Value**	**SM**
M_Z (GeV)	91.1876 ± 0.0021	91.1882 ± 0.0020
Γ_Z(GeV)	2.4955 ± 0.0023	2.4941 ± 0.0009
A_e	0.15138 ± 0.00216	0.1468 ± 0.0003
	0.1544 ± 0.0060	
	0.1498 ± 0.0049	

Table 2.3: Experimental values as SM predictions for, Z-boson pole mass M_Z, the decay rate Γ_Z and asymmetric polarization A_e in Z boson production. For more values and details see [24].

[4]The mass term of a scalar field can also be obtained through the second derivative of the potential evaluated at the minimum, in this case the minimum is equal to extremize and find $\langle h \rangle = 0$,

$$m_h^2 = \left.\frac{\partial^2 V}{\partial h^2}\right|_{\min} = \left[-\mu^2 + 3\lambda v^2 \left(1 + \frac{h}{v}\right)^2\right]_{\min} \to m_h^2 = 2\lambda v^2 = 2\mu^2. \tag{2.30}$$

Chapter 3

Dirac, Weyl, and Majorana Spinors

Originally in the SM, neutrinos are massless, but neutrino oscillation experiments indicate otherwise. By including mass to neutrinos, the SM must be extended. To construct mass terms for neutrinos, Weyl spinors are used. These spinors are irreducible representations of the Lorentz group. From the Weyl spinors, mass terms are constructed for the so-called Dirac and Majorana fermions.

Paul Dirac derived in 1928 a relativistic wave equation for the electron. This equation describes all spin 1/2 fermionic particles, which written in covariant notation is given by

$$(i\gamma^\mu \partial_\mu - m)\,\psi = 0, \tag{3.1}$$

where ψ is a Dirac spinor, which has four components and γ^μ are a set of matrices that satisfy a Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \tag{3.2}$$

The Dirac equation can be derived using the Euler-Lagrange equations, from the Lagrangian

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi. \tag{3.3}$$

Among the successes of the Dirac equation are the prediction of the existence of antiparticles and that spin is contained in the theory as a consequence of relativistic invariance.

3.1 Weyl spinors

The Dirac spinor can be written in terms of two-component spinors

$$\psi = \begin{pmatrix} \eta \\ \chi \end{pmatrix}. \tag{3.4}$$

The two-component spinors η and χ are called Weyl spinors. So the Dirac spinors form a reducible representation of the Lorentz group and the Weyl spinors form an irreducible representation of the Lorentz group.

If we set η or χ to zero in ψ, eigenstates of the matrix γ_5[1] are produced

$$\gamma_5 \begin{pmatrix} \eta \\ 0 \end{pmatrix} = + \begin{pmatrix} \eta \\ 0 \end{pmatrix}, \quad \gamma_5 \begin{pmatrix} 0 \\ \chi \end{pmatrix} = - \begin{pmatrix} 0 \\ \chi \end{pmatrix}. \tag{3.5}$$

[1] Where the matrix γ_5 is defined by a product of the γ_μ matrices, as $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$.

The eigenvalue of γ_5 is called the chirality of a spinor. Thus η has a chirality of $+1$ and χ has a chirality of -1. We will refer to a Weyl spinor with positive chirality as right-chiral spinor η_R, and to one with negative chirality as left-chiral spinor χ_L.

The chirality will specify the representation of the Lorentz group to which a spinor belongs. We can define the Weyl spinors as the components of the eigenstates of the chirality operator, which are irreducible representations of the Lorentz group. Weyl spinors η_R and χ_L transform under rotations and boosts as follows

$$\eta_R \to \eta_R' = U_R \eta_R = e^{\frac{1}{2}(i\boldsymbol{\alpha}-\boldsymbol{\beta})\cdot\boldsymbol{\sigma}}\eta_R \approx \left(\mathbb{1} + \frac{i}{2}\boldsymbol{\alpha}\cdot\boldsymbol{\sigma} - \frac{1}{2}\boldsymbol{\beta}\cdot\boldsymbol{\sigma}\right)\eta_R, \tag{3.6}$$

$$\chi_L \to \chi_L' = U_L \chi_L = e^{\frac{1}{2}(i\boldsymbol{\alpha}+\boldsymbol{\beta})\cdot\boldsymbol{\sigma}}\chi_L \approx \left(\mathbb{1} + \frac{i}{2}\boldsymbol{\alpha}\cdot\boldsymbol{\sigma} + \frac{1}{2}\boldsymbol{\beta}\cdot\boldsymbol{\sigma}\right)\chi_L. \tag{3.7}$$

The spinors η_R and χ_L transform independently under the Lorentz group. These spinors are the fundamental building blocks, from which any other spinor representation can be constructed. Note that the matrices $U_{L,R}$ are not unitary, instead we have

$$U_L^{-1} = U_R^\dagger, \quad U_R^{-1} = U_L^\dagger. \tag{3.8}$$

Now we define the sets composed by the Pauli matrices and the identity as

$$\sigma^\mu = (\mathbb{1}, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu = (\mathbb{1}, -\boldsymbol{\sigma}), \tag{3.9}$$

with which the matrices γ^μ can be written as

$$\gamma^\mu = \begin{pmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{pmatrix}. \tag{3.10}$$

If we substitute these definitions into the Dirac equation, we obtain

$$(\gamma^\mu p_\mu - m)\psi = \begin{pmatrix} -m & \bar{\sigma}^\mu p_\mu \\ \sigma^\mu p_\mu & -m \end{pmatrix}\begin{pmatrix} \eta_R \\ \chi_L \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{3.11}$$

so that we have two coupled equations for the Weyl spinors η_R and χ_L

$$\begin{aligned} \bar{\sigma}^\mu p_\mu \chi_L &= m\eta_R, \\ \sigma^\mu p_\mu \eta_R &= m\chi_L. \end{aligned} \tag{3.12}$$

Also, we can write the Dirac Lagrangian in terms of the Weyl spinors

$$\begin{aligned} \mathcal{L} = \bar{\psi}(\gamma^\mu p_\mu - m)\psi &= \left(\chi_L^\dagger, \eta_R^\dagger\right)\begin{pmatrix} \bar{\sigma}^\mu p_\mu \chi_L - m\eta_R \\ \sigma^\mu p_\mu \eta_R - m\chi_L \end{pmatrix} \\ &= \eta_R^\dagger \sigma^\mu p_\mu \eta_R + \chi_L^\dagger \bar{\sigma}^\mu p_\mu \chi_L - m\left(\chi_L^\dagger \eta_R + \eta_R^\dagger \chi_L\right). \end{aligned} \tag{3.13}$$

In this Lagrangian each term is invariant separately. The kinetic terms do not mix L and R components (this is called chiral symmetry), unlike the mass terms (which break it). We know that p_μ is a 4-vector, so the quantities $\eta_R^\dagger \sigma^\mu \eta_R$ and $\chi_L^\dagger \bar{\sigma}^\mu \chi_L$ must also be 4-vectors to have Lorentz invariance.

To construct invariant terms only from χ_L or η_R, we assume that $\chi_R^\dagger \eta_R$ and $\eta_R^\dagger \chi_L$ are invariant. We need to build an object from χ_L (η_R) that transforms like η_R (χ_L). For this purpose we use the equality $\sigma_2 \boldsymbol{\sigma}^* = -\boldsymbol{\sigma}\sigma_2$ and consider the terms $i\sigma_2\chi_L^{\dagger T}$ and $-i\sigma_2\eta_R^{\dagger T}$. Let us see if these terms transform as we wish

$$\begin{aligned} i\sigma_2\chi_L^{\dagger T} \to i\sigma_2\chi'^{\dagger T}_L &= i\sigma_2 \left(U_L \chi_L\right)^* \\ &= i\sigma_2 \left(\mathbb{1} + \frac{i}{2}\boldsymbol{\alpha}\cdot\boldsymbol{\sigma} + \frac{1}{2}\boldsymbol{\beta}\cdot\boldsymbol{\sigma}\right)^* \chi_L^* \\ &= i\sigma_2 \left(\mathbb{1} - \frac{i}{2}\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}^* + \frac{1}{2}\boldsymbol{\beta}\cdot\boldsymbol{\sigma}^*\right) \chi_L^{\dagger T} \\ &= \left(\mathbb{1} + \frac{i}{2}\boldsymbol{\alpha}\cdot\boldsymbol{\sigma} - \frac{1}{2}\boldsymbol{\beta}\cdot\boldsymbol{\sigma}\right) i\sigma_2\chi_L^{\dagger T} = U_R i\sigma_2 \chi_L^{\dagger T}. \end{aligned} \tag{3.14}$$

Now, let us look at the term $-i\sigma_2\eta_R^{\dagger T}$

$$\begin{aligned} -i\sigma_2\eta_R^{\dagger T} \to -i\sigma_2\eta'^{\dagger T}_R &= -i\sigma_2 \left(U_R \eta_R\right)^* \\ &= -i\sigma_2 \left(\mathbb{1} + \frac{i}{2}\boldsymbol{\alpha}\cdot\boldsymbol{\sigma} - \frac{1}{2}\boldsymbol{\beta}\cdot\boldsymbol{\sigma}\right)^* \eta_R^* \\ &= -i\sigma_2 \left(\mathbb{1} - \frac{i}{2}\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}^* - \frac{1}{2}\boldsymbol{\beta}\cdot\boldsymbol{\sigma}^*\right) \eta_R^{\dagger T} \\ &= -i \left(\mathbb{1} + \frac{i}{2}\boldsymbol{\alpha}\cdot\boldsymbol{\sigma} + \frac{1}{2}\boldsymbol{\beta}\cdot\boldsymbol{\sigma}\right) \sigma_2\eta_R^{\dagger T} = U_L \left(-i\sigma_2\eta_R^{\dagger T}\right) \end{aligned} \tag{3.15}$$

Thus, $i\sigma_2\chi_L^{\dagger T}$ transforms as a right-chirality Weyl spinor (with U_R), and $-i\sigma_2\eta_R^{\dagger T}$ as a left-chirality one (with U_L). In summary, we have the following types of transformation

$$\left.\begin{aligned} \eta_R &\to U_R \eta_R \\ i\sigma_2\chi_L^{\dagger T} &\to U_R \left(i\sigma_2\chi_L^{\dagger T}\right) \end{aligned}\right\} \text{Type I transformations (with } U_R\text{).}$$

$$\left.\begin{aligned} \chi_L &\to U_L \chi_L \\ -i\sigma_2\eta_R^{\dagger T} &\to U_L \left(-i\sigma_2\eta_R^{\dagger T}\right) \end{aligned}\right\} \text{Type II transformations (with } U_L\text{).}$$

Then, we have that the invariant mass-type terms will be of the form $(\text{type I})^\dagger\,(\text{type II})$ or $(\text{type II})^\dagger\,(\text{type I})$. The possible combinations are

$$\begin{aligned} &(\text{type I})^\dagger\,(\text{type II}) : \eta_R^\dagger \chi_L, \quad \eta_R^\dagger \left(-i\sigma_2\right) \eta_R^{\dagger T}, \quad \chi_L^T \left(-i\sigma_2\right) \chi_L. \\ &(\text{type II})^\dagger\,(\text{type I}) : \chi_L^\dagger \eta_R, \quad \chi_L^\dagger \left(i\sigma_2\right) \chi_L^{\dagger T}, \quad \eta_R^T \left(i\sigma_2\right) \eta_R. \end{aligned} \tag{3.16}$$

For clarity and convenience, we introduce the following notation[2]. For this, for example, we focus on invariants built from the left-chirality spinor, the subscript L is eliminated and two types of dot product between left-chiral Weyl spinors are defined, as follows

$$\chi\cdot\chi = \chi^T \left(-i\sigma_2\right) \chi^{\dagger T}, \qquad \bar{\chi}\cdot\bar{\chi} = \chi^\dagger \left(i\sigma_2\right) \chi^{\dagger T}. \tag{3.17}$$

The bar on Weyl spinors has nothing to do with the bar used to represent the conjugation of a Dirac spinor which, as we have mentioned, has four components.

It is very useful to introduce the operators P_L and P_R, given by

$$P_R = \frac{1}{2}\left(\mathbb{1} + \gamma_5\right), \qquad P_L = \frac{1}{2}\left(\mathbb{1} - \gamma_5\right). \tag{3.18}$$

[2]In the work and use of Weyl spinors, the Van der Waerden notation [34] (“dotted” and “dotless” or “undotted” spinors) is widely used, but is not employed in this work.

These operators[3] produce chiral projections of ψ, in other words

$$\psi_R = P_R\psi = \begin{pmatrix} \eta_R \\ 0 \end{pmatrix}, \quad \psi_L = P_L\psi = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix}. \tag{3.19}$$

Then we have the following eigenequation

$$\gamma_5\psi_{R,L} = \pm\psi_{R,L}. \tag{3.20}$$

Any Dirac spinor can be written in chiral projections as $\psi = \mathbb{1}\psi = (P_R + P_L)\,\psi = \psi_R + \psi_L$. Then the Dirac mass term can be written in terms of four-component spinors as

$$\mathcal{L}_{mass} = -m\bar{\psi}\psi = -m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right) \tag{3.21}$$

3.2 Helicity

In the limit case of massless particles, $m = 0$, the equations (3.12), for χ_L and η_R can be decoupled into two independent equations

$$\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{|\boldsymbol{p}|}\eta_R = \eta_R, \quad \text{and,} \quad \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{|\boldsymbol{p}|}\chi_L = -\chi_L, \tag{3.22}$$

which are known as Weyl equations. If we multiply these equations by $1/2$ and given that the spin operator is $\boldsymbol{S} = \boldsymbol{\sigma}/2$ and $\hat{\boldsymbol{p}} = \boldsymbol{p}/|\boldsymbol{p}|$, we have

$$\boldsymbol{S}\cdot\hat{\boldsymbol{p}}\,\eta_R = \frac{\eta_R}{2}, \quad \text{and,} \quad \boldsymbol{S}\cdot\hat{\boldsymbol{p}}\,\chi_L = -\frac{\chi_L}{2}. \tag{3.23}$$

where we have the helicity operator given by $\boldsymbol{S}\cdot\hat{\boldsymbol{p}}$ and its corresponding eigenvalue is the helicity of the state. Helicity is the projection of spin in the direction of motion of the particle. So, in the massless limit, the Weyl spinors are eigenstates of the helicity operator, in this limit the chiral spinors have well-defined helicities. Helicity is easier to imagine physically than chirality, but chiral eigenstates are those that have well-defined Lorentz transformation properties.

3.3 Majorana spinors

Before describing a Majorana spinor, we introduce the charge conjugation operation, which affects only the internal degrees of freedom of the state of a particle. This operation inverts all internal quantum numbers, such as electric charge, leptonic number, baryonic number, etc. That is, a particle is transformed into an antiparticle and vice versa. Given a Dirac spinor for a certain particle p,

$$\psi_p = \begin{pmatrix} \eta_p \\ \chi_p \end{pmatrix}. \tag{3.24}$$

The charge conjugation operation is defined as

$$\psi_a = \psi_p^c = C\bar{\psi}_p^T = \begin{pmatrix} \eta_a \\ \chi_a \end{pmatrix}, \tag{3.25}$$

[3] Being projectors, they satisfy the following properties

$$P_L + P_R = \mathbb{1}, \quad P_LP_R = P_RP_L = 0, \quad P_{L,R}^2 = P_{L,R}.$$

where C is the charge conjugation matrix, which satisfies the relations

$$C\gamma^{\mu T}C^{-1} = -\gamma^{\mu}, \quad \gamma_5 = C\gamma_5^T C^{-1}, \quad C = C^* = -C^{-1} = -C^T = -C^{\dagger}. \tag{3.26}$$

The right (left) chiral spinors for particles $\eta_p(\chi_p)$ are different from those for antiparticles $\eta_a(\chi_a)$. One representation of the charge conjugation matrix is to take $C = -i\gamma_2\gamma_0$, thus

$$\psi_p^c = \begin{pmatrix} i\sigma_2\chi_p^{\dagger T} \\ -i\sigma_2\eta_p^{\dagger T} \end{pmatrix} = \begin{pmatrix} \eta_a \\ \chi_a \end{pmatrix} \Rightarrow \eta_a = i\sigma_2\chi_p^{\dagger T}, \quad \text{and,} \quad \chi_a = -i\sigma_2\eta_p^{\dagger T} \tag{3.27}$$

So, for example, we can rewrite a Dirac spinor in terms of only left chiral fields

$$\psi_p = \begin{pmatrix} \eta_p \\ \chi_p \end{pmatrix} = \begin{pmatrix} i\sigma_2\chi_a^{\dagger T} \\ \chi_p \end{pmatrix}. \tag{3.28}$$

Ettore Majorana in 1938 discovered a theory [35], in which a fermionic particle is its own antiparticle. This means that Majorana fermions are completely neutral. These fermions are a solution to a version of the Dirac equation, where the γ-matrices are purely imaginary. By construction, the Majorana spinor ψ_M is not altered by the conjugation of the charge $\psi_M^c = \psi_M$, called the Majorana condition, which expresses the fact that $\chi_a = \chi_p = \chi$, and $\eta_a = \eta_p = \eta$. Majorana spinor consists of four components. Using the eq.(3.28) and the Majorana condition, the spinor ψ_M is given by

$$\psi_M = \begin{pmatrix} \eta \\ \chi \end{pmatrix} = \begin{pmatrix} i\sigma_2\chi^{\dagger T} \\ \chi \end{pmatrix}. \tag{3.29}$$

The Lagrangian that describes a free Majorana particle will be

$$\mathcal{L}_M = \frac{1}{2}\bar{\psi}_M \left(\gamma^{\mu}p_{\mu} - m\right)\psi_M, \tag{3.30}$$

where the factor 1/2 is for the purpose of obtaining canonical kinetic terms, also so that the parameter m corresponds to the mass of the particle. Let us write this Lagrangian only in terms of left-chiral spinors

$$\begin{aligned} \mathcal{L}_M &= \frac{1}{2}\left(\eta^{\dagger}, \chi^{\dagger}\right)\begin{pmatrix} \sigma^{\mu}p_{\mu} & 0 \\ 0 & \bar{\sigma}^{\mu}p_{\mu} \end{pmatrix}\begin{pmatrix} \eta \\ \chi \end{pmatrix} - \frac{m}{2}\left(\eta^{\dagger}, \chi^{\dagger}\right)\begin{pmatrix} \chi \\ \eta \end{pmatrix} \\ &= \frac{1}{2}\left(\chi^T\bar{\sigma}^{\mu T}p_{\mu}\chi^{\dagger T} + \chi^{\dagger}\bar{\sigma}^{\mu}p_{\mu}\chi\right) - \frac{m}{2}\left(\chi^T\left(-i\sigma_2\right)\chi + \chi^{\dagger}\left(i\sigma_2\right)\chi^{\dagger T}\right) \\ &= \chi^{\dagger}\bar{\sigma}^{\mu}p_{\mu}\chi - \frac{m}{2}\left(\chi\cdot\chi + \bar{\chi}\cdot\bar{\chi}\right). \end{aligned} \tag{3.31}$$

We can use chiral projection operators eq.(3.18) and act them on ψ_L^c and ψ_R^c, so that we will get the following

$$P_L\psi_L^c = 0, \quad P_R\psi_R^c = 0, \quad P_L\psi_R^c = \psi_R^c, \quad P_R\psi_L^c = \psi_L^c. \tag{3.32}$$

This is easy to see, using the relations satisfied by the charge conjugation operator, eq.(3.26), and the properties of the chiral projection operators. For example

$$\psi_L^c = C\overline{\psi}_L^T = C\left(\overline{\psi}P_R^2\right)^T = C\left(\overline{\psi}_L P_R\right)^T = CP_R^T\overline{\psi}_L^T,$$

but, $CP_R^T = \frac{C}{2}\left(\mathbb{1} + \gamma_5^T\right) = \frac{1}{2}\left(C + \gamma_5 C\right) = P_R C$, then $\psi_L^c = P_R C\overline{\psi}_L^T = P_R\psi_L^c$, from this equality it is easy to see that $\left(\mathbb{1} - P_R\right)\psi_L^c = 0 \Rightarrow P_L\psi_L^c = 0$, and similarly for ψ_R^c.

Now the Majorana condition $\psi = \psi^c$ is broken down into left and right components with the chiral projection operators, $\psi_L + \psi_R = \psi_L^c + \psi_R^c$, and we operate on both sides of this equality with any of the projectors P_L or P_R. Using the results of eq.(3.32), we get

$$\psi_R = \psi_L^c, \quad \text{and,} \quad \psi_L = \psi_R^c. \tag{3.33}$$

We can see that the left (right) component ψ_L (ψ_R) of the Majorana field is not independent, it is related to the right component ψ_R (ψ_L) through charge conjugation. Then we can write the Majorana field as

$$\psi = \psi_L + \psi_L^c, \quad \text{or,} \quad \psi = \psi_R + \psi_R^c. \tag{3.34}$$

This is how we have three types of mass terms, Dirac, Majorana right and Majorana left

$$\begin{aligned}
\mathcal{L}_D^{mass} &= -m_D \left(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L\right), \\
\mathcal{L}_{M,R}^{mass} &= -\frac{m_R}{2} \left(\overline{\psi}_R^c \psi_R + \overline{\psi}_R \psi_R^c\right), \\
\mathcal{L}_{M,L}^{mass} &= -\frac{m_L}{2} \left(\overline{\psi}_L^c \psi_L + \overline{\psi}_L \psi_L^c\right).
\end{aligned} \tag{3.35}$$

Chapter 4

Massive neutrinos

As mentioned, based on experimental evidence, neutrino oscillations imply that they are massive particles (see appendix A). There is an enormous number of mechanisms to describe the mass of neutrinos. Each model involves an energy scale, which is associated with grand unification or the Planck scale (string theories) and therefore difficult to observe directly. Other mechanisms lead to deviations from the SM, which may be observable at the energy scales of current experiments.

The simplest extension to SM is to add right-handed fields, ν_R, for the three neutrino flavors. Dirac-type masses are generated by the Higgs mechanism, in the same way as for other fermions (charged leptons and quarks). However, the key difference with respect to these is that, for neutrinos, once ν_Rs are introduced, a Majorana mass term build up only from right-handed fields is possible, so it must happen unless a symmetry (lepton number conservation) forbids it, and there is not a clear reason for that, unless the lepton number L is gauged (typically in combination with the baryon number, B, as $B - L$ is preserved by anomalies in the SM whereas $B + L$ is not).

These right-handed neutrinos should not be charged under the symmetry group $\mathcal{G}_{SM}$, that is they are singlets, so there is no symmetry to protect their masses and, therefore, they can be very large $m_R \gg v$.

Since neutrinos have no electric charge, they can be Dirac or Majorana particles. Thus, once right-handed neutrinos are introduced, there is no obstacle to including Majorana mass terms, as advanced before.

We can use the different types of fermion masses, cf. eq.(3.35), and write their most general mass term. In the case of only one family of neutrinos, we have

$$-2\mathcal{L}_m = 2m_D\left(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L\right) + m_L\left(\bar{\nu}_L\nu_L^c + \bar{\nu}_L^c\nu_L\right) + m_R\left(\bar{\nu}_R\nu_R^c + \bar{\nu}_R^c\nu_R\right). \tag{4.1}$$

We use the relation eq.(3.33) and note that $2m_D\bar{\nu}_L\nu_R = m_D\left(\bar{\nu}_L\nu_R + \bar{\nu}_R^c\nu_L^c\right)$, and similarly $2m_D\bar{\nu}_R\nu_L = m_D\left(\bar{\nu}_R\nu_L + \bar{\nu}_L^c\nu_R^c\right)$, then

$$\begin{aligned}
-2\mathcal{L}_m &= m_D\left(\bar{\nu}_L\nu_R + \bar{\nu}_R^c\nu_L^c\right) + m_L\bar{\nu}_L\nu_L^c + m_R\bar{\nu}_R\nu_R^c + h.c \\
&= \left(\bar{\nu}_L, \bar{\nu}_R^c\right)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c \\
&= \bar{\boldsymbol{\chi}}_L\mathcal{M}\boldsymbol{\chi}_L^c + h.c.,
\end{aligned} \tag{4.2}$$

where $\boldsymbol{\chi}_L = \left(\nu_L, \nu_R^c\right)^T$ and $\mathcal{M}$ is the mass matrix. The eigenvalues of the mass matrix are the masses of the physical neutrino states. The eigenvalues are obtained from the equation $\det(\mathcal{M} - m\mathbb{1}) = 0$, so the masses of the physical neutrinos are

$$m_{1,2} = \frac{1}{2}\left[m_L + m_R \mp \sqrt{(m_L - m_R)^2 + 4m_D^2}\right], \tag{4.3}$$

with m_1 and m_2 the mass of the light and heavy neutrino, respectively.

4.1 Type I Seesaw Mechanism

This mechanism is based on the following assumptions: (i) a Majorana mass term for ν_L is not allowed by the SM symmetries, then $m_L = 0$ [1], (ii) the Dirac mass term, is generated by SEWSB, so m_D is of the order of the electroweak scale, (iii) the Majorana mass is much larger than the Dirac mass, $m_R \gg m_D$. Then the eigenvalues given in eq.(4.3) are as follows

$$m_{1,2} \approx \frac{m_R}{2}\left[1 \mp \left(1 + \frac{1}{2}\left(\frac{2m_D}{m_R}\right)^2 + \cdots\right)\right] \Rightarrow m_1 \approx -\frac{m_D^2}{m_R}, \quad \text{and}, \quad m_2 \approx m_R. \tag{4.4}$$

Through the definition of the fields (a phase shift) the negative sign of the mass of the light neutrino can be absorbed. In this mechanism, the higher m_R, the lower m_1, hence the name seesaw.

The mass matrix is symmetric, so it can be diagonalized orthogonally, such that

$$\hat{M} = \mathcal{O}^T \mathcal{M} \mathcal{O} \Rightarrow \mathcal{M} = \mathcal{O}\hat{M}\mathcal{O}^T, \quad \text{with,} \quad \mathcal{O}^{-1} = \mathcal{O}^T \tag{4.5}$$

Here $\hat{M}$ is the diagonal matrix, with elements m_1 and m_2. The matrix $\mathcal{O}$ is constructed from the eigenvectors of $\mathcal{M}$, i.e. $\mathcal{O} = [\boldsymbol{a}_1, \boldsymbol{a}_2]$. The eigenvalues of $\mathcal{M}$ and the matrix $\mathcal{O}$ are

$$\boldsymbol{a}_1 = \frac{1}{f}\begin{pmatrix} m_R \\ -m_D \end{pmatrix}, \quad \boldsymbol{a}_2 = \frac{1}{f}\begin{pmatrix} m_D \\ m_R \end{pmatrix} \Rightarrow \mathcal{O} = \frac{1}{f}\begin{pmatrix} m_R & m_D \\ -m_D & m_R \end{pmatrix}; \quad f = \sqrt{m_D^2 + m_R^2} \tag{4.6}$$

We can also define a mixing angle θ, where $\tan\theta = \frac{m_D}{m_R}$, so that the orthogonal matrix becomes

$$\mathcal{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{4.7}$$

If we want to obtain real and positive eigenvalues, the matrix $\mathcal{O}$ must be multiplied by a diagonal matrix of complex phases $\mathcal{C}$ of the form

$$\mathcal{C}_{ij} = \sqrt{\mathcal{C}_i}\delta_{ij}, \quad \mathcal{C}_i = \text{Sign(eigenvalue of } \mathcal{M}). \tag{4.8}$$

Then the diagonalization is performed through the unitary matrix, $\mathcal{U} = \mathcal{O}\mathcal{C}$, such that $\hat{\mathcal{M}} = \mathcal{U}^T \mathcal{M} \mathcal{U}$. Where $\hat{\mathcal{M}}$ is diagonal with positive and real eigenvalues, which is equivalent to the redefinition of the field, $\nu_k \to i\nu_k$, for the k-field of negative eigenvalue. In this case

$$\mathcal{U} = \mathcal{O}\mathcal{C} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} i\cos\theta & \sin\theta \\ -i\sin\theta & \cos\theta \end{pmatrix}. \tag{4.9}$$

[1]This is consistent with the fact that, within the original SM, neutrino masses need to arise from a dimension five operator, as pointed out by Weinberg [36], see appendix B.

The physical states of the neutrinos (mass eigenvectors) are obtained from $\mathcal{O}^T \boldsymbol{\chi}_L^c$, $\mathcal{O}^T \boldsymbol{\chi}_L$ and are

$$\begin{aligned} \nu_1 &= (\nu_L + \nu_L^c)\cos\theta - (\nu_R + \nu_R^c)\sin\theta, \\ \nu_2 &= (\nu_R + \nu_R^c)\cos\theta + (\nu_L + \nu_L^c)\sin\theta. \end{aligned} \tag{4.10}$$

Given that $m_R \gg m_D \Rightarrow \cos\theta \approx 1$, and $\sin\theta \approx \frac{m_D}{m_R}$, then we have that the light state ν_1 is practically left-handed and the massive state ν_2 is practically right-handed. We have that the ν_2 state does not participate in the weak charged or neutral currents, and the couplings of the light neutrinos to the weak charged currents are practically as in the SM.

In the generalization to n flavors, the Weyl spinors are vectors of n components in flavor space, in the following way

$$\nu_L = (\nu_{1L}, \cdots, \nu_{nL})^T, \quad N_R = (N_{1R}, \cdots, N_{nR})^T, \tag{4.11}$$

where ν_{iL} and N_{iR} are Weyl spinors with flavor i. The masses m_D and m_R are now matrices M_D and M_R, with complex elements, and $M_R = M_R^T$.

In a seesaw model, in addition to the three active neutrinos, ν_{Li}, of the SM, we add n new singlets fields. The total number of states is $n' = 3 + n$, and thus, the symmetric matrix of masses has dimension $n' \times n'$ and is given by

$$\mathcal{M}_{n' \times n'} = \begin{pmatrix} 0_{3\times 3} & M_{D_{3\times n}} \\ M^T_{D_{n\times 3}} & M_{R_{n\times n}} \end{pmatrix}. \tag{4.12}$$

It can be shown [8, 10, 37] that, the matrix of the light neutrinos M_ν and heavy neutrinos M_N are given by

$$M_\nu = -M_D M_R^{-1} M_D^T, \quad M_N = M_R. \tag{4.13}$$

Note the analogy with m_1. In the type I seesaw mechanism, the masses of the light neutrinos are suppressed by factor m_D^2/m_R. This mechanism provides a good hypothesis for the small mass of neutrinos, but it is only a hypothesis. This hypothesis would be placed on solid grounds if neutrinos are shown to be Majorana particles.

There are also type II and III seesaw mechanisms, which are not discussed in this dissertation. In the former a weak-scalar triplet is added to the SM, while in the latter a $SU(2)_L$ triplet fermion with zero hypercharge is included. See also the appendix B.

4.2 Inverse Seesaw

Since in the type I seesaw mechanism, the mass of the right-handed neutrinos N_R is very large, on the order of $\mathcal{O} >> 1$ TeV, experimental tests of this mechanism are very complicated, if not impossible.

Thus, low-scale seesaw mechanisms (where the new particles can have masses in the currently testable TeV scale) are attractive from a phenomenological point of view, such as the inverse seesaw mechanism, ISS.

In the ISS mechanism, the addition of three right-handed neutrinos N_{iR} and three singlet neutral fermions S_{iL} is required, along with the three active neutrinos ν_{iL}. We have the following mass effective Lagrangian

$$\begin{aligned}-2\mathcal{L} &= \bar{\nu}_L m_D N_R + \overline{S}_L M N_R + \overline{S}_L \mu S_L^c + h.c. \\ &= \left(\bar{\nu}_L^c, \overline{N}_R, \overline{S}_L^c\right) \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \\ S_L \end{pmatrix} + h.c. \\ &= \overline{\boldsymbol{\chi}}_L^c \mathcal{M} \boldsymbol{\chi}_L + h.c,\end{aligned} \tag{4.14}$$

where m_D, M and μ are generic 3×3 mass matrices. The mass matrix $\mathcal{M}$ can be written as

$$\mathcal{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}; \quad \text{With:} \quad M_D = (0, m_D), \quad M_R = \begin{pmatrix} 0 & M \\ M^T & \mu \end{pmatrix}, \tag{4.15}$$

with the hierarchy $|\mu| \ll |m_D| \ll |M|$. The diagonalization of this 9×9 matrix provides the matrix for the mass of the light and sterile neutrinos

$$m_\nu = m_D \left(M^T\right)^{-1} \mu M^{-1} m_D^T, \quad m_N = M_R. \tag{4.16}$$

This produces light masses of the order of $\mathcal{O} = (\mu m_D^2/M^2)$ are generated. For m_D at electroweak scale, with an M scale of TeV and μ at keV scale, light neutrino masses in the eV range are obtained.

4.3 Non-unitary mixing

We consider a mixing matrix, that manifestly factors the parameters associated with the neutral heavy leptons, NHL, from the parameters describing the neutrino oscillations.

In the case of three light neutrinos and $n' = n - 3$ NHL, the matrix $U^{n\times n}$ (which diagonalizes the mass matrix) can be decomposed according to ref. [38] as

$$U^{n\times n} = \begin{pmatrix} N_{3\times3} & S_{3\times n'} \\ V_{n'\times3} & T_{n'\times n'} \end{pmatrix}, \tag{4.17}$$

we have that the N is a 3×3 matrix of the light neutrino sector, and S describes the coupling parameters for the extra singlet states.

The matrix N can be expressed in several ways, but the most convenient parametrization [39] for describing the current neutrino experiments is

$$N = N^{NP} U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U, \tag{4.18}$$

where U is the 3×3 lepton mixing matrix, better known as PMNS, which has been tested in the study of neutrino oscillation. The N^{NP} matrix characterizes the unitary violation and the new physics.

In addition to the parameters characterizing the unitary mixing, there are four more parameters: three of them are real α_{11}, α_{22}, α_{33} and the complex parameter α_{21} which contains a single CP violating phase. Regardless of the number of extra singlets, only one combination of phase parameters enters the important neutrino oscillation experiments.

As mentioned above, the diagonal elements are real, and are expressed as follows

$$\begin{aligned} \alpha_{11} &= c_{1n}c_{1n-1}c_{1n-2}\dots c_{14}, \\ \alpha_{22} &= c_{2n}c_{2n-1}c_{2n-2}\dots c_{24}, \\ \alpha_{33} &= c_{3n}c_{3n-1}c_{3n-2}\dots c_{34}, \end{aligned} \tag{4.19}$$

where $c_{ij} = \cos\theta_{ij}$. The off-diagonal terms α_{21} and α_{32} are expressed as a sum of n' terms

$$\begin{aligned} \alpha_{21} &= c_{2n}c_{2n-1}\dots c_{25}\eta_{24}\bar{\eta}_{14} + c_{2n}\dots c_{26}\eta_{25}\bar{\eta}_{15}c_{14} + \dots + \eta_{2n}\bar{\eta}_{1n}c_{1n-1}c_{1n-2}\dots c_{14}, \\ \alpha_{32} &= c_{3n}c_{3n-1}\dots c_{35}\eta_{34}\bar{\eta}_{24} + c_{3n}\dots c_{36}\eta_{35}\bar{\eta}_{25}c_{24} + \dots + \eta_{3n}\bar{\eta}_{2n}c_{2n-1}c_{2n-2}\dots c_{24}. \end{aligned} \tag{4.20}$$

A similar expression for α_{31} is

$$\begin{aligned} \alpha_{31} = {} & c_{3n}c_{3n-1}\dots c_{35}\eta_{34}c_{24}\bar{\eta}_{14} + c_{3n}\dots c_{36}\eta_{35}c_{25}\bar{\eta}_{15}c_{14} + \dots + \eta_{3n}c_{2n}\bar{\eta}_{1n}c_{1n-1}c_{1n-2}\dots c_{14} \\ & + c_{3n}c_{3n-1}\dots c_{35}\eta_{35}\bar{\eta}_{25}\eta_{24}\bar{\eta}_{14} + c_{3n}\cdots c_{36}\eta_{36}\bar{\eta}_{26}c_{25}\eta_{24}\bar{\eta}_{14} \\ & + \dots + \eta_{3n}\bar{\eta}_{2n}\eta_{2n-1}\bar{\eta}_{1n-1}c_{1n-2}\cdots c_{14}. \end{aligned} \tag{4.21}$$

Phases $\eta_{ij} = e^{-i\phi_{ij}}\sin\theta_{ij}$ and $\bar{\eta}_{ij} = -e^{i\phi_{ij}}\sin\theta_{ij}$ contain all phases that violate CP. We have a parametrization in which all the information about the additional leptons is separated in a compact and simple way, with three zeros.

Let us look at the expressions for α_{ij} in the case of two extra NHL

$$\begin{aligned} &\alpha_{11} = c_{15}c_{14}, \quad \alpha_{22} = c_{25}c_{24}, \quad \alpha_{33} = c_{35}c_{34}, \\ &\alpha_{21} = c_{25}\eta_{24}\bar{\eta}_{14} + \eta_{25}\bar{\eta}_{15}c_{14}, \quad \alpha_{32} = c_{35}\eta_{34}\bar{\eta}_{24} + \eta_{35}\bar{\eta}_{25}c_{24}, \\ &\alpha_{31} = c_{35}\eta_{34}c_{24}\bar{\eta}_{14} + \eta_{35}c_{25}\bar{\eta}_{15}c_{14} + c_{14}\eta_{35}\bar{\eta}_{25}\eta_{24}\bar{\eta}_{14}. \end{aligned} \tag{4.22}$$

According to ref. [11], the following square matrix describes the couplings of n neutrino states in the charged current weak interaction

$$K = (N, S), \tag{4.23}$$

the unitarity condition is of the form

$$KK^\dagger = NN^\dagger + SS^\dagger = \mathbb{1}, \tag{4.24}$$

where the terms that characterize the new physics are

$$NN^\dagger = \begin{pmatrix} \alpha_{11}^2 & \alpha_{11}\alpha_{21}^* & \alpha_{11}\alpha_{31}^* \\ \alpha_{11}\alpha_{21} & \alpha_{22}^2 + |\alpha_{21}|^2 & \alpha_{22}\alpha_{32}^* + \alpha_{21}\alpha_{31}^* \\ \alpha_{11}\alpha_{31} & \alpha_{22}\alpha_{32} + \alpha_{31}\alpha_{21}^* & \alpha_{33}^2 + |\alpha_{31}|^2 + |\alpha_{32}|^2 \end{pmatrix}. \tag{4.25}$$

All the information of the extra states is encoded in the α_{ij} parameters in a compact way, with this parameterization. The mixing of the active light neutrinos with the heavy states implies non-unitary effects and thus a modification of several SM observables, as discussed in ref. [39].

Another parameterization widely used in encoding non-unitarity effects due to heavy-light neutrino mixing, together with the N^{NP} or α-matrix, is the Hermitian parameterization o η-matrix [40], such that

$$N = (\mathbb{1} - \eta)\,U_\eta, \quad \text{or} \quad N = (\mathbb{1} - \alpha)\,U_\alpha \tag{4.26}$$

Where α is a lower triangular matrix see eqs.(4.18), and η is a Hermitian matrix given by

$$\eta = \begin{pmatrix} \eta_{ee} & \eta_{e\mu} & \eta_{e\tau} \\ \eta^*_{\mu e} & \eta_{\mu\mu} & \eta_{\mu\tau} \\ \eta^*_{\tau e} & \eta^*_{\tau\mu} & \eta_{\tau\tau} \end{pmatrix} . \tag{4.27}$$

The matrices U_α and U_η are unitary and equivalent to PMNS. We should note that matrices U_α and U_η are different. Although these matrices are identified with PMNS in each parameterization, this is only accurate up to small corrections due to deviations from unitarity. To relate U_α and U_η we perform a unitary rotation V

$$N = (\mathbb{1} - \alpha)\, U_\alpha = (\mathbb{1} - \eta)\, V V^\dagger U_\eta, \tag{4.28}$$

then

$$\mathbb{1} - \alpha = (\mathbb{1} - \eta)\, V, \quad \text{and} \quad U_\alpha = V^\dagger U_\eta. \tag{4.29}$$

The elements of V to linear order are identified as

$$V = 1 - \tilde{\eta} = \mathbb{1} - \begin{pmatrix} 0 & -\eta_{e\mu} & -\eta_{e\tau} \\ \eta^*_{\mu e} & 0 & -\eta_{\mu\tau} \\ \eta^*_{\tau e} & \eta^*_{\tau\mu} & 0 \end{pmatrix} , \tag{4.30}$$

then, the relation between the elements of the two matrices is

$$\begin{gathered} \mathbb{1} - \alpha = (\mathbb{1} - \eta)\,(\mathbb{1} - \tilde{\eta}) \simeq \mathbb{1} - \tilde{\eta} - \eta \Rightarrow \alpha = \eta + \tilde{\eta} \\ \alpha = \begin{pmatrix} \alpha_{ee} & 0 & 0 \\ \alpha_{\mu e} & \alpha_{\mu\mu} & 0 \\ \alpha_{\tau e} & \alpha_{\tau\mu} & \alpha_{\tau\tau} \end{pmatrix} = \begin{pmatrix} \eta_{ee} & 0 & 0 \\ 2\eta^*_{\mu e} & \eta_{\mu\mu} & 0 \\ 2\eta^*_{\tau e} & 2\eta^*_{\tau\mu} & \eta_{\tau\tau} \end{pmatrix} . \end{gathered} \tag{4.31}$$

Through eqs.(4.31) we can make a mapping between the two parametrizations.

4.4 The model

We use the model developed in ref. [20], in which five Majorana fields $\chi_i = \chi_{Li} + \chi^c_{Li}$ with $\chi_L \equiv (\nu_{L1}, \nu_{L2}, \nu_{L3}, N_L, N^c_R)$ are assumed. In this case $n = 2$, and the following forms are chosen for the matrices of eq. (4.12)

$$M_{D_{3\times 2}} = \begin{pmatrix} 0 & m_1 \\ 0 & m_2 \\ 0 & m_3 \end{pmatrix} , \quad M^T_{D_{2\times 3}} = \begin{pmatrix} 0 & 0 & 0 \\ m_1 & m_2 & m_3 \end{pmatrix} , \quad M_{R_{2\times 2}} = \begin{pmatrix} 0 & M \\ M & \mu \end{pmatrix} . \tag{4.32}$$

Thus, the mass matrix will be given by

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & 0 & m_1 \\ 0 & 0 & 0 & 0 & m_2 \\ 0 & 0 & 0 & 0 & m_3 \\ 0 & 0 & 0 & 0 & M \\ m_1 & m_2 & m_3 & M & \mu \end{pmatrix} . \tag{4.33}$$

The diagonalization of the mass matrix eq.(4.33), by a unitary matrix, produces the eigenvalues $m_{\chi_{1,2,3}} = 0$, which correspond to the active neutrinos and are identified with the light neutrinos $\nu_{1,2,3}$, which are exactly massless in this model. This also produces the positive real eigenvalues

$$m_{N_1} = m_{\chi_4} = \frac{1}{2}\left(\sqrt{4M'^2 + \mu^2} - \mu\right) , \quad m_{N_2} = m_{\chi_5} = \frac{1}{2}\left(\sqrt{4M'^2 + \mu^2} + \mu\right) , \tag{4.34}$$

where $M' = \sqrt{m_1^2 + m_2^2 + m_3^2 + M^2}$ has been defined. The neutrino mass eigenstates are obtained from

$$\chi_{Li} \to \sum_{j=1}^{5} U_{ij} \chi_{Lj}, \tag{4.35}$$

with U the unitary matrix, which is

$$U = \begin{pmatrix} -\frac{m_2}{\sqrt{m_1^2+m_2^2}} & -\frac{m_1 m_3}{m\sqrt{m_1^2+m_2^2}} & -\frac{m_1 M}{m M'} & -i\frac{m_1 m_{\chi_5}}{M'\sqrt{m_{\chi_5}^2+M'^2}} & \frac{m_1}{\sqrt{m_{\chi_5}^2+M'^2}} \\ \frac{m_1}{\sqrt{m_1^2+m_2^2}} & -\frac{m_2 m_3}{m\sqrt{m_1^2+m_2^2}} & -\frac{m_2 M}{m M'} & -i\frac{m_2 m_{\chi_5}}{M'\sqrt{m_{\chi_5}^2+M'^2}} & \frac{m_2}{\sqrt{m_{\chi_5}^2+M'^2}} \\ 0 & \frac{\sqrt{m_1^2+m_2^2}}{m} & -\frac{m_3 M}{m M'} & -i\frac{m_3 m_{\chi_5}}{M'\sqrt{m_{\chi_5}^2+M'^2}} & \frac{m_3}{\sqrt{m_{\chi_5}^2+M'^2}} \\ 0 & 0 & \frac{m}{M'} & -i\frac{M m_{\chi_5}}{M'\sqrt{m_{\chi_5}^2+M'^2}} & \frac{M'}{\sqrt{m_{\chi_5}^2+M'^2}} \\ 0 & 0 & 0 & i\frac{M'}{\sqrt{m_{\chi_5}^2+M'^2}} & \frac{m_{\chi_5}}{\sqrt{m_{\chi_5}^2+M'^2}} \end{pmatrix} \tag{4.36}$$

The three heavy-light mixings (components of the three neutrinos along the sterile flavor space) are

$$s_{\nu_k} = \frac{m_k}{\sqrt{m_{N_1} m_{N_2}}} = \frac{m_k}{M'}. \tag{4.37}$$

The mass parameter μ takes into account the mass splitting of the two heavy Majorana neutrinos. This mass parameter is the only one that breaks the leptonic number.

For $\mu = 0$ the two heavy neutrinos $\chi_{4,5}$ form a heavy Dirac field of mass M'. Also, if we deform the texture of the matrix (4.33), by a small input λ at position $\mathcal{M}_{44}$, we are able to give a mass to one of the light neutrinos. This mass will be of the order of $m_\nu \approx \lambda (m/M)^2$, very similar to what happens in the inverse seesaw model. However, the λ term does not change the heavy-light mixings (it has no effect on flavor physics). In this model, the effects on flavor physics are due to heavy neutrinos (see appendix C for a sketch of the phenomenological consequences in charged lepton flavor violating processes). An analogous reasoning applies to possible Dirac mass terms in the fourth row/column of the matrix (4.33). This mass pattern is approximate, including only the dominant mass terms, with all others being sub-eV. Vanishing entries are not symmetry protected, and loop corrections will contribute to them [41]. The actual generation of the masses and mixing of the three light active known particles would require the addition of extra singlets (in the minimally extended SM or type I seesaw models) or the small shifts mentioned above (in inverse seesaw models).

The key is that these deformations will not change significantly the heavy-light mixings, and therefore will be negligible in the charged lepton flavor violating observables analyzed in refs. [20, 21]. Thus, it will be feasible to predict $\ell \to \ell'\gamma, \ell \to 3\ell, \ell \to \ell'$ conversions in nuclei, $H/Z \to \ell\ell'$ decays, ... in a generic and simple model where this connection is possible (which is not the case in the general framework of e.g. ref. [19]). This is our main motivation to consider this simplified framework in our analysis, instead of taking a full-fledged model for lepton mixing and neutrino mass generation. Alternatively, we also preferred this representative model to the effective field theory treatment (see appendix B), where the number of operators contributing to charged lepton flavor violation [42] rises to 19 (without accounting for flavor structure, see [43]), complicating substantially the interpretation of the underlying physics lacking measurements of lepton flavor violation in the charged sector.

The charged current involving neutrinos at tree level, introduced by the U matrix, is described by

$$\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} W^-_\mu \sum_{i=1}^{3} \sum_{j=1}^{5} B_{ij} \bar{\ell}_i \gamma^\mu P_L \chi_j + h.c., \tag{4.38}$$

where B is a matrix of dimension 3×5 that defines the mixtures in the lepton sector, and it is in the diagonal charged lepton mass basis $\delta_{ik} = U^\ell_{ik}$

$$B_{ij} = \sum_{k=1}^{3} \delta_{ik} U_{kj}. \tag{4.39}$$

As a consistency test, we remark that we recover the first of eqs. (2.26) replacing $B_{ij} \to U^\nu_{ij}$, and restricting i, j from five to three, as it must be.

The elements of the B matrix can be expressed in terms of the squared mass ratio parameter $r = m^2_{N_2}/m^2_{N_1}$ and the heavy-light mixtures, as follows

$$B_{kN_1} = -i \frac{r^{\frac{1}{4}}}{\sqrt{1 + r^{\frac{1}{2}}}} s_{\nu_k}, \quad B_{kN_2} = \frac{1}{\sqrt{1 + r^{\frac{1}{2}}}} s_{\nu_k}, \quad \text{For: } k = 1, 2, 3. \tag{4.40}$$

In a realistic low-scale seesaw scenario, in addition to the restrictions on s_{ν_k}, we must be consistent with perturbative unitarity. Such a condition in this model is given by

$$m_{N_1} r^{\frac{1}{4}} < \frac{\sqrt{2\pi} v}{\max\{s_{\nu_i}\}}. \tag{4.41}$$

We consider, on the heavy-light mixtures, indirect limits (at 90% C.L.) that come from the global fits to electroweak precision observables [44]

$$s_{\nu_e} < 0.050, \quad s_{\nu_\mu} < 0.021, \quad s_{\nu_\tau} < 0.075. \tag{4.42}$$

Taking the maximal values of s_{ν_i}, then eq.(4.41) implies that $m_{N_1} r^{\frac{1}{4}} < 8.2$ TeV.

We use eq.(4.39) and we can identify the S matrix of the eq.(4.23), which will be of dimension 3×2 and given by

$$S = \begin{pmatrix} B_{14} & B_{15} \\ B_{24} & B_{25} \\ B_{34} & B_{35} \end{pmatrix}. \tag{4.43}$$

But we need $SS^\dagger$, so now we use eq.(4.40) and get the elements of that matrix, for example for the following elements we have

$$\begin{aligned} (SS^\dagger)_{11} &= |B_{14}|^2 + |B_{15}|^2 = \frac{r^{1/2} s^2_{\nu_1}}{1 + r^{1/2}} + \frac{s^2_{\nu_1}}{1 + r^{1/2}} = s^2_{\nu_1}, \\ (SS^\dagger)_{23} &= B_{24} B^*_{34} + B_{25} B^*_{35} = \frac{r^{1/2} s_{\nu_2} s_{\nu_3}}{1 + r^{1/2}} + \frac{s_{\nu_2} s_{\nu_3}}{1 + r^{1/2}} = s_{\nu_2} s_{\nu_3}, \\ (SS^\dagger)_{31} &= B_{34} B^*_{14} + B_{35} B^*_{15} = \frac{r^{1/2} s_{\nu_1} s_{\nu_3}}{1 + r^{1/2}} + \frac{s_{\nu_1} s_{\nu_3}}{1 + r^{1/2}} = s_{\nu_1} s_{\nu_3}. \end{aligned} \tag{4.44}$$

We proceed similarly, for the other components, and $SS^\dagger$ turns out to (expectedly) be

$$SS^\dagger = \begin{pmatrix} s^2_{\nu_1} & s_{\nu_1} s_{\nu_2} & s_{\nu_1} s_{\nu_3} \\ s_{\nu_1} s_{\nu_2} & s^2_{\nu_2} & s_{\nu_2} s_{\nu_3} \\ s_{\nu_1} s_{\nu_3} & s_{\nu_2} s_{\nu_3} & s^2_{\nu_3} \end{pmatrix}. \tag{4.45}$$

We make use of the unitarity relation given in eq.(4.24), then $SS^\dagger = \mathbb{1} - NN^\dagger$, so that

$$SS^\dagger = \begin{pmatrix} 1-\alpha_{11}^2 & -\alpha_{11}\alpha_{21}^* & -\alpha_{11}\alpha_{31}^* \\ -\alpha_{11}\alpha_{21} & 1-\alpha_{22}^2-|\alpha_{21}|^2 & -\alpha_{22}\alpha_{32}^*-\alpha_{21}\alpha_{31}^* \\ -\alpha_{11}\alpha_{31} & -\alpha_{22}\alpha_{32}-\alpha_{31}\alpha_{21}^* & 1-\alpha_{33}^2-|\alpha_{31}|^2-|\alpha_{32}|^2 \end{pmatrix}. \tag{4.46}$$

Through this equality we can identify the α_{ij} coefficients for certain values of the s_{ν_k} mixtures, as we do in the next chapter.

Chapter 5

Non-unitary constraints on the α_{ij}

The bounds on lepton non-unitarity for the case in which the three light neutrinos mix with two much heavier ones is presented in [19] (see appendix D for more details). For the normal ordering (NO) and inverse ordering (IO) at 95% confidence level the limits are

$$\begin{aligned}
&\text{NO:}\quad |\mathbb{1}-\alpha| < \begin{pmatrix} 9.4\cdot 10^{-6} & 0 & 0 \\ 2.4\cdot 10^{-5} & 1.3\cdot 10^{-4} & 0 \\ 4.4\cdot 10^{-5} & 2.6\cdot 10^{-4} & 2.1\cdot 10^{-4} \end{pmatrix}, \\
&\text{IO:}\quad |\mathbb{1}-\alpha| < \begin{pmatrix} 5.5\cdot 10^{-4} & 0 & 0 \\ 2.6\cdot 10^{-5} & 3.2\cdot 10^{-5} & 0 \\ 2.8\cdot 10^{-4} & 7.0\cdot 10^{-5} & 4.5\cdot 10^{-5} \end{pmatrix}.
\end{aligned} \tag{5.1}$$

As shown in eq.(4.46), there is a direct way to relate the alpha coefficients to the admixtures of the active neutrinos and the two heavy neutrinos. So that if we equal the first component $(SS^\dagger)_{11}$ and $(\mathbb{1}-NN^\dagger)_{11}$ we get

$$s_{\nu_1}^2 = 1-\alpha_{11}^2 \Longrightarrow \alpha_{11} = \sqrt{1-s_{\nu_1}^2}. \tag{5.2}$$

In order to compare with the quoted limits on the alpha coefficients, we are interested in the modulus of these coefficients. After obtaining coefficient α_{11}, it is easy to obtain the other ones

$$\begin{aligned}
&\alpha_{21} = -\frac{s_{\nu_1}s_{\nu_2}}{\alpha_{11}}, \qquad \alpha_{22} = \sqrt{1-\alpha_{21}^2-s_{\nu_2}^2}, \qquad \alpha_{31} = -\frac{s_{\nu_1}s_{\nu_3}}{\alpha_{11}}, \\
&\alpha_{32} = -\frac{1}{\alpha_{22}}\left(s_{\nu_2}s_{\nu_3}+\alpha_{21}\alpha_{31}\right), \qquad \alpha_{33} = \sqrt{1-\alpha_{31}^2-\alpha_{32}^2-s_{\nu_3}^2}.
\end{aligned} \tag{5.3}$$

It is also useful to write the heavy-light mixings in terms of the α_{ij}, as follows:

$$\begin{aligned}
&s_{\nu_1} = \sqrt{1-\alpha_{11}^2}, \quad s_{\nu_2} = -\frac{\alpha_{11}\alpha_{21}}{\sqrt{1-\alpha_{11}^2}}, \\
&s_{\nu_3} = \sqrt{1-\alpha_{31}^2-\alpha_{32}^2-\alpha_{33}^2}.
\end{aligned} \tag{5.4}$$

We will determine the coefficients that parameterize the non-unitarity deviations in two ways in the following (the first being a simplified, though informative, case).

First, we evaluate the expressions of eq.(5.3) at the limiting values of s_{ν_k} given in ref. [20]. We will see below that there are correlations in the limits for the $s_{\nu i}$ coefficients, so these bounds are only illustrative. We form the expression $|\mathbb{1}-\alpha|$ and obtain

$$|\mathbb{1}-\alpha| < \begin{pmatrix} 1.25\cdot 10^{-3} & 0 & 0 \\ 1.05\cdot 10^{-3} & 2.21\cdot 10^{-4} & 0 \\ 3.75\cdot 10^{-3} & 1.58\cdot 10^{-3} & 2.82\cdot 10^{-3} \end{pmatrix}. \tag{5.5}$$

With the exception of the 22 entry for the NO, we see that these bounds are less restrictive than those given in eqs. (5.1). This suggests than non-unitarity bounds can be more restrictive on the $s_{\nu i}$ mixing than the charged lepton flavor violating processes examined in ref. [20].

This is in agreement with a quick estimation using eq. (5.2), from which one gets (using the bound in eq. (4.42))

$$1-\alpha_{11} \sim s_{\nu_1}^2/2 \lesssim 1.25\cdot 10^{-3}\,. \tag{5.6}$$

Using eqs. (4.31), in terms of the η parameters, we have

$$\begin{aligned} &\eta_{11}=\alpha_{11}=9.987\cdot 10^{-1}, \quad \eta_{21}=\frac{\alpha_{21}}{2}=-5.257\cdot 10^{-4}, \quad \eta_{22}=\alpha_{22}=9.998\cdot 10^{-1} \\ &\eta_{31}=\frac{\alpha_{31}}{2}=-1.877\cdot 10^{-3}, \quad \eta_{32}=\frac{\alpha_{21}}{2}=-7.896\cdot 10^{-4}, \quad \eta_{33}=\alpha_{33}=9.972\cdot 10^{-1} \end{aligned} \tag{5.7}$$

To verify the previous estimates, in the second procedure, a set of random values is generated for the s_{ν_k}, within a region bounded by possible processes where the leptonic flavor violation occurs. The contour of these processes is taken from [20] and is shown in Figure 5.1.

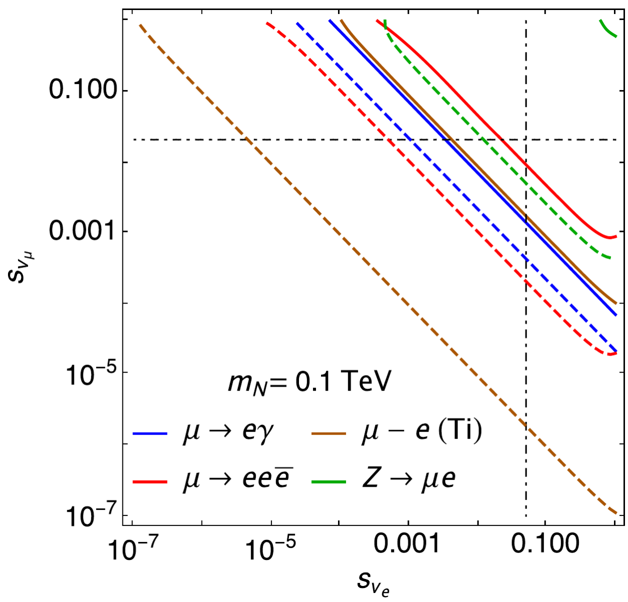


Figure 5.1: Contour in the $s_{\nu_e} - s_{\nu_\mu}$ plane where the different restrictions coming from charged lepton flavor violating processes are displayed (solid lines for current limits and dashed ones for future projections) in the case $m_{N_1} = m_{N_2} = 0.1$ TeV (Dirac field). The actual indirect limits are shown by the black dot-dashed lines. Plot taken from ref. [20].

The region which contains the possible values of s_{ν_e} and s_{ν_μ}, is formed by the indirect limits of $s_{\nu_e} < 0.050$ and $s_{\nu_\mu} < 0.021$ (black dot-dashed lines) and the process constraint

$\mu \to e\gamma$ (blue line). The s_{ν_τ} values are generated in the interval $0 \leq s_{\nu_\tau} < 0.075$.

The randomly generated points in the region described above are shown in Figure 5.2.

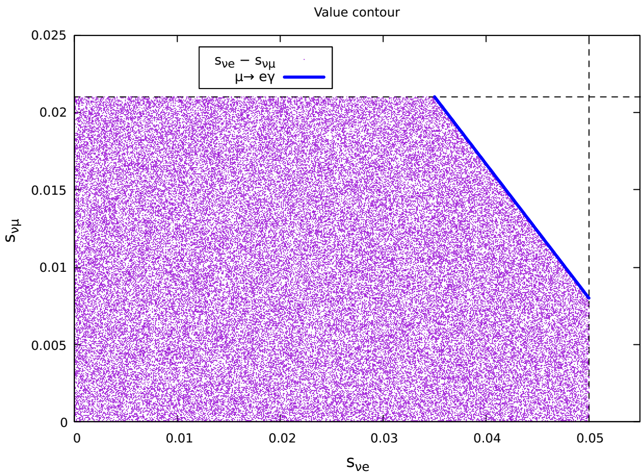


Figure 5.2: Points generated in the region bounded by the indirect limits of s_{ν_1} and s_{ν_2} (black dot-dashed lines) and the $\mu \to e\gamma$ process that were employed in our numerical analysis.

From this set of generated data we obtain the average values of the s_{ν_k} as well as their standard deviations (below them the confidence interval at 2σ is indicated), which are given by

$$\begin{aligned} \bar{s}_{\nu_1} =& 0.02 \pm 0.01, & \bar{s}_{\nu_2} &= 0.010 \pm 0.006, & \bar{s}_{\nu_3} &= 0.04 \pm 0.02. \\ & [0.00, 0.04] & & [-0.002, 0.022] & & [0, 0.08]\,. \end{aligned} \tag{5.8}$$

In terms of the η parameters the corresponding mean values are

$$\begin{aligned} \eta_{11} &= 1.0 \cdot 10^{-1}, & \eta_{21} &= -1.0 \cdot 10^{-4}, & \eta_{22} &= 1.0 \cdot 10^{-1} \\ \eta_{31} &= -4.0 \cdot 10^{-4}, & \eta_{32} &= -2.0 \cdot 10^{-4}, & \eta_{33} &= 1.0 \cdot 10^{-1} \end{aligned} \tag{5.9}$$

Subsequently, we use the formulae of eq.(5.3) and construct the expression $|\mathbb{1} - \alpha|$ and obtain (at 90% confidence level)

$$|\mathbb{1} - \alpha| < \begin{pmatrix} 2.0 \cdot 10^{-4} & 0 & 0 \\ 2.0 \cdot 10^{-4} & 5.0 \cdot 10^{-5} & 0 \\ 8.0 \cdot 10^{-4} & 4.0 \cdot 10^{-4} & 8.0 \cdot 10^{-4} \end{pmatrix}. \tag{5.10}$$

These limits are -as expected- more constraining than those in eq. (5.5) by factors in the interval $[3.5, 6.3]$, which is reasonable.

In this general case all constraints -but again the 22 entry in the NO case- are less restrictive than those in eq. (5.1), which confirms that non-unitarity bounds set more stringent limits on the s_{ν_i} coefficients than the upper bounds on charged lepton flavor violation processes examined in ref. [20]. Therefore, the predictions in this reference for $\mu \to e\gamma$, $\mu \to 3e$, etc. (and also of ref. [21] for charged lepton flavor violating Higgs decays, within the same model, which are any how much smaller than current limits on them) shall be reviewed taking this into account, which we plan to do in the future.

A final interesting observation is that the improvement of the limits in eq. (5.1) over those in eq. (5.10) does depend on flavor: indeed, for the NO case, the difference is maximal for the electron-related coefficients; while for the IO this happens mostly in the 33 entry, followed by the 32 and 21. In the first instance, it is order 20 for 11 and 31 ($\sim$ 8 for 21); and in the second one it is $\sim$ 20 for 33, and $[6, 8]$ for 21 and 32. It will be relevant to investigate how this translates into the different predictions for the diverse charged lepton flavor violating processes considered in [20] [1].

[1] It does not appear possible to exploit this big difference in the 33 entry for the IO case, which would need much better measurements of lepton flavor conserving processes with tau (anti)neutrinos.

Chapter 6

Conclusions and prospects

One of the main unanswered questions by the extremely successful SM is how the baryon asymmetry of the universe was generated. There are certainly quite some theoretically attractive possible explanations, one of which produces baryogenesis from an initial leptogenesis. In this paradigm, the required CP violation is produced at high-energy scales, in the decays of heavy right-handed Majorana neutrinos. These naturally link to a plausible explanation of the tiny (compared to the other known massive elementary particles) neutrino masses, through the so-called seesaw mechanism (this, specifically is of Type I). Unfortunately, this beautiful theory generally has little prospects of experimental verification in current or forthcoming experiments, which has motivated the introduction of the low-scale seesaws (in which Grand Unification is much more challenging than in the traditional ones). Among them, the inverse seesaw is the simplest possibility, which has attracted a lot of attention for its bright experimental prospects.

With these motivations in mind, in this Master Thesis we have studied if the constrictions from non-unitarity (with respect to the three light neutrinos scenario) are more restrictive than those coming from charged lepton flavor violation in the context of a simple model [20] which captures the main features of the Type I as well as of the inverse seesaw mechanisms, in two given limits. This framework features two sterile Majorana neutrinos, which fall into a single Dirac field in the degenerate case (therefore being able to consider both possible neutrino natures within a common setup). The interest of this setting resides in its flexibility to accommodate these two extreme cases of relevance (varying the associated new physics scale, which sets the mass of the heavy neutrinos). Although we are mostly interested in the low-scale mechanism, where the heavy neutrinos mass is $\mathcal{O}(\text{TeV})$, nothing prevents to increase them and end up in the canonical seesaw, with $\mathcal{O}(10^{12}\ \text{TeV})$ right-handed neutrino masses, even though the phenomenological interest would be limited (at most) to neutrinoless double beta decay signatures in this high-energy realization.

Ref. [20] shows that the most stringent constraints on the model heavy-light mixings come currently from $\mu \to e\gamma$ decays [1], where -for heavy neutrino masses in the TeV, or higher- the predicted branching ratio does not depend on these masses, in such a way that the upper limit on the non-observation of this process sets $s_{\nu_e}^2 s_{\nu_\mu}^2 < 5.1 \cdot 10^{-10}$ [20]. On the other hand, ref. [19] -within Type-I seesaw (related-)models- updated and improved global fit analysis of current flavour and electroweak precision observables to derive bounds on unitarity deviations of the leptonic mixing matrix and on the mixing of heavy neutrinos

[1] In the near future, these will be superseded by those imposed by muon-to-electron conversion in nuclei [20].

with the active ones, which we used for the two heavy neutrino case in both mass ordering scenarios. In any case, the largest deviation from unitarity is of the order of $[1, 5] \cdot 10^{-4}$, at 90% confidence level (depending on the neutrino mass ordering) in a parameter which is $\mathcal{O}(s_{\nu_i} s_{\nu_j})$.

This motivated the research undertaken in this thesis, where we wanted to see if the restrictions from non-unitarity (that start generally at tree level) could be more powerful than those from charged lepton flavor violation (which is a loop process in the SM and most of their viable extensions, including the class of models considered here). We have verified that this is indeed the case, which calls for a reanalysis of the results in refs. [20] and [21] accounting for our improved constraints on the heavy-light mixings. It is not immediate if this could be a general feature extending to more general mechanisms generating neutrino mixing by means of heavy neutrinos than the model specifically considered in this work (which, we recall, encompasses the Type I and inverse seesaw mechanisms as extreme relevant cases). Studying this question is an obvious interesting extension of our results.

Appendix A

Neutrino oscillations in vacuum

In this appendix we make a short recap of the main features of neutrino oscillations in vacuum, which have been the main guidance in our current knowledge of massive neutrino physics (see the PDG review and references therein [24]).

Neutrino oscillation is a completely complex phenomenon and its proper treatment requires quantum field theory calculations. The deduction followed here, which uses quantum mechanics, has a computational validity that depends on the energy regime in which the experiments are being performed.

The neutrino oscillation phenomenon relies on the fact that we can measure events in which some charged lepton or antilepton is created along with the emission of a neutrino of a certain flavor, and these neutrinos are detected far away from the source in a different flavor. In other words, the $|\nu_k\rangle$, $k = 1, 2, 3$, mass eigenstates (neutrino propagation in space-time) with well-defined energy do not coincide with the $|\nu_\alpha\rangle$, $\alpha = e, \mu, \tau$, interaction eigenstates (neutrino detection).

A neutrino of ν_α flavor, created in a charged weak current interaction, will be a superposition of the 3 eigenstates of mass ν_k

$$|\nu_\alpha\rangle = U_{\alpha k} |\nu_k\rangle \,, \tag{A.1}$$

where $U_{\alpha k}$ are the elements of the unitary leptonic mixing matrix, known as Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS) [28, 29]. For vacuum propagation, we can take the neutrino as a free particle. Given the initial state $|\nu_\alpha\rangle$, its evolution will be given by the evolution operator e^{-iHt}. Since the Hamiltonian is free, the eigenstates of mass are also eigenstates of energy

$$\begin{aligned} t = 0: \quad & |\nu_\alpha(0)\rangle = |\nu_\alpha\rangle \,, \\ t \neq 0: \quad & |\nu_\alpha(t)\rangle = e^{-iHt} U_{\alpha k} |\nu_k\rangle = e^{-iE_k t} U_{\alpha k} |\nu_k\rangle \,. \end{aligned} \tag{A.2}$$

We assume that all the components of the eigenstates of mass occur with the same momentum $|\boldsymbol{p}| \approx E$, so the energy E_k is given by the relativistic dispersion relation

$$E_k = \sqrt{\boldsymbol{p}^2 + m_k^2}. \tag{A.3}$$

In practice neutrinos are relativistic, $E_k \gg m_k$, so we can approximate the energy E_k as

$$E_k \approx |\boldsymbol{p}| + \frac{m_k^2}{2|\boldsymbol{p}|} = E + \frac{m_k^2}{2E}. \tag{A.4}$$

Thus, after a distance $L = t$, the neutrino ν_α can oscillate to a flavor ν_β with a probability given by

$$\begin{aligned}\mathcal{P}\left(\nu_\alpha \to \nu_\beta; L\right) = \left|\langle \nu_\beta | \nu_\alpha; L\rangle\right|^2 = \left|\sum_k U^*_{\beta k} U_{\alpha k} e^{-iE_k L}\right|^2 \\ = \sum_{kj} U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j} e^{i2\phi_{kj}}, \quad \phi_{kj} = -\frac{\Delta m^2_{kj} L}{4E},\end{aligned} \tag{A.5}$$

where $\Delta m^2_{kj} = m^2_k - m^2_j$. We make use of the relation $e^{i2\phi} = 1 - 2\sin^2\phi + i\sin 2\phi$, then

$$\mathcal{P}\left(\nu_\alpha \to \nu_\beta; L\right) = \sum_{kj} J^{\alpha\beta}_{kj}\left(1 - 2\sin^2\phi_{kj} + i\sin 2\phi_{kj}\right), \quad J^{\alpha\beta}_{kj} = U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}. \tag{A.6}$$

We use the unitarity relation of the PMNS, $\delta_{\alpha\beta} = U^*_{\alpha k} U_{\beta k}$, and we obtain

$$\mathcal{P}\left(\nu_\alpha \to \nu_\beta; L\right) = \delta_{\alpha\beta} - 4\sum_{k>j} Re\left(J^{\alpha\beta}_{kj}\right)\sin^2\phi_{kj} - 2\sum_{k>j} Im\left(J^{\alpha\beta}_{kj}\right)\sin 2\phi_{kj}. \tag{A.7}$$

An expression for antineutrinos is obtained if we make $U \to U^*$, which would change the sign in the term proportional to $Im\left(J^{\alpha\beta}_{kj}\right)$. This oscillation probability satisfies properties such as

- CPT invariance, $\mathcal{P}\left(\nu_\alpha \to \nu_\beta\right) = \mathcal{P}\left(\bar{\nu}_\beta \to \bar{\nu}_\alpha\right)$.
- Conservation of probability,

$$\sum_\beta \mathcal{P}\left(\nu_\alpha \to \nu_\beta\right) = \sum_\beta \mathcal{P}\left(\bar{\nu}_\alpha \to \bar{\nu}_\beta\right) = 1.$$

By means of the oscillation probability eqs.(A.7), let us see the survival probability of the electron neutrino

$$\begin{aligned}\mathcal{P}\left(\nu_e \to \nu_e\right) = 1 - 4\left|U_{e1}\right|^2\left|U_{e2}\right|^2\sin^2\phi_{21} \\ - 4\left|U_{e1}\right|^2\left|U_{e3}\right|^2\sin^2\phi_{31} - 4\left|U_{e2}\right|^2\left|U_{e3}\right|^2\sin^2\phi_{32}.\end{aligned} \tag{A.8}$$

Although this survival probability depends on the three mass differences, Δm^2_{21}, Δm^2_{31} and Δm^2_{32}, only two of them are independent. So the difference Δm^2_{32} can be expressed in terms of the other two

$$\Delta m^2_{31} = \Delta m^2_{32} + \Delta m^2_{21}. \tag{A.9}$$

The probability of oscillation depends on the quadratic difference of the masses, so for oscillation to occur, at least one mass eigenvalue must be non-zero. Experimental evidence shows such oscillations, so neutrinos must have mass. In experiments, information is obtained about mass differences, not about the absolute masses of the states.

Current neutrino oscillation experiments provide difference of the square of the masses of order

$$\Delta m^2_{21} \sim 8 \cdot 10^{-5}\text{eV}^2, \quad |\Delta m^2_{32}| \sim 2 \cdot 10^{-3}\text{eV}^2. \tag{A.10}$$

Since we do not know the absolute scale of the neutrino mass, there are two possible hierarchies for the neutrino mass. The normal ordering in which we have $m_1 < m_2 < m_3$, and the inverse ordering where $m_3 < m_1 < m_2$. Current experiments are not sensitive enough to discern which ordering is actually present.

In the case of two-flavor neutrino oscillation, U is a real orthogonal 2×2 matrix with parameter θ which encodes the mixing. The oscillation and survival probabilities are

$$\begin{aligned} \mathcal{P}\left(\nu_\alpha \to \nu_\beta\right) &= \sin^2\left(2\theta\right)\sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right), \\ \mathcal{P}\left(\nu_\alpha \to \nu_\alpha\right) &= 1 - \sin^2\left(2\theta\right)\sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right). \end{aligned} \tag{A.11}$$

The standard parameterization of the PMNS matrix is given in terms of three angles θ_{12}, θ_{13}, θ_{23}, one CP-violation phase δ and two additional phases α_{21}, α_{31} if the neutrinos are of Majorana type. Explicitely,

$$\begin{aligned} U &= \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{21}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{13} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}. \end{aligned} \tag{A.12}$$

We must say that neutrino oscillation experiments are not sensitive to Majorana phases and therefore unable to discern whether neutrinos are Dirac or Majorana particles. This requires an experiment to check the conservation of the leptonic number which is violated by the Majorana terms (like, par excellence, neutrinoless double beta decay).

Current knowledge of the two independent mass splittings, the three mixing angles and the Dirac CP violating phase is summarized in the 'Neutrino Masses, Mixing, and Oscillations' PDG review [24] (see also refs. [45–47]).

Appendix B

Weinberg operator

If we allow for non-renormalizable terms in the SM Lagrangian, the mass of neutrinos could be generated from a five-dimensional operator. The only five-dimensional operator that is invariant under the SM symmetry group is the so-called Weinberg operator [36]. This operator can be written in several ways, all of which are equivalent, one of them is as follows

$$\mathcal{L}_{5D} = \frac{c_{jk}}{\Lambda}\left(\overline{L}^c_j \tilde{H}\right)\left(L^T_k \tilde{H}\right) + h.c., \tag{B.1}$$

where Λ is the energy scale at which the operator is generated [1]. This Weinberg operator breaks the leptonic number by two units. In the unitary gauge, the Weinberg operator is given by

$$\mathcal{L}_{5D} = \frac{c_{jk}}{2\Lambda}\bar{\nu}^c_{L,j}\nu_{L,k}\left(h+v\right)^2 + h.c, \tag{B.2}$$

and after the SEWSB, yields a mass for neutrinos

$$\mathcal{L}^{mass}_{5D} = \frac{m^{jk}_\nu}{2}\bar{\nu}^c_{L,j}\nu_{L.k} + h.c, \quad m^{jk}_\nu = \frac{v^2}{\Lambda}c_{jk}, \tag{B.3}$$

which corresponds to a Majorana mass term. Note that unlike the rest of the fermions, where the dependence of their mass is linear with v (including a possible neutrino Dirac mass coming from the Higgs mechanism), the mass of neutrinos m_ν is quadratic in v.

At the tree level there are only three ways to build the Weinberg operator, which are

- L_j and H combine to form a fermionic singlet (type I seesaw), the intermediate particle is also a singlet.
- L_j and H combine to form a scalar triplet of $SU(2)_L$ (type II seesaw), the intermediate particle is a scalar that must belong to $SU(2)_L$.
- L_j and H combine to form a fermionic triplet (type III seesaw), the intermediate particle is a fermionic triplet as well.

Although the same operator is generated at the tree level, the different realizations imply different mechanisms in the high energy regime.

An interesting and valuable way to understand the significance of the Weinberg operator comes from the point of view of quantum effective field theories [48–51], that can be applied to the Standard Model itself [36,52–54]. From this point of view, one complements

[1]From the perspective of the Wilson renormalization group, we can interpret quantum field theories as manifestations of high-energy physical phenomena below a Λ cutoff scale.

the SM Lagrangian with a tower of operators of increasing mass dimensions, which are built using only the SM fields and gauge symmetries. Every additional operator dimension is compensated by a corresponding power of the new physics scale, $\Lambda >> v_{EW} \sim 246$ GeV. In this construction, the Weinberg operator is the single operator which appears at dimension five. If there is no symmetry reason that suppresses it compared to the dimension six operators, the Weinberg operator effects at low energies will be enhanced by a relative factor of E/Λ (where E is the characteristic energy scale of the process under consideration) with respect to the Fermi-type operators (and others) that appear at dimension six. Thus, it is natural that the first unequivocal manifestation of beyond the SM physics comes from the lowest operator dimension, eq. (B.1). This standpoint singles out Majorana neutrino masses as the most sensitive phenomenon to the ultraviolet completion of the SM (assuming Λ is a common scale for all new physics, which does not need to be true).

Interestingly, seesaw mechanisms contribute at dimension six only through non-unitary effects in lepton mixing [55]. In high-scale seesaws both dimension five and six operators are expected to be suppressed enough to likely prevent any associated testability. On the contrary, its low-scale variants suppress the Weinberg operator but not the dimension-six one [2], so deviations from unitarity could be measurable. This reasoning further motivates the research tackled in this thesis.

[2]Precisely, η (4.26) is half of the coefficient of this dimension six operator [55].

Appendix C

Restrictions from charged lepton flavor violating processes on the heavy-light mixings of the model

In this appendix we summarize the main constraints that are obtained [20] on the model introduced in section 4.4 from the current upper limits on charged lepton flavor violating processes.

In addition to the charged current involving heavy neutrinos, eq. (4.38) [1], these also appear in neutral current interactions, that read [20]

$$\mathcal{L}_Z = -\frac{g}{4c_W} Z_\mu \sum_{i,j=1}^{5} \bar{\chi}_i \gamma^\mu (C_{ij} P_L - C^*_{ij} P_R) \chi_j \,, \tag{C.2}$$

where we note that the couplings involve different flavors with both chiralities. In eq. (C.2) C is a 5×5 matrix. Explicitely, it is

$$C_{ij} = \sum_{k=1}^{3} (U^\nu_{ki})^* U^\nu_{kj} \,, \tag{C.3}$$

whose elements involving heavy neutrinos can again be written in terms of the heavy-light mixing and the mass ratio between the two heavy states (see also refs. [56, 57]):

$$C_{N_1 N_1} = \frac{\sqrt{r}}{1+\sqrt{r}} \sum_{k=1}^{3} s^2_{\nu_k} \,, \; C_{N_2 N_2} = \frac{1}{1+\sqrt{r}} \sum_{k=1}^{3} s^2_{\nu_k} \,, \; C_{N_1 N_2} = -C_{N_2 N_1} = \frac{i r^{1/4}}{1+\sqrt{r}} \sum_{k=1}^{3} s^2_{\nu_k} \,. \tag{C.4}$$

The model is renormalizable thanks to the following identities fulfilled by the B (4.39)

[1] We omitted there the piece corresponding to the would-be Goldstone bosons ($G^\pm$), which is

$$\mathcal{L}_{G^\pm} = -\frac{g}{\sqrt{2} M_W} G^- \sum_{i=1}^{3} \sum_{j=1}^{5} B_{ij} \bar{\ell}_i \left(m_{\ell_i} P_L - m_{\chi_j} P_R \right) \chi_j + h.c. \tag{C.1}$$

As expected, we recover the last two eqs. (2.26) in the SM limit, with: $i,j = 1,2,3$, $B_{ij} \to U^\nu_{ij}$, $C_{ij} \to \delta_{ij}$ and $C^*_{ij} = 0$.

and C matrices (C.3)

$$\sum_{k=1}^{5} B_{ik}B^*_{jk} = \delta_{ij}\,, \;\sum_{k=1}^{3} B^*_{ki}B_{kj} = \sum_{k=1}^{5} C_{ik}C^*_{jk} = C_{ij}\,, \;\sum_{k=1}^{5} B_{ik}C_{kj} = B_{ij}\,,$$
$$\sum_{k=1}^{5} m_{\chi_k}C_{ik}C_{jk} = \sum_{k=1}^{5} m_{\chi_k}B_{ik}C^*_{kj} = \sum_{k=1}^{5} m_{\chi_k}B_{ik}B_{jk} = 0\,. \tag{C.5}$$

With these interactions, one has effective lepton flavor violating contributions involving a gauge boson and a lepton pair, as well as those amid four-fermions, which are sketched in the following figure.

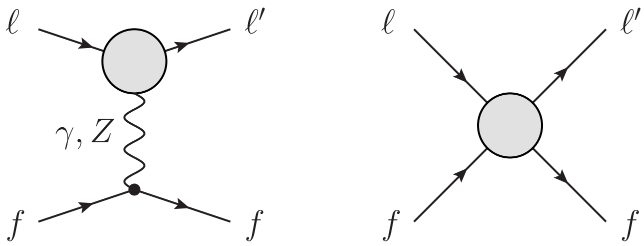


Figure C.1: Generic penguin and box diagrams contributing to $\ell \to 3\ell'$ decays (for all flavor combinations in the final-state) and $\ell \to \ell'$ conversion in nuclei. Figure taken from ref. [20].

We do not detail the different contributions to the effective boxes (see fig. 3 in ref. [20]) [2]. The diverse contributions to the $V\ell\ell'$ vertices are represented in the following diagrams (where would-be Goldstone bosons, appearing in the Feynman-'t Hooft gauge, are not displayed).

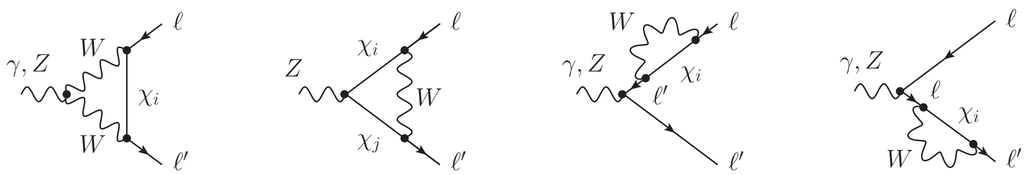


Figure C.2: One-loop diagrams contributing to the $V\ell\ell'$ effective vertex in our model. Figure taken from ref. [20].

For simplicity, we will focus now on the $\mu \to e\gamma$ case, that will suffice here, for our illustrative purposes. The relevant effective vertex can be written ($q^2 = 0$ for the real photon in this process)

$$\Gamma^\gamma_\mu(q^2) = e\left[F^\gamma_L(q^2)\gamma_\mu P_L + 2iF^\gamma_M(q^2)P_R\sigma_{\mu\nu}q^\nu\right]\,, \tag{C.6}$$

in terms of the two form factors $F^\gamma_{L,M}(q^2)$, although only the dipole one, F^γ_M contributes for real photons, due to gauge invariance. The corresponding amplitude is simply

$$\mathcal{M}(\mu \to e\gamma) = iem_\mu A_{2R}\bar{u}(p_e)P_R\sigma^{\mu\nu}q_\nu u(p_\mu)\epsilon^\gamma_\mu(q)\,, \tag{C.7}$$

[2]These include lepton number violating processes.

where we introduced $F_M^\gamma(0) \equiv \frac{m_\mu}{2} A_{2R}$, the electron mass was neglected and $\epsilon_\mu^\gamma(q)$ is the photon (transverse) polarization. Thus, the partial decay width reads

$$\Gamma(\mu \to e\gamma) = \alpha m_\mu^3 |F_M^\gamma(0)|^2 \,. \tag{C.8}$$

The required form factor is

$$F_M^\gamma(0) = \frac{\alpha_W}{8\pi M_W^2} \frac{m_\mu}{2} \sum_{i=1}^{5} B_{\mu i}^* B_{ei} f_M^\gamma(x_i) \,, \tag{C.9}$$

where $\alpha_W s_W^2 = \alpha$, $x_i M_W^2 \equiv m_{\chi i}^2$ and

$$f_M^\gamma(x) = \frac{3x^3 \ln x}{2(x-1)^4} - \frac{2x^3 + 5x^2 - x}{4(x-1)^3} + \frac{5}{6} \,. \tag{C.10}$$

From the previous equation it is straightforward to see that in the limit where $x_i \to \infty$ (heavy neutrino masses much larger than the electroweak scale), $f_M^\gamma(x \to \infty) \to \frac{1}{3}$, which explains why the corresponding constraint on $s_{\nu_e} s_{\nu_\mu}$ is a constant for large enough ($\gtrsim 1$ TeV) sterile neutrino masses. Specifically, this restriction becomes

$$BR(\mu \to e\gamma) \sim \frac{3\alpha}{8\pi} s_{\nu_e}^2 s_{\nu_\mu}^2 \,, \tag{C.11}$$

resulting in

$$s_{\nu_e}^2 s_{\nu_\mu}^2 < 5.1 \times 10^{-10} \,, \tag{C.12}$$

at 95% confidence level, which is the main constraint used in fig. 5.1, beyond the individual limits $s_{\nu_e} < 0.050$ and $s_{\nu_\mu} < 0.021$ (also $s_{\nu_\tau} < 0.075$) used in ref. [20], taken from the global fit to electroweak precision observables and lepton flavor conserving processes of ref. [58], which was recently superseded by the analysis in ref. [19] (see appendix D).

Appendix D

Phenomenological constraints on lepton non-unitarity

In this appendix we will briefly review how ref. [19] used different neutrino data to constrain possible deviations from lepton unitarity, that we employed here directly from their analysis for two heavy sterile states (that is pioneering). This corresponds to the minimal possible setting, where the dimension five and six operators are fully correlated, since these one (and consequently $\alpha \sim \eta$) can be completely reconstructed from the former up to a global scale [59].

The following observables were considered in this global fit (see the full list of references in ref. [19]):

- The W-boson mass measurements from LEP, Tevatron, LHCb and ATLAS. The controversial CDF-2 measurement is discussed, but not included in the reference results, as it is disfavored in the consistency tests.

- The LHC and Tevatron effective weak-mixing angle determinations. Other lower-energy and less precise measurements are disregarded.

- Five lepton universality tests obtained from weak decays ratios: $R^{\pi}_{\mu e}$, $R^{\pi}_{\tau\mu}$, $R^{K}_{\mu e}$, $R^{\tau}_{\mu e}$ and $R^{\tau}_{\mu\tau}$. The first three divide the so-called $P_{\ell 2}, P \to \ell\nu_\ell$ $(P = \pi, K)$ decays, for the different lepton flavors indicated; whereas the last two are the ratios among the leptonic tau decays, and between the τ and μ decays into $e\bar{\nu}\nu$, respectively.

- Ten CKM unitarity tests in weak decays: V_{ud} from superallowed Fermi β decays, V_{us} from eight different inputs (with assorted dependence on the $\eta_{\ell\ell}$ parameters) and the ratio between the $K_{\ell 2}$ and $\pi_{\ell 2}$ decays, from which the $|V_{us}/V_{ud}|$ ratio is extracted.

- Charged lepton flavor violating observables. Currently, the most constraining ones are the radiative decays, three-body leptonic decays and $\mu \to e$ conversion in nuclei. All of them, for heavy enough neutrino masses, depend mostly on the modulus of the relevant non-diagonal η element.

Apart from the latter group of processes, all others are lepton flavor conserving. Relation between diagonal and non-diagonal processes in flavor space include the use of the Schwarz inequality (which in this particular case is saturated, i. e. $|\eta_{\alpha\beta}| = \sqrt{\eta_{\alpha\alpha}\eta_{\beta\beta}}$).

The SM inputs for the analysis are the PDG values of M_Z, G_F and $\alpha_{em}(0)$ [1], where it must be noted that G_F and G_μ are no longer the same in presence of non-unitary mixing. Explicitly, $G_F = G_\mu(1 + \eta_{ee} + \eta_{\mu\mu} + \mathcal{O}(\eta^2))$. The same dependence on the η parameters arises from the modification of the relations between M_W and M_Z as well as amid s_W^2 and $\alpha/(G_\mu M_Z^2)$ with respect to the SM case. This happens analogously for the invisible/full Z^0 decay width, the e^+e^- hadronic cross-section at the Z^0-pole and in the ratios between the different leptonic Z decay widths over their hadronic counterpart (R_ℓ). All previous precision observables (but the R_ℓ ratios) depend also on $\eta_{\tau\tau}$, in different ways for each. $R^\pi_{\mu e}$, $R^K_{\mu e}$ and $R^\tau_{\mu e}$ depend on $\eta_{\mu\mu} - \eta_{ee}$, while $R^\pi_{\tau\mu}$ and $R^\tau_{\mu\tau}$ are functions of $\eta_{\tau\tau} - \eta_{\mu\mu}$.

We quote in the following the $\eta_{\ell\ell}$ dependence of the different CKM inputs used in the analysis. It is on $\eta_{\mu\mu}$ in the standard V_{ud} determination, as well as in the V_{us} extractions from the ratio among one-meson tau decays, and from the different $(K_L/K_S/K^\pm) \to \pi e\nu$ processes. It is on η_{ee} in the $(K_L/K_S/K^\pm) \to \pi\mu\nu$ decays. The $|V_{us}/V_{ud}|$ ratio, which is obtained by dividing the K over the π $\ell 2$ decays, is independent of the η parameters, as they cancel our in the ratio, since both numerator and denominator involve the same final states. This ratio is still a useful η-independent constraint on the analysis. Finally, the one-Kaon tau decays are functions of $\eta_{ee} + \eta_{\mu\mu} - \eta_{\tau\tau}$, which is sensitive to a new direction in parameter space.

The main results of this global fit are summarized in eqs. (5.1), that we used in this thesis. Very interesting are the regions in the right-handed neutrino mixing flavor space shown in their figure 3 [19], which are consistent with neutrino oscillation data, fixing mixings to the best fit values, and floating phases and absolute neutrino mass. In this minimal case, a preference is seen for non-vanishing $\eta_{\mu\mu}$ (trying to alleviate the Cabibbo angle anomaly). A non-unitary best fit point is found for normal ordering, but not for the inverse one.

[1] Appropriate running is included, requiring m_t, M_H, $\alpha_S(M_Z)$ and $\Delta\alpha(M_Z)$ as well.

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