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Centro de Investigación y de Estudios Avanzados del IPN
Unidad Guadalajara

Observabilidad en los Sistemas de Eventos Discretos modelados con Redes de Petri Interpretadas

Tesis presentada por

Luis Isidro Aguirre Salas

Para obtener el grado de
Doctor en Ciencias

en la especialidad de
Ingeniería Eléctrica

Guadalajara, Jal., Agosto 2001.

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Maestro en Ciencias

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Observability in Discrete Event Systems modeled by Interpreted Petri Nets

Summary

The aim of this work is to study the observability and observer design problems in Discrete Event Systems (DES) modeled by Interpreted Petri Nets (IPN). More precisely, we investigate the following three problems: 1) determine under which conditions an IPN is observable, 2) given a non-measured IPN, compute a minimal cost sensor configuration such that the net is observable and, 3) provide an algorithm to design asymptotic observers.

A structural and sufficient condition for observability in IPN is provided based on the notion of input and output sequence invariants of an IPN. This condition is similar to the one stated by M. Wonham for linear continuous systems using a geometric approach. For live, conservative and cyclic IPN, it is shown that observability can be tested through the properties of event-detectability and marking-detectability. Event-detectability is a structural property so it can be tested in polynomial time. On the other hand, some sufficient conditions for marking-detectability are provided based on the concepts of marking conservative laws and the synchronic distance among the transitions of the net.

The notions of W-observability and observability with respect to a firing language are introduced. These kinds of observability are less restrictive than the notion of observability. W-observability is related to the possibility of computing the marking of a net even if it enters to execute an infinite behavior. On the other hand, observability with respect to a firing language, or L-observability, implies the possibility of computing the marking of a net whose behavior is known to be confined into a given sublanguage of the firing language. Some sufficient conditions are given for W-observability and L-observability in live, conservative and cyclic IPN. These conditions are based on the concepts of event-detectability and W-marking-detectability, in the first case, and L-event-detectability and L-marking-detectability, in the second case.

For live, conservative and cyclic IPN, a simple algorithm to choose a minimal cost sensor configuration is developed. This algorithm assumes that only state sensors are available and that a sensor can be assigned to several places of the net.

Finally, a procedure to design an asymptotic observer is presented. This procedure is derived from the analysis of the observer estimation error, resulting in an asymptotic observer that can be represented as an IPN.

Observabilidad en los Sistemas de Eventos Discretos modelados con Redes de Petri Interpretadas

Resumen

Este trabajo presenta un estudio de los problemas de observabilidad y diseño de observadores asintóticos en los Sistemas de Eventos Discretos (SED) modelados con Redes de Petri Interpretadas (RPI). En particular, se abordan los siguientes problemas: 1) determinar bajo que condiciones una RPI dada es observable, 2) dada una RPI no medible, calcular un conjunto de sensores mínimo en costo tal que se preserve la propiedad de observabilidad y, 3) desarrollar un algoritmo para el diseño e implementación de observadores asintóticos.

Una condición suficiente y necesaria para la observabilidad en RPI es presentada. Esta condición está basada en la noción de invariantes de secuencias de entrada y de salida de una RPI. Esta condición es similar a la presentada por M. Wonham para los sistemas lineales continuos usando el enfoque geométrico. Dada la complejidad de esta condición, para RPI vivas, conservativas y cíclicas se demuestra que la observabilidad puede ser verificada a través de las propiedades de evento-detectabilidad y marcado-detectabilidad. La propiedad de evento-detectabilidad depende de la estructura de la red y puede ser verificada en tiempo polinomial. Por otro lado, varias condiciones suficientes para marcado-detectabilidad son presentadas. Estas están basadas en los conceptos de leyes conservativas de marcado y la distancia sincrónica entre las transiciones de la red.

Las nociones de W-observabilidad y observabilidad con respecto a un lenguaje fueron introducidas. La W-observabilidad está relacionada con la posibilidad de calcular el marcado de una red aún si esta se mantiene ejecutando un comportamiento infinito. En cambio, la observabilidad con respecto a un lenguaje, o simplemente L-observabilidad, implica la posibilidad de calcular el marcado de la red cuando se sabe que su comportamiento es restringido a un sublenguaje su lenguaje de disparos. Así, se presentan varias condiciones suficientes para W-observabilidad y L-observabilidad. Dichas condiciones están basadas en los conceptos de evento-detectabilidad y W-marcado-detectabilidad, para el primer caso, y en los conceptos de L-evento-detectabilidad y L-marcado-detectabilidad, en el segundo caso.

También se presenta un algoritmo simple para calcular una configuración de sensores mínima en costo para RPI vivas, conservativas y cíclicas. El algoritmo propuesto supone que sólo hay disponibles sensores de estado y que un sensor puede ser asignado a varios lugares de la red.

Finalmente, se presenta un procedimiento para diseñar observadores asintóticos para RPI observables. Este procedimiento se derivó del análisis realizado al modelo del error de estimación, resultando en un observador asintótico que puede representarse como una RPI.

Contents

Chapter 1

Introduction

1.1 Overview

A DES is a dynamic system, whose space state is countable and the state changes in response to event occurrences. Examples of DES are the flexible manufacturing systems, data management systems, traffic systems, computer nets and communication protocols, among others.

A continuous system also can be modeled as DES when the states are discretized, [13]. In order to discretize the state, relevant values of the state variables are fixed; thus, if a variable reaches a value within a preestablished interval, it is though that the system is in a given state, say q_i.

The change from a state to another is given by the occurrence of events. The event occurrences depend on the internal system dynamics or can be forced by actuator signals. When a system reaches a state, some sensors are activated. These sensor signals are present at the system output as long as the system remains at that state. Generally, the actuator and sensor signals are modeled as input and output alphabets, respectively, [23].

In several DES applications, it is necessary to have a complete knowledge of the system state, in order to perform control or monitoring tasks. Unfortunately, in most of real-time applications, due to a poor instrumentation or the existence of variables that cannot be directly measured, it is impossible to perform all the required measurements in order to completely determine the actual system state. However, if the knowledge of the system structure and the input and output sequences suffices to compute the system state, then the system is said to be observable. This leads to the observability issue: determine the system state through the information provided by its structure and the sequences of input and output symbols.

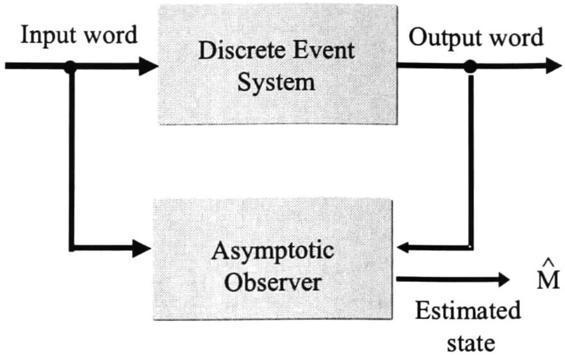

Figure 1.1: General scheme of the pair system-observer.

The task of determining the state of an observable system is performed by an observer. Figure 1.1 shows a general block diagram of the pair $DES - Observer$. The use of an observer allows to reduce the number of required sensors, since it is not necessary to measure the states that can be inferred from the others. This represents a sensor cost reduction. Moreover, since the number of signals is reduced in the communication system, hard traffic and high complex telemetric systems can be avoided, [44]. The observers had been used in fault tolerant systems, [12],[18]; monitoring and alarm systems, [12], [14]; and in estimated-state feedback control, [8], [12], [17].

There exist some similar concepts to observability such as diagnosability, testability and invertibility. The concept of diagnosability is used in fault tolerant systems. It is related to the possibility of detecting the unobservable fault event occurrences within a finite interval of events, [32]. Since the fault event set is partitioned into equivalence classes, it is not required to detect every system event; diagnosability only implies the detection of fault events belonging to different classes. On the contrary, observability requires the detection of every system event in order to reconstruct the actual system state.

Similarly, the property of testability is related to the possibility of distinguishing the coset to which belongs the states of a system under partial event observations, [4]. The cosets are formed according to a given set of possible faults. For testability, the order of the observations is irrelevant as long as the observations are sufficient to discern among the cosets of failure states. On the contrary, since observability implies possibility of exactly determining the actual system state, the order of the observations is relevant.

Similarly, the property of testability is related to the possibility of distinguishing among the failure states of a system under partial event observations, The set of system events is partitioned according to a given set of possible faults. This leads to define a partition of the system state space. For testability, the order of the observations is irrelevant as long as the observations are sufficient to discern among the cosets of failure states. On the contrary, since observability implies possibility of exactly determining the actual system state, the order of the observations is relevant.

On the other hand, the concept of invertibility is related to the possibility of completely reconstructing every sequence of system events from the knowledge of the observable event sequences, [25]. As it will be seen later, this concept is closely related to the concept of event-detectability in Petri nets. However, in the general case, invertibility does not implies observability, since the knowledge of the system event sequences is not sufficient to reconstruct the initial or actual system state.

This work deals with the observability problem, the exact computation of the actual system state. In particular, it is devoted firstly, to find out a characterization of observable DES. Next, to provide a methodology to select a minimal cost sensor configuration to preserve the observability property and finally, to provide a procedure for the design of asymptotic observers.

1.2 Relevant literature

The observability problem in DES has been addressed by several authors, most of them in the framework of DES modeled by Finite State Machines (FSM). For example, in [42], M. Wonham studies the observability problem from the supervisory control point of view in DES modeled by FSM under partial observations. The behavior of the FSM modelling a DES is constrained to a sublanguage K of its marked language L_m. The sublanguage K is observable with respect to L_m, if the information provided by the observed event sequences suffices to compute a control law such that the system can be constrained to K.

This notion of observability has been extended by several authors in order to solve the controller design problem. For example, in [44], the authors show that observability of a language is a necessary condition for the existence of a supervisor. However, since observability is not preserved under the union of languages, it is introduced the concept of normal languages. A language is normal if two partial observer words can be distinguished using the knowledge of the FSM structure. It is shown that normality implies observability and the supremal normal language always exists. Thus, formulae for computing the supremal normal sublanguage are presented in [5] and [19].

In [19], the authors extend the notion of observability proposed by Wonham to the context of decentralized supervisory control and introduces the concept of co-observability for a set of decentralized supervisors. In [22], [26] and [44] , the authors use the concept of co-observability to show the existence of decentralized controllers in DES modeled by FSM under partial event observations.

In [7], Cieslax et. al. extend the Wonham's works to DES modeled by non-deterministic FSM under partial event observations. The authors introduce the concepts of invariant languages, controllable languages and distinguishable languages in order to solve the supervisory design problem.

Another extension to the Wonham's works is presented, in [37], where the observability problem is addressed from the augmented language point of view and it is shown that observability is a necessary condition for the existence of a supervisor for systems whose desired behavior is given by non-regular languages. Similarly, in [20], Kumar et. al. introduces the concepts of $\omega -$ *observability* and $\omega -$ *normality* in DES modeled by sequential behaviors under partial observations.

In most of the previous works, the observability of a DES is characterized in terms of the language generated by the FMS modeling the system in order to establish existence conditions of a supervisor. However, the exact computation of the system state is not addressed. Only few works in FSM address the problem of determining the actual system state. For example, in [28], P. Ramadge studies the problem of determining the current system state of a non-deterministic FSM under full event observation and partial state observation. The observability problem is stated as the exact reconstruction the system state after the occurrence of every event. In [8], Özveren and Willski address the observability and observer design problems for DES modeled by FSM under event partial observations. The authors introduce the concept of stable languages to characterize observable FSM and provide a procedure to design observers, which allow the existence of state ambiguities, i.e. the system state is uniquely known only in certain time intervals. Also the notion of resilient observers is introduced. This kind of observers are able to recover the system state even under communication burns (e.g. wrong measurements).

Although the FSM formalism is suitable for describing DES, their application is limited to small size systems, since the models explicitly take into account all the possible system states, resulting in quite large models when the size of the system grows. In order to cope with the state explosion problem, Petri Nets (PN) are being adopted as DES modeling formalism. Besides compact models, PN provide clear graphical descriptions and a simple mathematical support, allowing to represent causal relationships, process synchronization, resource allocation and concurrence, inherent to DES behavior ([11], [24]).

The observability problem also has been studied from the PN point of view. One of the earliest reported works on observability using PN is [17]. In that paper, Ichikawa and Hiraishi dealt with the observability problem in a class of augmented PN, where a set of external places represent the sensor signals. Thus, if a external place gets marked then it means that the event associated with its input transition occurred. In this context the authors divide the observability problem into two subproblems: firstly, to compute the sequence of event occurrences σ_e given a sequence of markings σ_M in the external places; secondly, to compute the set of possible initial state given the sequence of event occurrences σ_e. The authors provide a necessary and sufficient condition to guarantee that a sequence of event occurrences can be uniquely determined. Also, based on the state equation of a PN, a procedure to compute the set of possible initial states is provided.

More recently, in [12] and [14], Giua and Fanni introduce a kind of asymptotic observers for DES under complete event observation. The proposed observer consists of an augmented PN model with an algorithm representing the observer dynamics. This kind of observers uses an incremental method for estimating the system state: the observer initial marking is equal to zero and, as the system evolves, it increments the number of tokens, thus, the observation error is non decreasing. On the other hand, if the number of total resources is known, the upper bounds for the state error can be given. This kind of observers are able to determine the marking of a PN when the firing of all transitions is known and the lower marking bound of every place is zero. An extension to this work is presented in [33]. The authors provide several notions of observability such as marking observability, uniform observability and structural observability, which are related to the existence of event sequences that lead the estimation error to zero.

On the other hand, in [2], the observability problem is addressed from the Interpreted Petri Net (IPN) point of view. A DES is modeled by a live, 1-bounded and cyclic IPN where every event is assumed to be measurable and controllable. For this kind of IPN, the authors provide an algorithm to design an asymptotic observer represented as an IPN; thus, further analysis can be realized on the pair system-observer using the well-known PN techniques. An extension to that work is presented in [29], where an event can be uncontrollable and a controllable event is not necessarily measurable. In [31], a generalization of the previous works is presented. The provided observer consists of an extended system model, where input and output transitions has been added to the observer places in order to regulate the estimation error. The convergency of the observer marking to the system state is shown in terms of the system event sequences. In [30], the authors present a methodology to design asymptotic observers for DES modeled by cyclic, live and conservative IPN where the lower marking bound of every place can be greater than zero. The proposed methodology is derived from a convergency analysis on the estimation error model, which is also represented as an IPN. The last four works form part of the research presented in this dissertation.

A common assumption in the previous works (in FSM and PN) is that the sensor configuration is given a priori. However, while designing a system, it is important to determine the set of measurable state variables in order to preserve the observability property. This problem is referred as the minimal sensor choice for observability. This problem has been the subject of several investigation in the literature, mainly for DES modeled by FSM under partial observations. For example, in [44], Young and Garg present a greedy algorithm to select a set of observable events. Similarly, in [15], Haji-Valizadeh and Loparo provide algorithms to define sufficient observable spaces, i.e. a set of events whose observation by a supervisor is enough to realize a desired behavior. In these two works, the observability property is defined in supervisory control terms like in [42] and [44]. From the PN net point of view, in [3], an algorithm to compute a minimal cost sensor choice for observability in live, conservative and cyclic Interpreted Free-Choice nets is presented. This last work also forms part of the research presented in this dissertation.

1.3 General Objectives

This work is devoted to study the observability and observer design problem in Discrete Event Systems (DES). The Interpreted Petri Nets (IPN), an extension to the Petri Net (PN) formalism, were selected as modeling formalism since they preserve all the advantages of the PN formalism and allow to represent the input signals associated to controllable events (transitions) and the output signals generated when a state (marking) is reached. Thus, an IPN model is suitable for the study the observability property in a natural way.

The main objectives of this thesis are:

1. To establish necessary and sufficient conditions for observability in DES modeled by IPN.
2. To provide a polynomial algorithm to obtain a minimal cost sensor configuration such that a given IPN preserves the observability property.
3. To provide an procedure for the design of asymptotic observers for observable IPN.

1.4 Outline of the thesis

Chapter 2 briefly presents the Interpreted Petri Net (PN) formalism and the required notation. Firstly, the definitions of PN and some related properties are presented. This allows to introduce the Interpreted Petri Nets (IPN), an extended PN formalism, and some concepts related to the input and output languages of an IPN.

In Chapter 3, a definition of observability in IPN terms is presented. Then, the concepts of input and output sequence invariants are presented. This sequence invariants are used to characterize observable IPN. However, since the computation of the sequence invariants represent a NP problem, the second part of this chapter is devoted to obtain a structural characterization of observable IPN. Thus, the concepts of event-detectability and marking-detectability are presented. Also, a necessary and sufficient condition for event-detectability and several sufficient conditions for marking-detectability are presented. It is shown that, these two properties represent a necessary and sufficient condition for observability in live, cyclic and conservative IPN.

Chapters 4 and 5 introduces the concepts of W-observability and L-observability. The first one for systems whereat least a firing sequence leading to a distinguishable marking exists and the second for IPN confined into a specific behavior. Characterizations of these two notions of observability are also provided. Based on the concepts of W-marking-detectability, L-event-detectability and L-marking-detectability.

Chapter 6 addresses the minimal cost sensor choice problem for observability in IPN. Firstly, the concepts of non-measurable IPN and sensor configuration are defined. These concepts are used to formally establish the minimal cost sensor choice problem for observability in IPN. Then, a characterization of the measurable nodes preserving the event-detectability property of an IPN is provided. This characterization leads to a simple algorithm to compute a minimal cost sensor configuration preserving the observability property. However, in order to improve the computation complexity of this algorithm, a reduction operation over the set of sensor configurations is performed. This leads to an improved algorithm. The presented algorithm can be used also to compute minimal cost sensor configurations for W-observability or L-observability.

In Chapter 7, a methodology to build asymptotic observers for systems modeled by observable, W-observable or L-observable IPN is presented. This procedure is derived from a convergency analysis realized on the observer estimation error model, which is also represented as an IPN. This analysis allows to find out conditions that must be satisfied to achieve an asymptotic convergency of the observer state to the actual system state.

Finally, in Chapter 8, conclusions are provided together with a discussion of original contribution and directions for further work.

Chapter 2

The Interpreted Petri Net Formalism

.**SUMMARY**. This chapter briefly introduces the main concepts, properties and the notation related to the PN and IPN modelling formalisms.

2.1 Introduction

The Petri nets (PN) are being used as modeling formalism for DES since they provide clearer graphical descriptions and, simple and sound mathematical support, allowing to represent DES properties such as process synchronization, concurrence, causal relationships, etc. [34]. In particular, the Interpreted Petri Net (IPN) formalism, an extension of PN, is used to model DES. The IPN formalism uses input and output languages to capture the control and sensor signals of a DES, respectively; this allows a more realistic representation of the DES behavior and to study certain system properties such as controllability, stability and observability. This chapter briefly introduces the main concepts and the notation related with PN, [9],[24], and IPN, [23].

2.2 Petri Nets

Definition 2.1 *A Petri net structure is a 4-tuple* $N = (P, T, I, O)$, *where*

- $P = \{p_1, p_2, ..., p_n\}$ *is a finite set of n places, depicted as circles,*
- $T = \{t_1, t_2, ..., t_m\}$ *is a finite set of m transitions, depicted as lines or bars,*
- $I : P \times T \longrightarrow \{0, 1\}$ *is a function representing the arcs going from places to transitions, and*
- $O : P \times T \longrightarrow \{0, 1\}$ *is a function representing the arcs going from transitions to places.*

A PN structure N can be also represented by its *incidence matrix* $C = [c_{ij}]_{n \times m}$, where $c_{ij} = O(p_i, t_j) - I(p_i, t_j)$.

Definition 2.2 *Let* x, $y \in T \cup P$, *then* ${}^\bullet(x) = \{y \mid$ *there is an arc from* y *to* $x\}$ *and* $(x)^\bullet = \{y \mid$ *there is an arc from* x *to* $y\}$ *are the sets of predecessors and successors of the node* x*, respectively.*

Definition 2.3 *A subnet of a net* $N = (P, T, I, O)$ *is a 4-tuple* $Q = (P', T', I', O')$, *where* $P' \subseteq P$, $T' \subseteq T$ *and* $\forall p \in P'$ $\forall t \in T'$ *it holds that* $I'(p, t) = I(p, t)$ *and* $O'(p, t) = O(p, t)$. *The net* Q *is also called the subnet generated by the set of nodes* $X = P' \cup T'$

Definition 2.4 *A p-invariant* Y *of a* PN *is a rational-valued solution of equation* $Y^T C = 0$. *The support of a p-invariant* Y_i *is the set* $||Y_i|| = \{p_j | Y(p) \neq 0\}$.

Definition 2.5 *The marking of a PN is a function $M : P \to \mathbb{Z}^+$, where $\mathbb{Z}^+ = \mathbb{N} \cup \{0\}$, that assigns to each place of N a non-negative number of tokens, depicted as black dots inside the places.*

The marking at the k-th instant is often represented by a vector $M_k = [M_k(p_1)\dots M_k(p_n)]^T$ or by a list $M_k = [a_1^{b_1}, a_2^{b_2}, \dots, a_n^{b_n}]$, where a_i is the index of the $i-th$ place, $b_i = M_k(p_i)$ and, if $M_k(p_i) = 0$ then a_i is omitted and if $M_k(p_i) = 1$ then b_i is omitted (see the example at the end of this section).

Definition 2.6 *A PN system (N, M_0) is a PN structure N with an initial marking M_0.*

The marking of a PN system evolves according to the following rules:

1. A transition $t_j \in T$ of a given PN system is *enabled* at a marking M_k if $\forall p_i \in P$, $M_k(p_i) \geqslant I(p_i, t_j)$. The set of enabled transitions at a marking M_k is

$$E(M_k) = \{t | \forall p \in P, M_k(p) \geqslant I(p,t)\}$$

2. If t_j is enabled at a marking M_k then it can be fired. In this case, a new marking M_{k+1} is reached. M_{k+1}, which can be computed using the equation:

$$M_{k+1} = M_k + C\vec{v}_k \tag{2.1}$$

where $\vec{v}_k$ is the firing vector of the enabled transition t_j, [9], which is defined as

$$v_k(i) = \begin{cases} 1, & i = j \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.7 *A firing sequence of a PN system (N, M_0) is a sequence $\sigma = t_i t_j \dots t_k$ such that $M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \dots \xrightarrow{t_k} M_k$.*

Observe that, a firing sequence is an enabled sequence at M_0; thus, the fact of reaching M_k from M_0 by firing the sequence is denoted by $M_0 \xrightarrow{\sigma} M_k$.

Definition 2.8 *Let (N, M_0) be a PN system.*

- *The set of all firing sequences of* (N, M_0)

$$\pounds(N, M_0) = \{\sigma = t_i t_j ... t_w | M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} ... \xrightarrow{t_k} M_k \xrightarrow{t_w} M_w\}$$

is called firing language of (N, M_0).

- *A firing sequence* $\sigma \in \pounds(N, M_j)$ *where* $M_j \xrightarrow{\sigma} M_k$ *is called blocking sequence if* M_k *is a blocking marking, i.e. no transition is enabled at it.*

Definition 2.9 *The set* $\mathbf{R}(N, M_0) = \{M_k | M_0 \xrightarrow{\sigma} M_k\}$ *of* (N, M_0) *is called reachability set of a PN system* (N, M_0).

Boundeness and liveness are two important properties of a DES. Boundeness is often interpreted as stability and it is used to identify the existence of system overflows; while, liveness means that, for every system event e_k and for any system state, it is always possible to reach a state, at which e_k can occur. Moreover, if a SED is live then it is deadlock-free, [45]. In PN terms, boundeness and liveness are defined as follows:

- A place p_i of a PN is b-bounded, if $\forall M_k \in \mathbf{R}(N, M_0)$, $M_k(p_i) \leq b$, where b is an nonnegative integer. A PN is b-bounded, if all its places are b-bounded.

- A transition t_k is said to be live if $\forall M_j \in \mathbf{R}(N, M_0)$, $\exists \sigma$ such that $M_j \xrightarrow{\sigma} M_r$ and $t_k \in \sigma$. A PN is live if all its transitions are live.

Another desired property of a SED is a cyclic behavior. A SED has a cyclic behavior if there exists a sequence of events that allows to reach the initial state from any reachable state, it means that any task could be infinitely often performed. In PN terms a global cyclic behavior is defined as follows, [34].

- A PN is cyclic if $\forall M \in \mathbf{R}(N, M_0)$, $\exists \sigma$ such that $M \xrightarrow{\sigma} M_0$.

A cyclic PN is also usually known in the literature as "a reversible net"

The following example illustrates some of the above concepts.

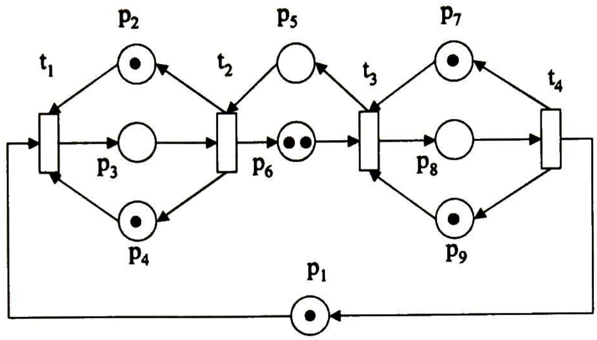

Figure 2.1: A PN system.

Example 2.10 *Consider the PN system of Figure 2.1. The incidence matrix of this net is*

$$C = \begin{bmatrix} -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

The sets of predecessors and successors of t_2 are $^\bullet(t_2) = \{p_3, p_5\}$ and $(t_2)^\bullet = \{p_2, p_4, p_6\}$, respectively. A p-invariant of the net is $y_1 = [1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0]^T$. The support of y_1 is $||y_1|| = \{p_1, p_3, p_6, p_8\}$. The initial marking is $M_0 = [1\ 1\ 0\ 1\ 0\ 2\ 1\ 0\ 1]^T$ or, in list form, $M_0 = [1, 2, 4, 6^2, 7, 9]$. It is easy to verify that, (N, M_0) is a 3-bounded, live and cyclic PN system. The set of enabled transitions at M_0 is $E(M_0) = \{t_1, t_3\}$. Suppose that, the transition t_3 is fired. The firing vector of $\sigma = t_3$ is $\vec{\sigma} = [0\,0\,1\,0]^T$ In this case, after firing $\vec{\sigma}$, the marking $M_1 = [1, 2, 4, 5, 6, 8]$ is reached.

2.3 Interpreted Petri Nets

The Interpreted Petri Net (IPN) formalism is an extension to PN. An IPN consists of a PN system plus input and output alphabets assigned to the transitions and places of the net by certain functions. These alphabets represent the actuator and sensor signals of the system. Thus, an IPN has the possibility of modeling the sequences of commands given to the actuators and the sensor signals emitted each time a new state is reached. This modelling capacity of an IPN allows to study system properties such as stability, controllability and observability. An IPN is formally defined as follows.

Definition 2.11 *An Interpreted Petri Net (IPN) is a 5-tuple* $Q = (N', \Sigma, \Phi, \lambda, \varphi)$ *where*

- $N' = (N, M_0)$ *is a PN system,*
- $\Sigma = \{a_1, a_2, ..., a_u\}$ *is the input alphabet, where* a_i *is an input symbol,*
- $\Phi = \{\phi_1, \phi_2, ..., \phi_s\}$ *is the marking output alphabet, where* ϕ_i *is an output symbol,*
- $\lambda : T \rightarrow \Sigma \cup \{\varepsilon\}$ *is a function that assigns an input symbol to each transition of the net, where* ε *represents an internal system event. This function has the following restriction:* $\forall t_j, t_k \in T,\ j \neq k$ *if* $I(p_i, t_j) = I(p_i, t_k) \neq 0$ *and both* $\lambda(t_j),\ \lambda(t_k) \neq \varepsilon$*, then* $\lambda(t_j) \neq \lambda(t_k)$*,*
- $\varphi : \mathbf{R}(N, M_0) \rightarrow \Phi$ *is a function that assigns an output symbol to each reachable marking of the net.*

Remark 2.12 *To enhance the fact that there exists an initial marking in an IPN, hereafter it will be written* (Q, M_0) *instead of* Q*.*

The transition input alphabet Σ of an IPN can be thought as actuator signals attached to the transitions of the net. Similarly, the marking output alphabet Φ of the net can be thought as state sensor signals. In this context, it is possible to distinguish between controllable and uncontrollable transitions, and between measurable and non-measurable places of the net as established in the following definitions.

Definition 2.13 *A transition* $t_j \in T$ *is said to be controllable, if* $\lambda(t_i) \neq \varepsilon$*, and uncontrollable, otherwise.*

The set $T_c = \{t | \lambda(t) \neq \varepsilon\}$ *is the set of controllable transitions and* $T_u = \{t | \lambda(t) = \varepsilon\}$ *is the set of uncontrollable transitions. Observe that,* $T = T_c \cup T_u$ *and* $T_c \cap T_u = \emptyset$*.*

Graphically, a controllable transition is represented as a unfilled bar, while a uncontrollable transition as a filled bar.

A controllable transition represents an event which occurrence can be prevented (e.g. open a valve, turn off a power source), while a uncontrollable transition represent an event which occurrence depends on the internal dynamics of the system.

This work focuses in the case where $\Phi = [\mathbb{Z}^{+}]^{r}$ and φ is a linear function defined as $\varphi : [\mathbb{Z}^{+}]^{n} \longrightarrow [\mathbb{Z}^{+}]^{r}$, where $n = |P|$ and r is the number of available marking outputs. In this case, the function φ can be represented as a matrix

$$\varphi = [\varphi_{ij}]_{r \times n} = \begin{bmatrix} e_u^T \\ e_v^T \\ \vdots \\ e_w^T \end{bmatrix}$$

where, $\{p_u, p_v, \ldots, p_w\}$ is the set of places whose marking is available at the output and e_i is the i-th elementary vector ($e_i\,[i] = 1$ and $e_i[j \neq i] = 0, \forall i = 1, 2, \ldots, n$). The places whose marking is available at the output are called measurable places. Formally these places are defined as follows.

Definition 2.14 *A place $p_i \in P$ is said to be measurable, if $\exists j \in 1, 2, \ldots, r$ such that $\varphi_j = e_i^T$, where φ_j is the j-th row of φ; otherwise, p_i is called non-measurable.*

The set $P_m = \{p | \exists j \in 1, 2, \ldots, r$ such that $\varphi_j = e_i^T\}$ is the set of measurable places and $P_{nm} = P \backslash P_m$ is the set of non-measurable places.

A measurable place is depicted as a unfilled circle, while a non-measurable place is depicted as a filled circle.

The marking of an *IPN* evolves according to the following rules:

1. A transition $t_j \in T$ is *enabled* at a marking M if $\forall p_i \in P$, $M(p_i) \geqslant I(p_i, t_j)$.
2. If t_j is an enabled uncontrollable transition then it can be fired, but if t_j is an enabled controllable transition and the symbol $\lambda(t_j) = a_i \neq \varepsilon$ is present at the input of the net then t_j must fire. In both cases, if t_j fires at a marking M_k, then a new marking M_{k+1} is reached, which can be computed using the equation 2.1.

Observe that in the definition of an *IPN*, the function λ does not allows that two transitions sharing the same input place have the same input symbol different from ε. This is due to the firing rules of an *IPN* : if these transitions get enabled at the same marking and the input symbol is present, then both transitions must be fired, resulting in a negative marking. On the other hand, suppose that these transitions are labelled with ε. Since the symbol ε represents internal events then their firing depends on the system internal dynamics, so without loss of generality it can be thought that one of them can occur before the other.

Since the firing of enabled uncontrollable transitions depends on the internal dynamic of the modeled SED and the firing of enabled controllable transitions depends of the presence of the input symbols attached to them, the incidence matrix C of the net can be split into two matrices: $C = [C^{\varepsilon} \vdots C^{c}]$, where $C^{\varepsilon} = [c^{\varepsilon}_{ij}]_{n\times|T_{nm}|}$ and $C^{c} = [c^{c}_{ij}]_{n\times|T_m|}$ are formed by the columns of the uncontrollable and controllable transitions, respectively. Similarly, the firing vector of a sequence at the k-th event can be split into vectors $\vec{v}_k = [\vec{v}^{\varepsilon}_k \mid \vec{v}^{c}_k]$, where

$$\begin{aligned} v^{\varepsilon}_k(i) &= \begin{cases} v_k(i), & t_i \in T_{nm} \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \\ v^{c}_k(i) &= \begin{cases} v_k(i), & t_i \in T_m \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

Hence, the state equation of an IPN can be written as:

$$\begin{aligned} M_{k+1} &= M_k + C^{\varepsilon}\vec{v}^{\varepsilon}_k + C^{c}\Xi(M_k, \vec{v}^{c}_k) \\ y_{k+1} &= \varphi M_{k+1} \end{aligned} \tag{2.2}$$

where y_{k+1} is the output signal vector of the measurable places at the $k+1$ instant and $\Xi : \mathbf{R}(N, M_0) \times R^m \to R^m$ is a function that returns the firing vector of the enabled transitions contained in $\vec{v}^{c}_k$; thus, $\Xi(M_k, \vec{v}^{c}_k) = \vec{v}'^{c}_k$ such that

$$v'^{c}_k(i) = \begin{cases} v^{c}_k(i), & \text{if } \forall p_j \in P,\ M(p_j) \geqslant I(p_j, t_i) \\ 0, & \text{otherwise} \end{cases}$$

The function Ξ represents the fact of accepting or not an input word given to the system, i.e. preventing the firing of non-enabled controllable transitions. Thus, if an input symbol is given to the system and the corresponding transition is not enabled then it is not fired. Figure 2.2 shows the block diagram of an IPN. The block named "*uncontrollable firings*" represents the occurrences of events due to the internal dynamics of the system.

Due to the input and output alphabets, an IPN can generate the following languages.

Definition 2.15 *The input language of* (Q, M_0) *is*

$$\mathcal{L}_{in}(Q, M_0) = \{\omega = \lambda(t_i)\lambda(t_j)...\lambda(t_k) | \sigma = t_i t_j ... t_k \in \mathcal{L}(Q, M_0)\},$$

while its output language is

$$\mathcal{L}_{out}(Q, M_0) = \{z = \varphi(M_0)\varphi(M_1)...\varphi(M_k) | M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} ... \xrightarrow{t_k} M_k \wedge t_i t_j ... t_k \in \mathcal{L}(Q, M_0)\}.$$

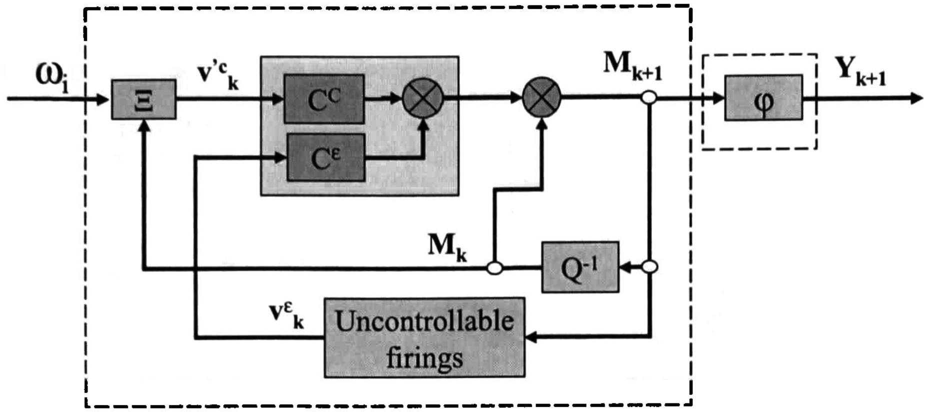

Figure 2.2: Block diagram representation of an *IPN*.

Definition 2.16 *Let (Q, M_0) be an IPN and $\omega \in \pounds_{in}(Q, M_0)$ be an accepted input word. The set of possible firing sequences generated when ω is accepted is*

$$\pounds_{\omega}(Q, M_0) = \{\sigma = t_i t_j ... t_v | \lambda(\sigma) = \lambda(t_i)\lambda(t_j)...\lambda(t_v) = \omega\}$$

The set of possible reached markings when ω is accepted is

$$Ł_{\omega}(Q, M_0) = \{M | M_0 \xrightarrow{\bar{\sigma}} M \wedge \sigma \in \pounds_{\omega}(Q, M_0)\}$$

where $\bar{\sigma}$ is a prefix of σ.

The set of possible output signal sequences generated when ω is accepted is

$$\Phi_{\omega}(Q, M_0) = \{\varphi(M_j)\varphi(M_k)...\varphi(M_h) | M_j, M_k, \dots, M_h \in Ł_{\omega}(Q, M_0)\}$$

Observe that, $\pounds_{\omega}(Q, M_0) \subseteq \pounds(Q, M_0)$ and $\Phi_{\omega}(Q, M_0) \subseteq \pounds_{out}(Q, M_0)$.

To illustrate the above concepts, consider the manufacturing cell of the following example, adopted from [45].

Example 2.17 *A production cell consists of two machines, M_1 and M_2, two robots, R_1 and R_2, for loading and unloading parts from M_1 and M_2 respectively. An incoming conveyor (IC) carries a part to be pre-processed by M_1. The pre-processed part is unloaded by R_1 and left in a intermediate 2-slot buffer B. Then R_2 takes it and loads M_2. A finished part is deposited in an outgoing conveyor (OC). Figure 2.3 shows a scheme of the production cell and Figure 2.4, an IPN model for it.*

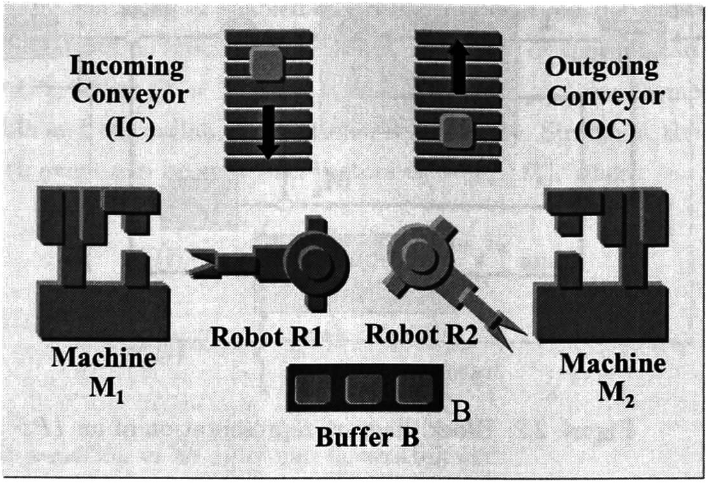

Figure 2.3: Scheme of a typical automated production cell.

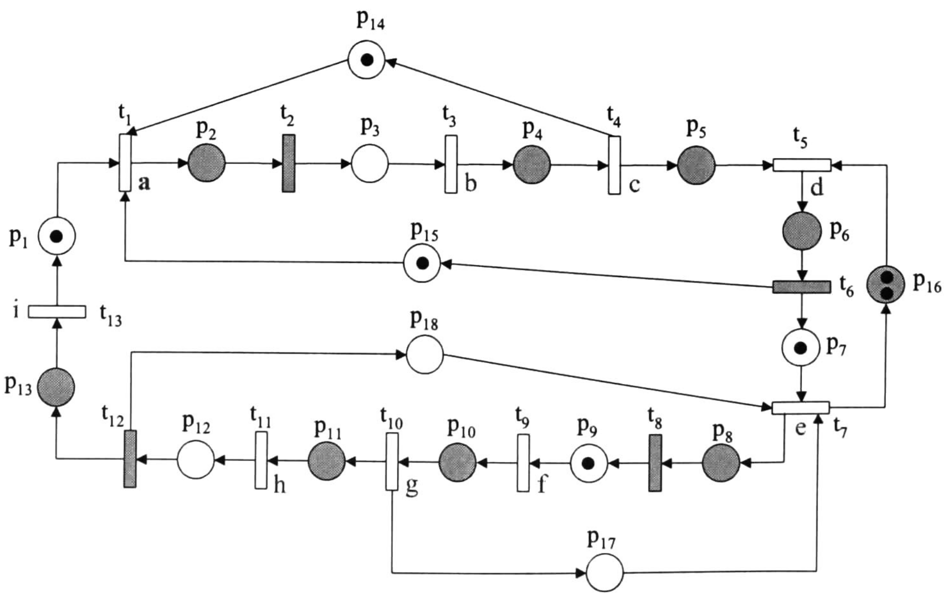

Figure 2.4: An *IPN* model of the automated production cell.

The interpretation of each place and transition, and their respective input and output symbols are the following

Place	*Interpretation*	*Transition*	*Interpretation*
p_1	*Part available*	t_1	*Load* M_1
p_2 (p_8)	*Loading* $M_1(M_2)$	t_2	*End loading* M_1
p_3 (p_9)	M_1 (M_2) *loaded*	t_3	*Start* M_1
p_4 (p_{10})	$M_1(M_2)$ *working*	t_4	*Stop* M_1
p_5	*Part preprocessed*	t_5	*Unload* M_1
p_6	*Unloading to buffer*	t_6	*End unloading* M_1
p_7	*Parts buffered*	t_7	*Load* M_2
p_{11}	*Part processed*	t_8	*End loading* M_2
p_{12}	*Unloading to the OC*	t_9	*Start* M_2
p_{13}	*Part outgoing*	t_{10}	*Stop* M_2
p_{14} (p_{17})	M_1 (M_2) *idle*	t_{11}	*Unload* M_2
p_{15} (p_{18})	R_1 (R_2) *available*	t_{12}	*End unloading* M_2
p_{16}	*Available slots*	t_{13}	*Enter a new part*

The input and output labeling functions are

t_i	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}
$\lambda(t_i)$	a	ε	b	c	d	ε	e	ε	f	g	h	ε	i

$$\varphi = [e_1\ e_3\ e_7\ e_9\ e_{12}\ e_{14}\ e_{15}\ e_{17}\ e_{18}]^T$$

In this case, the input alphabet is $\Sigma = \{a,\ b,\ c,\ \ldots,\ i\}$, $T_c = \{t_1,\ t_3,\ t_4,\ t_5,\ t_7,\ t_9,\ t_{10},\ t_{11},\ t_{13}\}$ *and* $T_u = \{t_2,\ t_6,\ t_8,\ t_{12}\}$. *On the other hand,* $P_m = \{p_1,\ p_3,\ p_7,\ p_9,\ p_{12},\ p_{14},\ p_{15},\ p_{17},\ p_{18}\}$ *and* $P_{nm} = \{p_2,\ p_4,\ p_5,\ p_6,\ p_8,\ p_{10},\ p_{11},\ p_{13},\ p_{16}\}$. *The initial marking is* $M_0 = [1,\ 7,\ 9,\ 14,\ 15,\ 16^2]$, *which means that a part is available, there is a buffered part and a part in* M_2, *the* M_1 *is idle, the* R_1 *is available and two buffer slots are empty. At this marking, the net yields the output* $y_0 = \varphi(M_0) = [101101100]^T$ *It is easy to verify that,* (Q, M_0) *is live, cyclic and 3-bounded. An input word is* $\omega = abc$, *which yields the firing sequence* $\sigma = t_1t_2t_3t_4$ *and the* *output word*

$$z = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2.4 Discussion

In this chapter, the Petri Net and Interpreted Petri Net formalisms were reviewed in order to introduce the necessary notation for the remaining of this manuscript. As it will be seen in next chapters, due to its capability of representing the input and output languages of a Discrete Event System, the Interpreted Petri Nets represent a suitable formalism not only to model a Discrete Event System, but also to study some qualitative system properties; in particular, to study the observability and asymptotic observer design problems.

Chapter 3

Observability in IPN

SUMMARY. This chapter addresses the observability problem in DES modeled by IPN. A definition of the observability property is presented in IPN terms. A characterization of observable IPN is provided based on the concepts of input and output sequence invariants. Several characterizations of observable IPN are provided for live, bounded and cyclic IPN,

3.1 Introduction

Observability is an important property of dynamic systems since it implies the possibility of estimating the system states that cannot be measured. Observability is defined in linear continuous systems as the possibility of determining the initial state of the system in a finite time by using the knowledge of its inputs, outputs and structure [6]. In the DES area, some definitions of observability has been proposed. Most of these definitions have been stated in regular language terms, where the concept of observability has been tied to the supervisor existence problem. Only in a few number of works the observability has been independently addressed. On the other hand, from the Petri net point of view, the concept of observability has been established depending of the method used to estimate the system state.

In this chapter, the observability property is studied from the PN point of view. Firstly, a definition of observability is provided. This definition is similar to the one used in continuous system theory. Then, the concepts of input and output sequence invariants are presented. This sequence invariants are used to characterize observable IPN. However, since the computation of the sequence invariants represent a NP problem, the second part of the first section is devoted to obtain a structural characterization of observable IPN.

3.2 Basic definitions

While modeling a DES in IPN terms, the knowledge of resources contained in the system and the number of codifications of the state variables is usually known. For example, the number of available robots, machines, etc. is known; in the case of discretized systems, it is known that in the state variables of model are codified into different values (e.g. in a water tank three levels can be defined: low, normal, high). This knowledge reduces the number of possible initial markings of an IPN.

This leads to the concepts of conservative marking laws (CML), a set of equations that indicates the number of tokens contained in a set of places for any reachable marking.

Definition 3.1 *Let (Q, M_0) be an IPN and $Y = \{\vec{Y}_1\ \vec{Y}_2 \ldots \vec{Y}_s\}$ be the set of all the minimal p-semiflows of the net. The set*

$$CML(Q, M_0) = \left\{ \begin{array}{c} \vec{Y}_1^T M = k_1 \\ \vec{Y}_2^T M = k_2 \\ \cdots \\ \vec{Y}_s^T M = k_s \end{array} \right\}$$

is called conservative marking laws, where $k_i = \vec{Y}_i^T M_0$ *and* $M = [M(p_1)\ M(p_2) \ldots M(p_n)]^T$

A CML *is well-defined, if* $\forall p_j \in P_{nm}\ \exists \vec{Y}_i$ *such that* $Y_i(p_j) > 0$, *i.e. all non-measurable places of the net are contained in at least one equation.*

Definition 3.2 *Let* (Q, M_0) *be an* IPN *and* $\xi_i := [\vec{Y}_i^T M = k_i] \in CML(Q, M_0)$ *be a conservative marking law. The support of* ξ_i *is*

$$||\xi_i|| = \{p_j \in P | Y_i(p_j) \neq 0\}$$

Observe that a CML depends on the initial marking of the net. This dependency is related to the number of tokens initially contained in the net and on less degree to their location. In this sense, the concept of a CML is analogous to the concept of "macro-markings" in [14]. A CML can be determined by the knowledge of resources into the system, for instance, number of machines, number of workshops, buffer and machine capacities, etc. Note that, the number of machines is known, but the state of these machines (working, idle, etc.) could be unknown.

In IPN terms, the observability is defined as follows.

Definition 3.3 *An* IPN *given by* (Q, M_0) *is observable in* k *steps if* $\forall \omega \in \mathcal{L}_{in}(Q, M_0)$, *such that* $k \leq |\omega| < \infty$, *the information provided by the knowledge of the input word* ω, *the output word generated by* ω, *the system structure and* $CML(Q, M_0)$ *suffices to uniquely determine* M_0 *and the firing sequence generated by* ω, $\sigma_\omega = t_i t_j \ldots t_r$.

Observe that this definition implies that, after the occurrence of a k-length input word, it is possible to reconstruct the reached marking M_k and every reached marking from M_0 to M_k.

Example 3.4 *Consider the* IPN *of Figure 3.1.a). The CML of this net is* $M(p_1)+M(p_2) = 1$. *Suppose that the input word* $\omega = a$ *is observed; however, since no place is measurable, then no output word is observed. Thus, it is impossible to determine whether the transition* t_1 *is fired or not. Hence, it is impossible also to determine if* $M_0 = [10]^T$ *or* $M_0 = [01]^T$ *Therefore, the net is not observable.*

Now, consider the IPN *of Figure 3.1.b). The CML of this net is* $M(p_1)+M(p_2)+M(p_3) = 1$. *The initial output is* $z_0 = [0]$. *Suppose that when the input word* $\omega = a$ *is given, the output word* $z_1 = [1]$ *is observed. Hence, the actual marking is* $M_1 = [001]^T$ *Although, the current marking of the net can be determined, it cannot be determined which transition was fired. Thus, also it cannot be determined which was the initial marking* $M = [100]^T$ *or* $M' = [010]^T$

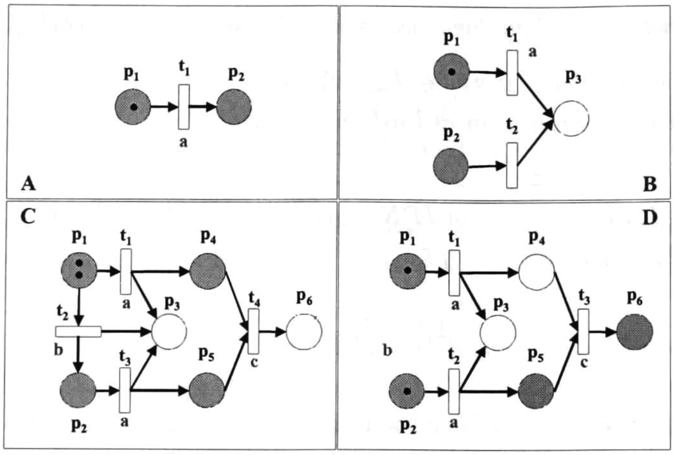

Figure 3.1: Several unobservable and observable IPN.

Consider the IPN of Figure 3.1.c). The CML of this net is $M(p_1) + M(p_2) + M(p_4) + M(p_5) + M(p_6) = 2$. Suppose that the input word $\omega = baac$ is given. In this case, the following sequence of outputs is observed:

$$z = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Thus, it is known that the current marking is $M = [5^3, 6]$ and the initial marking was $M_0 = [1^2]$. However, it cannot be determined whether $\sigma' = t_1t_2t_3t_4$ or $\sigma'' = t_1t_3t_2t_4$ was fired: Hence, the net is not observable.

Consider the net of Figure 3.1.d). The CML of this net is

$$\begin{aligned} M(p_1) + M(p_2) + M(p_3) &= 2 \\ M(p_1) + M(p_4) + M(p_6) &= 1 \\ M(p_2) + M(p_5) + M(p_6) &= 1 \end{aligned}$$

Suppose that the input word $\omega = aac$ is given. In this case, the following sequence of outputs is observed:

$$z = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

Thus, it is known that the current marking is $M = [3, 6]$, the initial marking was $M_0 = [1, 2]$ and the sequence $\sigma' = t_1t_2t_3$ was fired: Hence, the net is observable.

The knowledge of $CML(Q, M_0)$ allows to reduce the set of possible initial markings to those that agree with the knowledge of the plant resources and state codification.

Definition 3.5 *Let* (Q, M_0) *be an* IPN *and* $CML(Q, M_0)$ *be its set of conservative marking laws. The set of markings that agree with* M_0 *is*

$$\mathbb{R}(Q, M_0) = \{M \in (\mathbb{Z}^+)^m | \varphi(M) = \varphi(M_0) \wedge \forall \xi_i : [\vec{Y}_i^T M = k_i] \in CML(Q, M_0), \vec{Y}_i^T M = k_i\}$$

Definition 3.6 *Let* (Q, M_0) *be an* IPN,

1. *The sequence invariant of an input word* $\omega \in \mathcal{L}_{in}(Q, M_0)$ *is the set*

$$\begin{aligned} I_\omega = \{M_i M_j \ldots M_k | M_i \xrightarrow{t_q} M_j \xrightarrow{t_r} \ldots \xrightarrow{t_s} M_k \wedge \lambda(t_q)\lambda(t_r)\ldots\lambda(t_s) = \omega \wedge \\ M_i \in \mathbb{R}(Q, M_0)\} \end{aligned}$$

2. *The sequence invariant of an output word* $z \in \mathcal{L}_{out}(Q, M_0)$ *is the set*

$$\begin{aligned} O_z = \{M_i M_j \ldots M_k | M_i \xrightarrow{t_q} M_j \xrightarrow{t_r} \ldots \xrightarrow{t_s} M_k \wedge \varphi(M_i)\varphi(M_j)\cdots\varphi(M_k) = z \wedge \\ M_i \in \mathbb{R}(Q, M_0)\} \end{aligned}$$

Definition 3.7 *Let* (Q, M_0) *be an* IPN *and* $CML(Q, M_0)$ *be its set of conservative marking laws. The set of correlated input and output words of lenght greater or equal to* k *is*

$$\Delta_k = \{(\omega, z) | \omega \in \mathcal{L}_{in}(Q, M_0) \wedge |\omega| \geq k \wedge z \in \mathcal{L}_{out}(Q, M_0) \wedge z = \text{ is the output yielded by } \omega\}.$$

Using these sequence invariants, the following characterization of observable can be stated.

Theorem 3.8 *An* IPN *given by* (Q, M_0) *is observable in* k *steps if and only if* $\forall(\omega, z) \in \Delta_k$, *it holds that* $|I_\omega \cap O_z| \leq 1$.

Proof. (Necessity). Let (Q, M_0) be an observable IPN; thus, there exists a function $\Psi : \Delta_k \to \mathbb{R}(Q, M_0)$. Suppose that there exists an input invariant, say I_ω, with $|\omega| \geq k$, and an output invariant, say O_z, such that $|I_\omega \cap O_z| > 1$. It means that, there exist two marking sequences $M_i M_j \ldots M_k$, $M_q M_r \ldots M_s \in I_\omega$ such that $M_i M_j \ldots M_k, M_q M_r \ldots M_s \in O_z$. Thus, $\Psi((\omega, z)) = M_k$ and $\Psi((\omega, z)) = M_s$. In this case, Ψ is not a function, a contradiction.

(Sufficiency). Suppose that the biggest cardinality of an intersection of an input invariant and an output invariant of (Q, M_0) is equal to one for $\forall \omega \in \mathcal{L}_{in}(Q, M_0)$ such that $|\omega| \geq k$. Let $M_i M_j \ldots M_k, M_q M_r \ldots M_s \in O_z$ be marking sequences belonging to the invariant O_z

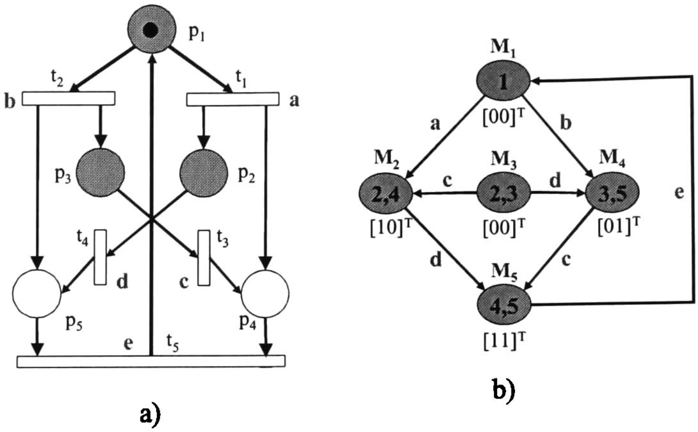

Figure 3.2: An IPN system and the merge of the reachability graphs of $M' = [1]$ and $M" = [2,3]$.

of the output word z. Since, by hypothesis they belong to different input invariants, say $M_iM_j \ldots M_k \in I_{\omega_i}$ and $M_qM_r \ldots M_s \in I_{\omega_j}$, where $|\omega_i| \geq k$ and $|\omega_j| \geq k$, then the function Ψ can be built as follows $\Psi((\omega_i, z)) = M_k$ and $\Psi((\omega_j, z)) = M_s$. Therefore, (Q, M_0) is observable. ■

It means that, the markings that cannot be distinguished through the output sequence, must be distinguished through the input word. Observe that, this characterization is similar to the one presented for linear continuous systems using the geometric approach, [41].

Example 3.9 *Consider the IPN system shown in Figure 3.2.a), where the set of conservative marking laws is the following*

$$CML(Q, M_0) = \left\{ \begin{array}{c} M(p_1) + M(p_2) + M(p_5) = 1 \\ M(p_1) + M(p_3) + M(p_4) = 1 \end{array} \right\}$$

In this case, since $y_0 = [00]^T$ *the set of markings that agree with* M_0 *is*

$$\mathbb{R}(Q, M_0) = \{[1], [2,3]\}$$

Figure 3.2.b), shows the graph resulting of merging the reachability sets of each marking of $\mathbb{R}(Q, M_0)$. *The 2-dimensional vector shown below each node corresponds to the output vector of the given marking.*

For short, a symbol is assigned to each output vector as follows:

$\varnothing = [00]^T$	$y = [01]^T$
$x = [10]^T$	$z = [11]^T$

In this case, some of the output invariants of the net are

i	z_i	O_{z_i}	i	z_i	O_{z_i}
1	$\varnothing$	$\{M_1, M_3\}$	9	xz	$\{M_2M_5\}$
2	$\varnothing x$	$\{M_1M_2, M_3M_2\}$	10	$xz\varnothing$	$\{M_2M_5M_1\}$
3	$\varnothing xz$	$\{M_1M_2M_5, M_3M_2M_5\}$	11	y	$\{M_4\}$
4	$\varnothing xz\varnothing$	$\{M_1M_2M_5M_1, M_3M_2M_5M_1\}$	12	yz	$\{M_4M_5\}$
5	$\varnothing y$	$\{M_1M_4, M_3M_4\}$	13	$yz\varnothing$	$\{M_4M_5M_1\}$
6	$\varnothing yz$	$\{M_1M_4M_5, M_3M_4M_5\}$	14	z	$\{M_5\}$
7	$\varnothing yz\varnothing$	$\{M_1M_4M_5M_1, M_3M_4M_5M_1\}$	15	$z\varnothing$	$\{M_5M_1\}$
8	x	$\{M_2\}$			

Observe that the marking sequences belonging to the same output invariant are indistinguishable from each other by the output. For example, the marking sequences M_1M_2, M_3M_2 and M_1M_4, M_3M_4 are indistinguishable by the output since they yield the same output word. Now, some of the input invariants of the net are

j	ω_j	I_{ω_j}	j	ω_j	I_{ω_j}	j	ω_j	I_{ω_j}
1	a	$\{M_1M_2\}$	7	b	$\{M_1M_4\}$	13	de	$\{M_2M_5M_1\}$
2	ad	$\{M_1M_2M_5\}$	8	bc	$\{M_1M_4M_5\}$	14	dea	$\{M_2M_5M_1M_2\}$
3	ade	$\{M_1M_2M_5M_1\}$	9	bce	$\{M_1M_4M_5M_1\}$	15	deb	$\{M_2M_5M_1M_4\}$
4	c	$\{M_3M_2, M_4M_5\}$	10	d	$\{M_2M_5, M_3M_4\}$	16	ce	$\{M_4M_5M_1\}$
5	cd	$\{M_3M_2M_5\}$	11	dc	$\{M_3M_4M_5\}$	17	cea	$\{M_4M_5M_1M_2\}$
6	cde	$\{M_4M_5M_1\}$	12	dce	$\{M_3M_4M_5M_1\}$	18	ceb	$\{M_4M_5M_1M_4\}$

Notice that, $M_1M_2 \in I_a$ and $M_3M_2 \in I_c$. It means that, although they are indistinguishable by the output (both yield the output word $\varnothing x$), they can be distinguished by the input.

In general, observe that $\forall(\omega, z) \in \Delta_2$, the intersection of the corresponding input and output invariants has at most one element. Therefore, by Theorem 3.8, the IPN system is observable in 2 steps.

Example 3.10 *Consider the IPN system shown in Figure 3.3.a), where the set of conservative marking laws is the following*

$$CML(Q, M_0) = \left\{ \begin{array}{c} M(p_1) + M(p_2) + M(p_5) = 1 \\ M(p_1) + M(p_3) + M(p_4) = 1 \end{array} \right\}$$

and the set of markings that agree with M_0 is

$$\mathbb{R}(Q, M_0) = \{[1], [2, 3]\}$$

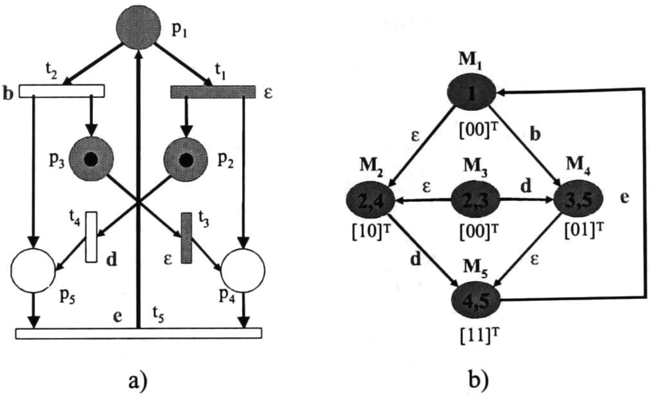

Figure 3.3: An IPN system and the merge of the reachability graphs of $M' = [1]$ and $M" = [2,3]$.

Figure 3.3.b), shows the graph resulting of merging the reachability sets of each marking of $\mathbb{R}(Q, M_0)$.

Observe that, the only difference between this IPN *system and the one of the previous example is that transitions* t_1 *and* t_3 *are uncontrollable. In this case, conserving the notation of the previous example, some of the output invariants are*

i	z_i	O_{z_i}
1	$\varnothing$	$\{M_1, M_3\}$
2	$\varnothing x$	$\{M_1M_2, M_3M_2\}$
3	$\varnothing xz$	$\{M_1M_2M_5, M_3M_2M_5\}$
4	$\varnothing(xz\varnothing)^+$	$\{M_1(M_2M_5M_1)^+, M_3(M_2M_5M_1)^+\}$
5	$\varnothing y$	$\{M_1M_4, M_3M_4\}$
6	$\varnothing yz$	$\{M_1M_4M_5, M_3M_4M_5\}$
7	$\varnothing yz\varnothing$	$\{M_1M_4M_5M_1, M_3M_4M_5M_1\}$
8	x	$\{M_2\}$

i	z_i	O_{z_i}
9	xz	$\{M_2M_5\}$
10	$xz\varnothing$	$\{M_2M_5M_1\}$
11	y	$\{M_4\}$
12	yz	$\{M_4M_5\}$
13	$yz\varnothing$	$\{M_4M_5M_1\}$
14	z	$\{M_5\}$
15	$z\varnothing$	$\{M_5M_1\}$

but some of the input invariants are

j	ω_j	I_{ω_j}
1	ε	$\{M_1M_2, M_3M_2, M_4M_5\}$
2	b	$\{M_1M_4, M_1M_4M_5\}$
3	d	$\{M_2M_5, M_1M_2M_5, M_3M_4, M_3M_4M_5, M_3M_2M_5\}$
4	e	$\{M_5M_1, M_5M_1M_2, M_4M_5M_1, M_4M_5M_1M_2\}$
5	$(\varepsilon de)^+$	$\{M_1(M_2M_5M_1)^+, M_3(M_2M_5M_1)^+\}$

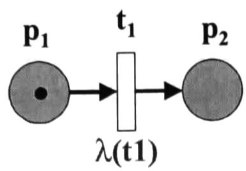

Figure 3.4: A unobservable IPN.

Notice that for ω_2 and z_5, the intersection of these invariants is

$$I_{\omega_2} \cap O_{z_5} = \{M_1(M_2M_5M_1)^+, M_3(M_2M_5M_1)^+\}$$

whose cardinality is bigger that one. Thus, M_1 and M_3 are indistinguishable. Moreover, since ω_2 is infinite, then it is not possible to find a k as stated in the definition 3.3; therefore, by Theorem 3.8, the IPN system is not observable in k steps.

Observe that, in the approach herein presented, the output of an IPN is only related to its measurable places. Hence, if an input symbol is given to the net, only a change in the output indicates that it was accepted. i.e. the corresponding transition was fired; similarly, the firing of an uncontrollable transition can be only detected through a change in the output. Thus, it has no sense to talk about the sequence invariant of a given input word if the firings yielded by it cannot be detected.

For example consider the IPN shown in Figure 3.4, where no place is measurable. Let $M_1 = [10]^T$ and $M_2 = [01]^T$ be the reachable markings of this net. Suppose that, $M_0 = M_1$ and the input symbol $\lambda(t_1)$ is given to the net. Since t_1 is enabled and $\lambda(t_1)$ is present, its firing leads to $M = [01]^T$; however, the output does not change. On the other hand, if $M_0 = M_2$ and $\lambda(t_1)$ is given, since t_1 is not enabled it cannot be fired; however the output is the same. Thus, in both cases the output does not provides information to determine the initial marking. Therefore the net is unobservable.

If the observability of this net is tested using the invariant approach, the following sequence invariants can be obtained: $\mathbf{O}_\varnothing = \{M_1, M_2, M_1M_2\}$ and $I_{\lambda(t_1)} = \{M_1M_2\}$. In this case, $\mathbf{O}_\varnothing \cap I_{\lambda(t_1)} = \{M_1M_2\}$, $|\mathbf{O}_\varnothing \cap I_{\lambda(t_1)}| = 1$ and, by Theorem 3.8, the net seems to satisfy observability. However, since the firing of t_1 does not yield a change in the output, it has no sense to talk about the invariant $I_{\lambda(t_1)}$, so the net is unobservable.

Therefore, in order to obtain a valid result in the observability test through sequence invariants it needs to be verified that the firing of any transition affects the output.

Unfortunately, finding out the sets of related markings by the input or by the output words of a given IPN implies a searching algorithm in its reachability set, which can be quite complex. However, for a certain class of IPN, the observability test is relatively simple and, in some cases, observability can be tested from the structure, as it is shown in the next section.

3.3 Characterization of observable IPN

As it will be seen in this section, from definition 3.3, to achieve observability in an IPN two points have to be fulfilled. Firstly, in order to reconstruct M_0 or any reachable marking, it is necessary to have the complete knowledge of the firing sequence generated by an input word given to the net, i.e. to detect the occurrence of each event. Secondly, it is necessary to completely determine the actual marking of the net, the marking M_k, in a finite number of input events, (i.e. $k < \infty$). This leads to the event-detectability and marking-detectability issues of an IPN.

3.3.1 Event-detectability

Definition 3.11 *An IPN given by (Q, M_0) is event-detectable (ED) if every transition firing can be uniquely determined.*

In a SED, the state changes due to the occurrence of events. Thus, if every change of state yields a change in the output symbol, then the occurrence of a system event can be detected through those changes. In IPN terms, it means that, the net has the sufficient amount of measurable places to distinguish the change from one state to another and, in consequence, the firing of all transitions. To proof this affirmation, suppose that M_j and M_k are two reachable consecutive markings, i.e. $M_0 \xrightarrow{\sigma} M_j \xrightarrow{t_k} M_k$. Since t_k is enabled at M_j,

$$\begin{array}{ll} M_k = M_j + C\vec{t_k} & \text{(state equation)} \\ \varphi(M_k) = \varphi(M_j + C\vec{t_k}\ ) & \text{(applying } \varphi \text{ to both sizes)} \\ \varphi(M_k) = \varphi(M_j) + \varphi(C\vec{t_k}\ ) & \text{(since } \varphi \text{ is linear)} \\ \varphi(C\vec{t_k}\ ) = \varphi(M_k) - \varphi(M_j) & \text{(solving for } \varphi(C\vec{t_k}\ )) \end{array}$$

where $\varphi(C\vec{t_k}\ )$ is the k-th column of the matrix formed by the rows of C corresponding to the measurable places of the net. Observe that, the column $\varphi(C\vec{t_k}\ )$ represents the change in the output (i.e. a change in the marking of the measurable places) due to the firing of the transition t_k. In other words, every difference in the system output corresponds to a column in the matrix φC.

Now, the following problems arise: what does it happen if there exists a transition, say t_j, having no effect in the marking of the measurable places, i.e. $\varphi C\vec{t_j} = 0$?; and what does it happen if there is two or more transitions, say t_r, t_s having the same effect in the marking of the measurable places; i.e. $\varphi C\vec{t_r} = \varphi C\vec{t_s}$?. In the first case, it means that, two consecutive markings have the same output symbol, so it is not possible to detect change of state and the firing of t_j. In the second case, the tracking of the system evolution gets ambiguous, since the firings of t_r and t_r may lead to different markings. However, if these transitions have associated different input symbols, then they can be distinguished from each other, precisely by the presence of the corresponding input symbol (remember that, since no time is attached to transitions, it is assumed that only one transition can occur at a time; in the case, that two transitions occur simultaneously, it can be thought that their occurrence is separated for a sufficient small time interval, so they occurred one after another). Thus, to achieve event-detectability, the matrix φC must have no null columns and the repetitive columns must have associated different input symbols as it is formally stated in the following proposition.

Proposition 3.12 *An IPN given by (Q, M_0) is event-detectable if and only if*

1. $\forall i \in [1, 2, \ldots, m]$, $\varphi C(\bullet, i) \neq 0$, *and*
2. $\forall j \neq k \in [1, 2, \ldots, m]$ *such that* $\varphi C(\bullet, j) = \varphi C(\bullet, k)$, $\lambda(t_j) \neq \lambda(t_k)$, *where* $\varphi C(\bullet, x)$ *is the column of* φC *corresponding to the transition* t_k.

Proof. (Sufficiency) Suppose that, $\forall i \in [1, 2, \ldots, m]$, $\varphi C(\bullet, i) \neq 0$. Let $M_r, M_s \in \mathbf{R}(N, M_0)$ be two consecutive markings, such that $M_r \xrightarrow{t_j} M_s$. From the state equation of (Q, M_0), it is easy to see that, $\varphi C(\bullet, j) = \varphi(M_s) - \varphi(M_r)$. Since $\varphi C(\bullet, j) \neq 0$, it follows that, $\varphi(M_s) - \varphi(M_r) \neq 0$ or $\varphi(M_s) \neq \varphi(M_r)$. Thus, $M_s \in O_{\varphi(M_s)}$ and $M_r \in O_{\varphi(M_r)}$. Hence, the firing of t_j produces a change in the net output, so its firing can be detected.

Now, suppose that, $\forall j, k \in [1, 2, \ldots, m]$ with $j \neq k$ such that $\varphi C(\bullet, j) = \varphi C(\bullet, k)$, $\lambda(t_j) \neq \lambda(t_k)$. Let $M_u, M_v, M_w, M_x \in \mathbf{R}(N, M_0)$ and t_j, t_k be two transitions such that $M_u \xrightarrow{t_j} M_v$ and $M_w \xrightarrow{t_k} M_x$. Since $\lambda(t_j) \neq \lambda(t_k)$, $M_u M_v \in I_{\lambda(t_j)}$ and $M_w M_x \in I_{\lambda(t_k)}$. Thus, whether $M_u M_v \in O_{\varphi(M_u)\varphi(M_v)}$ and $M_w M_x \in O_{\varphi(M_u)\varphi(M_v)}$ or not, the firing of t_j can be distinguished from the firing of t_k.

Observe that, in both cases, the firings of t_j and t_j can be uniquely determined. Therefore, (Q, M_0) is event-detectable.

(Necessity) Suppose that, (Q, M_0) is event-detectable. Let t_i be a transition such that $\varphi C(\bullet, i) = 0$ and t_j, t_k be two transitions such that $\varphi C(\bullet, j) = \varphi C(\bullet, k)$ and $\lambda(t_j) = \lambda(t_k)$. In the first case, suppose that $M_r \xrightarrow{t_i} M_s$ where $M_r, M_s \in \mathbf{R}(N, M_0)$. Thus, $\varphi C(\bullet, i) = 0$ implies that $\varphi(M_s) = \varphi(M_r)$, so the firing of t_i does not produces a change in the net output and, therefore, it cannot be detected. In the second case, $\varphi C(\bullet, j) = \varphi C(\bullet, k)$ and $\lambda(t_j) = \lambda(t_k)$ imply that the firing of t_j and t_k have the same effect in the output and both need the same input symbol to get fired. Thus, firing of t_j is confused with the firing of t_k. In both cases, (Q, M_0) is not event-detectable, a contradiction. ■

This result can be explained as follows: if the matrix φC has a null column, it means that the transition associated to this column has no measurable input and output places, so its firing has no effect in the output y and its firing cannot be detected. On the other hand, if there are two equal columns in φC, the corresponding transitions have one or more common measurable input or output places, so their firing produce the same effect in the output y and their firings cannot be distinguished from each other.

Example 3.13 *Consider the IPN of Figure 3.5, where the input and output functions are*

t_i	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}
$\lambda(t_i)$	a	e	b	c	d	g	a	b	c	d	g

$$\varphi = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Thus, the matrix φC is

$$\varphi C = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

In this case, all columns are no null and different from each other, except columns number 1 and 6. However, since $\lambda(t_1) = a$ and $\lambda(t_6) = g$, by Proposition 3.12, the net is event-detectable.

3.3.2 Marking-detectability

Definition 3.14 *An IPN given by (Q, M_0) is marking-detectable (MD) in k steps if and only if $\forall \omega \in \pounds_{in}(Q, M_0)$, such that $|\omega| \leq k < \infty$, the information provided by the knowledge of*

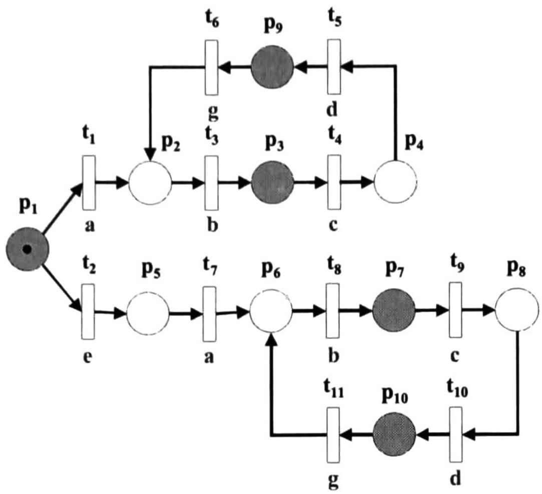

Figure 3.5: An *IPN*.

the input word ω, the output word generated by ω, the system structure and a CML suffices to uniquely determine $M_k \in \mathbf{R}(N, M_0)$, where $M_0 \xrightarrow{\sigma} M_k$ and σ is the firing sequence generated by ω.

In a bounded IPN given by (Q, M_0), the marking of a place has an upper and a lower bounds, which determine the range of different values that the marking of a place can reach.

In a conservative IPN given by (Q, M_0), the marking of a place has an upper and a lower bounds, which determine the range of different values that the marking of a place can reach. Formally, the upper and lower bounds of a place are defined as follows.

Definition 3.15 *Let (Q, M_0) be a conservative IPN. The upper and lower marking bounds that a place p_j can reach given an initial marking M_0 are*

$$\begin{aligned} M^{UB}(p_j) &= \max(M(p_j) | M \in \mathbf{R}(Q, M_0)) \quad \textit{and} \\ M^{LB}(p_j) &= \min(M(p_j) | M \in \mathbf{R}(Q, M_0)) \end{aligned}$$

The range of possible reachable markings of a place p_j is $\mathbb{D}(p_j) = M^{UB}(p_j) - M^{LB}(p_j)$.

In the case of marked graph and state machines, the upper and lower marking bounds of every place can be obtained from the CML by solving the following integer linear programming problems

$$\begin{array}{cc} M^{LB}(p_k) = \min M(p_k) & M^{UB}(p_k) = \max M(p_k) \\ s.t. & s.t. \\ \vec{Y}_1^T M = k_1 & \vec{Y}_1^T M = k_1 \\ \vec{Y}_2^T M = k_2 & \vec{Y}_2^T M = k_2 \\ \ldots & \ldots \\ \vec{Y}_s^T M = k_s & \vec{Y}_s^T M = k_s \end{array}$$

In some class of systems, it is possible to determine when a place reaches its upper or lower marking bound. Thus, its making becomes known, and from this moment, having the knowledge of the subsequent firing sequence the marking of the place remains known. If the above is true for every place and there exists a firing sequence such that all places reach their upper or lower bounds, then the net becomes marking-detectable.

Definition 3.16 *Let (Q, M_0) be an IPN and $G(Q, M_0) \subseteq \pounds(Q, M_0)$. The synchronic distance of a set of transitions $R \subseteq T$ with respect to another set of transitions $S \subseteq T$ in $G(Q, M_0)$ denoted by $SD(G(Q, M_0),\ R,\ S)$, is the maximum number of firings of the transitions of R that can occur in all possible firing sequences of $G(Q, M_0)$ without firing any transition of S.*

If $G(Q, M_0) = \pounds(Q, M_0)$, then the *synchronic distance between* R and S *is simply denoted by* $SD((Q, M_0),\ R,\ S)$.

Once obtained the knowledge of a CML, the upper and lower markings bounds of every place can be computed, [24]. Then, if synchronic distance between the sets of input and output transitions of a place holds, then it is possible to determine that the place has reached either its lower or upper marking bound.

Definition 3.17 *Let (Q, M_0) be an IPN. The set of reachable markings*

$$\mathbb{C} = \{M_i,\ M_j,\ \ldots,\ M_k | M_0 \xrightarrow{\sigma} M_i \xrightarrow{t_q} M_j \xrightarrow{t_r} \ldots \xrightarrow{t_s} M_k \xrightarrow{t_v} M_i\}$$

is called marking cycle and the firing sequence $\sigma_c = t_q t_r t_s t_v$ is called the firing sequence or repetitive stationary sequence of $\mathbb{C}$.

Observe that all the markings belonging to a marking cycle $\mathbb{C}$ are reachable from M_0.

Using the above definitions, the following theorem states that, given an IPN, if the firing sequence of every marking cycle of an IPN leads to the lower or upper marking bound of every non-measurable place then in every cycle there exist a marking that can be determined, so the IPN is marking-detectable.

Theorem 3.18 *Let (Q, M_0) be a live, cyclic, and conservative and event-detectable IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is marking-detectable in k steps if for every marking cycle $\mathbb{C}_i$ of the net it holds that $\forall p_j \in P_{nm}$ either*

i) $SD(\sigma_{C_i}, {}^\bullet(p_j), (p_j)^\bullet) = \mathbb{D}(p_j)$ *or*

ii) $SD(\sigma_{C_i}, (p_j)^\bullet \ {}^\bullet(p_j)) = \mathbb{D}(p_j)$
where $\mathbb{D}(p_j) = M^{UB}(p_j) - M^{LB}(p_j)$ and $SD(\sigma_{C_i}, R, S)$ is the synchronic distance between the set of transitions R and S in the sequence σ_{C_i}.

Proof. From the $CML(Q, M_0)$, $\forall p \in P_{nm}$ the marking bounds $M^{LB}(p)$ and $M^{UB}(p)$ can be computed. Let $p_k \in P_{nm}$ be a non-measurable place and suppose that the net enters to execute the marking cycle $\mathbb{C}_i$ at the marking $M_i \in \mathbb{C}_i$. Let set $M_0 = M_i$. Assume that *i)* holds in $\mathbb{C}_i$, then there exists a firing sequence $\sigma_k \in \overline{\sigma_{C_i}}$, such that the number of firings of transitions in ${}^\bullet(p_k)$ without firing any transition in $(p_k)^\bullet$ is equal to $\mathbb{D}(\mathrm{p}_k)$. The sequence σ_k will eventually occur since the net is executing the cycle $\mathbb{C}_i$. The sequence σ_k can be split into $\sigma_k = \sigma_1\sigma_2$, such that σ_2 does not contain any transition in $(p_j)^\bullet$ and the transitions in ${}^\bullet(p_j)$ appears $\mathbb{D}(\mathrm{p}_k)$ times. Then

$$\begin{aligned} M_j(p_k) &= M_0(p_k) + C(p_k, \bullet)\vec{\sigma_1} + C(p_k, \bullet)\vec{\sigma_2} \\ &= M_n(p_k) + \mathbb{D}(p_k) \\ &= M_n(p_k) + M^{UB}(p_k) - M^{LB}(p_k) \end{aligned}$$

We claim that, $M_n(p_k) = M^{LB}(p_k)$ and $M_j(p_k) = M^{UB}(p_k)$. To prove it, assume that $M_n(p_k) = M^{LB}(p_k) + \triangle M$, this implies that $\triangle M > 0$, then

$$\begin{aligned} M_j(p_k) &= M^{LB}(p_k) + \triangle M + M^{UB}(p_k) - M^{LB}(p_k) \\ &= M^{UB}(p_k) + \triangle M \\ &> M^{UB}(p_k) \end{aligned}$$

which is a contradiction. Thus, after firing σ_k the marking of p_k is $M^{UB}(p_k)$, which can be determined.

Now, assume that *ii)* holds, then there exists a firing transition sequence $\sigma'_k \in \pounds(Q, M_0)$ such that the number of firings of transitions in $(p_k)^\bullet$ without firing any transition in ${}^\bullet(p_k)$ is equal to $\mathbb{D}(\mathrm{p}_k)$. Following a similar procedure like in the previous case, after firing σ'_k the marking of p_k can be determined and remains known for any future evolution since the net is event-detectable.

Moreover, using this procedure, the marking of the remaining non-measurable places can be determined. Since the net is bounded, if an infinite sequence occurs, it means that the net is reaching at least one already visited marking, so the length of the largest marking cycle is finite, say h. Hence, the whole marking will be known after the firing of a transition sequence $\sigma \in \mathcal{L}(Q, M_0)$, which length is at most $k = h$; therefore, (Q, M_0) is marking-detectable. ■

Note that, the test for the existence of finite firing sequences of length equal to or less than k, as stated in the definition of marking-detectability, is made by inspection on the marking cycles. This approach seems to be useless, since the computation of the marking cycles is NP complex. However, for certain classes of PN, in [10], it has been shown that the marking cycles can be obtained from the net structure in polynomial time.

Theorem 3.19 *Let (Q, M_0) be a live, cyclic, conservative and event-detectable IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is marking-detectable in k steps if for every marking cycle $\mathbb{C}_i$ of the net it holds that $\forall \xi_i : Y_i^T M = k_i \in CML(Q, M_0)$, $\exists p_j \in P$ such that $Y_i(p_j) \neq 0$ and $SD((\sigma_{C_i}, {}^\bullet(p_j), (p_j)^\bullet) = k_i$, where $\mathbb{D}(p_j) = M^{UB}(p_j) - M^{LB}(p_j)$ and $SD(\sigma_{C_i}, R, S)$ is the synchronic distance between the set of transitions R and S in the sequence σ_{C_i}.*

Proof. The $CML(Q, M_0)$ can be arranged as:

$$\begin{array}{cccc} \alpha_1^1 M(p_1)+ & \dots & +\alpha_n^1 M(p_n) & = k_1 \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_1^w M(p_1)+ & \dots & +\alpha_n^w M(p_n) & = k_w \end{array} \tag{3.1}$$

where $\begin{bmatrix} \alpha_1^i & \cdots & \alpha_n^i \end{bmatrix}^T = Y_i$ is the i^{th} p-invariant.

Suppose that the net enters to execute the marking cycle $\mathbb{C}_i$ at the marking $M_i \in \mathbb{C}_i$. Let set $M_0 = M_i$. Let $\xi_i : Y_i^T M = k_i$ be the i-th law of $CML(Q, M_0)$ and $p_k \in P$ be a place such that $Y_i(p_j) \neq 0 \neq 0$ and $k_i = M^{UB}(p_k)$. Assume that i) holds, then there exists a firing sequence $\sigma_k \in \overline{\sigma_{C_i}}$ such that the number of firings of transitions in ${}^\bullet(p_k)$ without firing any transition in $(p_k)^\bullet$ is equal to $M^{UB}(\mathrm{p}_k)$. The sequence σ_k will eventually occur since the net is executing the cycle $\mathbb{C}_i$. Then, after the firing of σ_k the marking $M_j(p_k) = k_i$ is reached. Since in a live and bounded PN every reachable marking agrees with the p-invariants [9], $M^{LB}(p_k) = 0$. Thus, $\forall p_r \neq p_k$ such that $Y_i(p_r) \neq 0 \neq 0$, $M_j(p_r) = 0$. Hence, the marking of every place belonging to the support of the i-th p-invariant becomes known and remains known for any future evolution of the net since it is event-detectable. Following a similar procedure for the remaining CML, the whole marking of the net can be completely determined

Since the net is bounded, if an infinite sequence occurs, it means that the net is reaching at least one already visited marking, so the length of the largest marking cycle is finite, say h.

Hence, the whole marking will be known after the firing of a transition sequence $\sigma \in \mathcal{L}(Q, M_0)$ of length $k = h$; therefore, (Q, M_0) is marking-detectable. ■

The following theorem states that if every marking cycle contains a marking whose output is unique then the net is marking-detectable.

Theorem 3.20 *Let (Q, M_0) be a live, cyclic, bounded and event-detectable IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is marking-detectable in k steps if for every marking cycle $\mathbb{C}_i$ $\exists M \in \mathbb{C}_i$ such that $\varphi^{-1}(\varphi(M)) = \{M\}$.*

Proof. Since (Q, M_0) is conservative, by Proposition ??, a CML can be obtained. Suppose that the net enters to execute the marking cycle $\mathbb{C}_i$. Let $M \in \mathbb{C}_i$ be a reachable marking belonging to $\mathbb{C}_i$ such that $\varphi^{-1}(\varphi(M)) = \{M\}$. Thus, when the output $\varphi(M)$ is presented, the only solution for the $CML(Q, M_0)$ is the marking M. Thus, the marking of every non-measurable place becomes known and remains known for any future evolution of the net.

Since the net is conservative, if an infinite sequence occurs, it means that the net is reaching at least one already visited marking, so the length of the largest marking cycle is finite, say h. Hence, the whole marking will be known after the firing of a transition sequence $\sigma \in \mathcal{L}(Q, M_0)$ of length $k = h$; therefore, (Q, M_0) is marking-detectable. ■

Example 3.21 *Consider the live cyclic and bounded IPN and its reachability graph shown in Figure 3.6. The set of measurable places is $P_m = \{p_4, p_5\}$; thus*

$$\varphi C = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 \end{bmatrix}$$

Note that $\varphi C(\bullet, 1) = \varphi C(\bullet, 3)$ but $\varepsilon = \lambda(t_1) \neq \lambda(t_3) = S$. Similarly, $\varphi C(\bullet, 2) = \varphi C(\bullet, 4)$ but $T = \lambda(t_2) \neq \lambda(t_4) = \varepsilon$. Hence, by Proposition 3.12, the net is event-detectable.

The outputs of each marking are the following :

i	1	2	3	4	5	6	7	8
$\varphi(\mathbf{M}_i)$	$[00]^T$	$[10]^T$	$[01]^T$	$[20]^T$	$[11]^T$	$[21]^T$	$[00]^T$	$[10]^T$

Note that the outputs of the markings M_3, M_4, M_5 and M_6 are unique. From Figure 3.6, it is easy to see that at least one of these markings belongs to every marking cycle of the net. Hence, by Theorem 3.20, the net is marking-detectable in a finite number of steps, i.e. $\exists k < \infty$.

3.3.3 Observable IPN

Theorem 3.22 *An IPN given by (Q, M_0) is observable in k steps if and only if it is event-detectable and marking-detectable.*

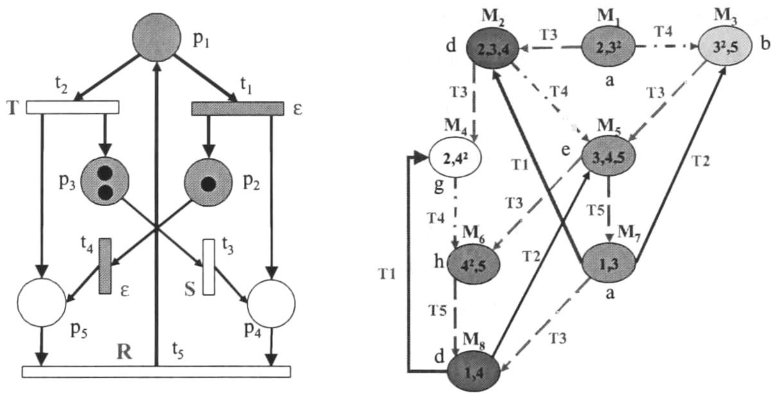

Figure 3.6: A live, bounded and cyclic IPN and its reachability graph.

Proof. (Sufficiency) Since (Q, M_0) is marking detectable in k steps, $\exists k < \infty$ such that M_k is known. Due to the event-detectability property, the firing sequence $\sigma_\omega = t_i t_j \ldots t_v$ yield by the input word ω and the output word $z = \varphi(M_0)\varphi(M_1)\ldots\varphi(M_k)$ are known, where $M_0 \xrightarrow{t_i} M_r \xrightarrow{t_j} \ldots \xrightarrow{t_v} M_k$. It means that, $|O_z \cap I_\omega| = \{M_0 M_r \ldots M_k\}$; otherwise, M_k were undistinguished yet. In this case, M_0 can be computed recursively solving the set of equations:

$$\begin{aligned} M_{i-1} &= M_i - C\overrightarrow{t_v} \\ &\ldots \\ M_1 &= M_2 - C\overrightarrow{t_j} \\ M_0 &= M_1 - C\overrightarrow{t_i} \end{aligned}$$

(observe that the terms $C\overrightarrow{t_i}, C\overrightarrow{t_j}, ..., C\overrightarrow{t_k}$ are known by the event-detectability property). Thus, M_0 can be computed using the information provided by the system structure, input and output sequences.

Now, suppose that, a transition sequence $\sigma_{\omega'} = t_u t_w \ldots t_x$ fires from M_k. Thus, the marking sequence $M_k M_{k+1} \ldots M_{k+i}$ is yield, where $M_k \xrightarrow{t_u} M_{k+1} \xrightarrow{t_w} \ldots \xrightarrow{t_x} M_{k+i}$, and $i = |\sigma_{\omega'}|$. Since the net is event-detectable, every transition firing is be detected and uniquely determined; thus, $M_k M_{k+1} \ldots M_{k+i}$ is the only sequence that yield the output word $y = \varphi(M_k)\varphi(M_{k+1}) \ldots \varphi(M_{k+i})$,.i.e. $O_y = \{M_k M_{k+1} \ldots M_{k+i}\}$. Hence, and $\forall I_{\omega_i}$, $|I_{\omega_i} \cap O_y| = 1$ and the marking M_{k+i} can be uniquely determined.

Therefore, (Q, M_0) is observable in k steps.

(Necessity) Suppose that, (Q, M_0) is not event-detectable. Although a marking M_k, such that $M_0 \xrightarrow{\sigma} M_k$, can be determined, the firing sequence $\sigma \in \mathcal{L}_\omega(Q, M_0)$ cannot be uniquely determined. Thus, it is not possible to compute the initial marking M_0. Hence, (Q, M_0) is not observable in k steps, a contradiction.

Now, suppose that, (Q, M_0) is not marking-detectable in k steps. It means that, for at least a marking $M_k \in \mathbf{R}(N, M_0)$, $\exists M_0 M_i \ldots M_j$ and $M_0 M_r \ldots M_k$ such that $M_0 M_i \ldots M_j$, $M_0 M_r \ldots M_k \in I_\omega$ and $M_0 M_i \ldots M_j$, $M_0 M_r \ldots M_k \in O_z$, where $\omega \in \pounds_{in}(Q, M_0)$ and $z \in \pounds_{out}(Q, M_0)$, with $|\omega|$ infinite. Thus, by Theorem 3.8, (Q, M_0) is unobservable, a contradiction. ■

The following corollaries are derived from the above theorem and from the results of the previous subsection.

Corollary 3.23 *Let (Q, M_0) be a live, cyclic and conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is observable in k steps if it is event-detectable and for every marking cycle $\mathbb{C}_i$ of the net it holds that $\forall p_j \in P_{nm}$ either*

1. $SD(\sigma_{C_i}, {}^\bullet(p_j), (p_j)^\bullet) = \mathbb{D}(p_j)$ *or*
2. $SD(\sigma_{C_i}, (p_j)^\bullet \ {}^\bullet(p_j)) = \mathbb{D}(p_j)$
   where $\mathbb{D}(p_j) = M^{UB}(p_j) - M^{LB}(p_j)$ and $SD(\sigma_{C_i}, R, S)$ is the synchronic distance between the set of transitions R and S in the sequence σ_{C_i}.

Corollary 3.24 *Let (Q, M_0) be a live, cyclic and bounded IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is observable in k steps if it is event-detectable and for every marking cycle $\mathbb{C}_i$ of the net it holds that $\forall \xi_i : Y_i^T M = k_i \in CML(Q, M_0)$ $\exists p_j \in P$ such that $Y_i(p_j) \neq 0$ and $SD((\sigma_{C_i}, {}^\bullet(p_j), (p_j)^\bullet) = k_i$.*
where $\mathbb{D}(p_j) = M^{UB}(p_j) - M^{LB}(p_j)$ and $SD(\sigma_{C_i}, R, S)$ is the synchronic distance between the set of transitions R and S in the sequence σ_{C_i}.

Corollary 3.25 *Let (Q, M_0) be a live, cyclic and conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is observable in k steps if it is event-detectable and for every marking cycle $\mathbb{C}_i$ $\exists M \in \mathbb{C}_i$ such that $\varphi^{-1}(\varphi(M) = \{M\}$.*

3.4 Examples

Example 3.26 *Consider the IPN of Figure 3.2. Since the measurable places are $P_m = \{p_4, p_5\}$,*

$$\varphi C = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 \end{bmatrix}$$

Observe that the 1st and 3th columns are equal, $\varphi C(\bullet, t_1) = \varphi C(\bullet, t_3)$, but $a = \lambda(t_1) \neq \lambda(t_3) = c$. Similarly, $\varphi C(\bullet, t_2) = \varphi C(\bullet, t_4)$, but $b = \lambda(t_2) \neq \lambda(t_4) = d$. Hence, by Proposition 3.12, the net is event-detectable.

It is easy to show that, the net is live, cyclic and 1-bounded. The marking $M_4 = [4,5]$ is the only marking in the reachability set that has the output $\varphi(M_4) = [11]^T$ Thus, by Corollary 3.25, the net is observable in a finite number of steps, as indicated in Example 3.9.

Example 3.27 *The IPN of Figure 3.3 is live, cyclic and 1-bounded; however, since $P_m = \{p_4, p_5\}$ and $T_c = \{t_2, t_4, t_5\}$, thus,*

$$\varphi C = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 \end{bmatrix}$$

and $\varphi C(\bullet, t_1) = \varphi C(\bullet, t_3)$ with $\lambda(t_1) = \lambda(t_3) = \varepsilon$. Hence, by Proposition 3.12, the net is not event-detectable. Therefore, by Theorem 3.22, the net is unobservable in k steps, the same result of Example 3.10.

3.5 Discussion

In this chapter, the observability problem in DES modeled by IPN was addressed. Firstly, a definition of observability in Interpreted Petri Net (IPN) terms was provided. To characterize observability, the concepts of input and output sequence invariants of an IPN were introduced. This characterization is similar to the one for linear continuous systems using a geometric approach. Although, the provided characterization represents a necessary and sufficient condition to determine whether or not a given IPN satisfies observability, it leads to algorithms with high computational complexity. However for live, cyclic and bounded IPN, the test for observability can be reduced to the verification of the event-detectability and marking-detectability. The property of event-detectability can be tested in polynomial time. Thus, several necessary and sufficient conditions for event-detectability, and sufficient conditions for marking-detectability were provided.

The conditions for marking-detectability are based on the concepts of the set of marking conservative laws of a IPN and the synchronic distance among the transitions of the net The computation of the synchronic distance in every firing sequence has a high complexity. However, in the approach herein presented the synchronic distance conditions are only tested in every elemental marking cycle that, for certain classes of PN, can be obtained from the net structure. Thus, the verification of the synchronic distance conditions can be performed in a polynomial time.

Chapter 4

W-observability

SUMMARY. In this chapter, the concepts of W-observability and W-marking-detectability are introduced. It is shown that, event-detectability and W-marking-detectability are necessary and sufficient conditions for W-observability. In this context, several sufficient conditions for W-marking-detectability are provided. In particular, it is shown that event-detectability is a necessary and sufficient condition for W-observability in live, cyclic and bounded interpreted free-choice nets.

4.1 Introduction

The notion of observability presented in the previous chapter seems to be too restrictive, since it implies the possibility of determine a reachable marking in every cyclic behavior of the net. This chapter addresses a less restrictive notion of observability called W-observability. This notion of observability is related with the possibility of determining at least a reachable marking M and, then the initial marking M_0 and every subsequent reachable marking from M. In order to establish a characterization of W-observable IPN the concept of W-marking-detectability is introduced. Several sufficient conditions for this property are presented. This conditions lead to some characterizations of W-observable IPN. In particular, it is shown that event-detectability is a necessary and sufficient condition for W-observability in live, cyclic and bounded interpreted free-choice nets.

4.2 Basic definitions

In the previous chapter, it has been shown that an IPN is observable if and only if it is event-detectable and marking-detectable. However, marking-detectability seems to be a very restrictive condition since it implies the possibility of determine the actual marking of the net in every cyclic behavior. In this case, only a few IPN are observable in the strict sense. In order to overcome this restriction, the concept of W-observability is introduced. the property of W-observability implies the existence of at least a finite input word ω such that the initial and every reached marking by ω can be completely determined.

Definition 4.1 *An IPN given by (Q, M_0) is W-observable in k steps if and only if $\forall\omega \in \pounds_{in}(Q, M_0)$, $\exists z$ such that $\omega z \in \pounds_{in}(Q, M_0)$ with $|z| \leq k < \infty$ and the information provided by the knowledge of the input word ωz, the output word generated by ωz the system structure and $CML(Q.M_0)$ suffices to uniquely determine M_0 and the firing sequence generated by ωz $\sigma = t_i t_j \ldots t_r$.*

It means that, there exists the possibility of complete every input word to a word such that the reached marking can be determined.

4.3 Characterization of W-observable IPN

Similarly as for observability, to achieve W-observability two points must be fulfilled. Firstly, to detect the occurrence of each system event; and, secondly, it is necessary to completely determine the actual marking of the net, the marking M_k, in a finite number of input events, (i.e. $k < \infty$). The first condition is related to the concept of event-detectability introduced in the previous chapter. The second point is related to the concept of marking-detectability. However, since W-observability only requires the existence of a finite input word, the concept of W-marking-detectability is introduced.

4.3.1 W-marking-detectability

Definition 4.2 *An IPN given by (Q, M_0) is w-marking-detectable (MD) in k steps if and only if $\forall \omega \in \mathcal{L}_{in}(Q, M_0)$, $\exists z$ such that $|z| \leq k < \infty$, $\omega z \in \mathcal{L}_{in}(Q, M_0)$ and the information provided by the knowledge of the input word ωz, the output word generated by ωz the system structure and $CML(Q, M_0)$ suffices to uniquely determine $M_k \in \mathbf{R}(N, M_0)$, where $M_0 \xrightarrow{\sigma} M_k$ and σ is the firing sequence generated by ωz.*

Observe that, W-observability implies the existence of a finite input word whose occurrence leads to determine the reached marking.

The following results represent sufficient conditions for W-marking-detectability. This conditions are similar to the ones for marking-detectability presented in the previous chapter, with the difference that they must hold for the whole net and not for every marking cycle.

Theorem 4.3 *Let (Q, M_0) be a live, cyclic, conservative and event-detectable IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is w-marking-detectable in k steps if $\forall p_j \in P_{nm}$ either 1) $SD((Q, M_0),\ ^\bullet(p_j),\ (p_j)^\bullet) = \mathbb{D}(p_j)$ or 2) $SD((Q, M_0),\ (p_j)^\bullet\ \ ^\bullet(p_j)) = \mathbb{D}(p_j)$, where $\mathbb{D}(p_j) = M^{UB}(p_j) - M^{LB}(p_j)$.*

Proof. Let $p_k \in P_{nm}$ be a non-measurable place. Assume that $i)$ holds, then there exists a firing sequence $\sigma_k \in \mathcal{L}(Q, M_0)$ such that the number of firings of transitions in $^\bullet(p_k)$ without firing any transition in $(p_k)^\bullet$ is equal to $\mathbb{D}(p_k)$. If σ_k does not occurs immediately from M_0, since (Q, M_0) is cyclic, then it will return to M_0 and eventually σ_k will occur. The sequence σ_k can be split into $\sigma_k = \sigma_1\sigma_2$, such that σ_2 does not contain any transition in $(p_j)^\bullet$ and the transitions in $^\bullet(p_j)$ appears $\mathbb{D}(p_k)$ times. Then

$$\begin{aligned} M_j(p_k) &= M_0(p_k) + C(p_k, \bullet)\vec{\sigma_1} + C(p_k, \bullet)\vec{\sigma_2} \\ &= M_n(p_k) + \mathbb{D}(p_k) \\ &= M_n(p_k) + M^{UB}(p_k) - M^{LB}(p_k) \end{aligned}$$

We claim that, $M_n(p_k) = M^{LB}(p_k)$ and $M_j(p_k) = M^{UB}(p_k)$. To prove it, assume that $M_n(p_k) = M^{LB}(p_k) + \triangle M$, this implies that $\triangle M > 0$, then

$$\begin{aligned} M_j(p_k) &= M^{LB}(p_k) + \triangle M + M^{UB}(p_k) - M^{LB}(p_k) \\ &= M^{UB}(p_k) + \triangle M \\ &> M^{UB}(p_k) \end{aligned}$$

which is a contradiction. Thus, after firing σ_k the marking of p_k is $M^{UB}(p_k)$, which can be determined.

Now, assume that *ii)* holds, then there exists a firing transition sequence $\sigma'_k \in \pounds(Q, M_0)$ such that the number of firings of transitions in $(p_k)^\bullet$ without firing any transition in ${}^\bullet(p_k)$ is equal to $\mathbb{D}(\mathrm{p}_k)$. Following a similar procedure like in the previous case, after firing σ'_k the marking of p_k can be determined and remains known for every future evolution since the net is event-detectable.

Using this procedure, the marking of the remaining non-measurable places can be determine after the firing of the sequence $\sigma = \sigma_k u \sigma_j \ldots u \sigma_c$, where σ_k, σ_j, $\ldots$, σ_c are the sequences satisfying the synchronic distance conditions for each non-measurable place respectively and $u, \ldots, v \in \pounds(Q, M_0)$. Hence, since $z = \lambda(\sigma) \in \pounds_{in}(Q, M_0)$ and $|z| < \infty$, then (Q, M_0) is w-marking-detectable in k steps. ■

Conditions i) and ii) of the previous theorem are called the synchronic distance conditions for W-marking-detectability (SD conditions). For example, the first condition indicates that, for every non-measurable place of the net p_j, there exists a firing sequence σ_j such that the number of occurrences of the transitions in ${}^\bullet(p_j)$ without firing any transition of $(p_j)^\bullet$ is equal to $M^{UB}(p_j) - M^{LB}(p_j)$, so after the firing of σ_j, it is known that the marking of the place p_j is equal to its upper bound, $M^{UB}(p_j)$.

Theorem 4.4 *Let (Q, M_0) be a conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is w-marking-detectable in k steps if $\exists M \in \mathbf{R}(N, M_0)$ such that $\varphi^{-1}(\varphi(M)) = \{M\}$.*

Proof. Let $M \in \mathbf{R}(N, M_0)$ such that $\varphi^{-1}(\varphi(M)) = \{M\}$. Thus, when the output $\varphi(M)$ is presented, the only solution for the CML is the marking M. Thus, the marking of every non-measurable place becomes known. Therefore, (Q, M_0) is w-marking-detectable. ■

Theorem 4.5 *Let (Q, M_0) be a live, cyclic, conservative and event-detectable IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is w-marking-detectable in k steps if $\forall \xi_i : Y_i^T M = k_i \in CML(Q, M_0)$, $\exists p_j \in P$ such that $Y_i(p_j) \neq 0$ and $SD((Q, M_0), {}^\bullet(p_j), (p_j)^\bullet) = k_i$.*

Proof. Let $\xi_i : Y_i^T M = k_i$ be the i-th conservative marking law and $p_k \in P$ be a place such that $Y_i(p_j) \neq 0$ and $SD((Q, M_0), {}^\bullet(p_j), (p_j)^\bullet) = k_i$. Then, there exists a firing sequence $\sigma_k \in \mathcal{L}(Q, M_0)$ such that the number of firings of transitions in ${}^\bullet(p_k)$ without firing any transition in $(p_k)^\bullet$ is equal to k_i. If σ_k does not occurs immediately from M_0, since (Q, M_0) is cyclic, then it will return to M_0 and eventually σ_k will occur. Then, after the firing of σ_k the marking $M_j(p_k) = k_i$ is reached. Thus, $\forall p_r \neq p_k$ such that $Y_i(p_r) \neq 0$, $M_j(p_r) = 0$. Hence, the marking of every place belonging to the support of the i-th conservative marking law becomes known and remains known for every future evolution since the net is event-detectable. Following a similar procedure for the remaining CML, the whole marking of the net can be completely determined after the firing of the sequence $\sigma = \sigma_k u \sigma_j \ldots u \sigma_c$, where σ_k, σ_j, ..., σ_c are the sequences satisfying the synchronic distance conditions for each non-measurable place respectively and $u, \ldots, v \in \mathcal{L}(Q, M_0)$. Hence, since $z = \lambda(\sigma) \in \mathcal{L}_{in}(Q, M_0)$ and $|z| < \infty$, then (Q, M_0) is w-marking-detectable in k steps. ■

Unfortunately, the synchronic distance conditions in the previous theorems depend on the initial marking of the net, which is unknown. However, for live, cyclic and 1-bounded *IPN* an for a large class of *IPN* called Interpreted Free-Choice Petri nets (*IFCN*), these conditions are implicitly satisfied as stated in the following corollary and theorem.

Corollary 4.6 *Let (Q, M_0) be a live and cyclic IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is w-marking-detectable in k steps if it is 1-bounded and event-detectable.*

Proof. Since the net is 1-bounded, the synchronic distance between the input and output transitions of each place is 0 or 1. Thus, $\forall p \in P$, $SD((Q, M_0), {}^\bullet(p), (p)^\bullet) = 1 = M^{UB}(p_j) - M^{LB}(p_j)$. Therefore, the SD conditions of Theorem 4.5 hold and, (Q, M_0) is marking-detectable.
■

Definition 4.7 *Let (Q, M_0) be an IPN. A subnet S_i of Q generated by a nonempty set of S of nodes is a S-component of Q if $\forall p \in S$, ${}^\bullet(p)^\bullet \subseteq S$, and S_i is a strongly connected state machine.*

Definition 4.8 *Let (Q, M_0) be an IPN and S_i be a S-component of Q. The characteristic vector of a S-component S_i is $\vec{\chi}_i$, where if $p_j \in S$ then $\chi_i(p_j) = 1$ and $\chi_i(p_j) = 0$, otherwise.*

Proposition 4.9 *Let (Q, M_0) be a live, cyclic, structurally bounded Interpreted Free-choice Petri Net (IFCN). Let $\mathbb{S}=\{S_1, S_2, \ldots, S_h\}$ be the set of all the S-components of Q and $\mathbb{Y}=\{\vec{Y}_1, \vec{Y}_2, \ldots, \vec{Y}_k\}$ the set of all the minimal p-invariant of Q. Then, $\forall p_j \in P$*

$$\min_{p_j}\{\chi_i^T M_0 | S_i \in \mathbb{S} \wedge \chi_i(p_j) = 1\} = \min_{p_j}\{Y_i^T M_0 | Y_i \in \mathbb{Y} \wedge Y_i(p_j) = 1\}$$

Proof. By Proposition 5.7 of [9]. $\forall S_i \in \mathbb{S}$, $\vec{\chi}_i$ is a minimal p-invariant of Q and, since (Q, M_0) is live and bounded, by Theorem 5.6 of [9], Q is covered by S-components, i.e. $\forall p \in P$ $\exists S_i \in \mathbb{S}$ such that $\chi_i(p) = 1$. Thus, $\mathbb{X} = \{\vec{\chi}_i | S_i \in \mathbb{S}\} \subseteq \mathbb{Y}$.

By Theorem 5.9 of [9], the upper marking bound of a place $p \in P$ is $M^{UB}(p) = r = \min\{\chi_i^T M_0 | S_i \in \mathbb{S} \wedge \chi_i(p) = 1\}$. Observe that, $\mathbf{R}(N, M_0) \subseteq \{M | Y^T M = Y^T M_0\}$. Thus, in the general case, $r \geq \min\{Y_i^T M_0 | Y_i \in \mathbb{Y} \wedge Y_i(p) = 1\}$.

We prove that $r > \min\{Y_i^T M_0 | Y_i \in \mathbb{Y} \wedge Y_i(p) = 1\}$ is false. Suppose that, it is true. By Theorem 5.9 of [9], the marking M_k, where $M_k(p) = r$, is a reachable marking. In this case, by Theorem 9.6 of [9], the marking M_k must agree with the p-invariants of the net, i.e. $Y^T M_k = Y^T M_0$. However, in this case, $Y^T M_k > Y^T M_0$. It means that, M_k is not reachable and $r > M^{UB}(p)$, a contradiction. Thus, $r \leq \min\{Y_i^T M_0 | Y_i \in \mathbb{Y} \wedge Y_i(p) = 1\}$.

Therefore, $\min_p\{\chi_i^T M_0 | S_i \in \mathbb{S} \wedge \chi_i(p) = 1\} = \min_p\{Y_i^T M_0 | Y_i \in \mathbb{Y} \wedge Y_i(p) = 1\}$ ■

Theorem 4.10 *Let (Q, M_0) be a live, cyclic, structurally bounded Interpreted Free-choice Petri Net ($IFCN$), where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is w-marking-detectable in k steps if it is event-detectable.*

Proof. Since (Q, M_0) is live and bounded, by Proposition 4.9, $\forall p \in P$, $M^{UB}(p) = \min\{Y_i^T M_0 | Y_i \in \mathbb{Y} \wedge Y_i(p) = 1\}$. By Theorem 5.9, the marking $M_k(p_r) = M^{UB}(p_r)$ can be reached and there exists $\xi_i : Y_i^T M = k_i \in CML$ such that $Y_i(p_r) \neq 0$ and $M^{UB}(p_r) = k_i$. In this case, when M_k is reached, $\forall p_j \neq p_i \in ||\xi_i||$, $M_k(p_j) = 0$. Hence, $\exists \eta = \sigma\sigma_r \in \mathcal{L}(Q, M_0)$ such that $M_0 \xrightarrow{\sigma} M \xrightarrow{\sigma_r} M_k$ where σ_r is the required sequence of Theorem 4.5.

Now, if M_k is set as the initial marking, then this procedure can be repeated until the marking of the all non-measurable places becomes known. Therefore, (Q, M_0) is w-marking-detectable. ■

Another important corollary of Theorem 3.18 states that if a IPN is live, cyclic and 1-bounded, then the synchronic distance conditions hold and, hence, it is w-marking-detectable.

Example 4.11 *Consider the strongly connected IPN of Figure 4.1. The set of measurable places is $P_m = \{p_1, p_3, p_5, p_7, p_9, p_{11}\}$; thus,*

$$\varphi C = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

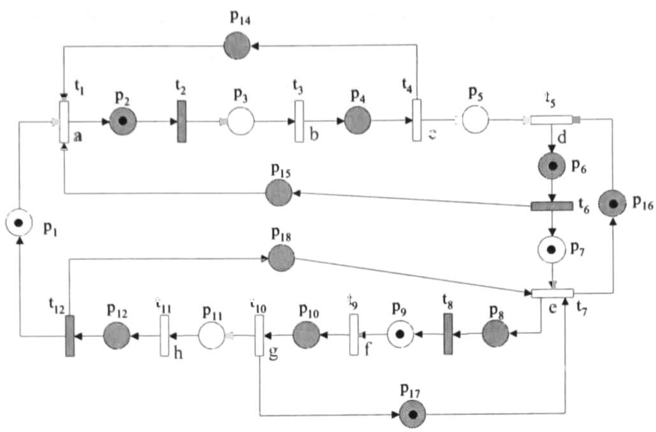

Figure 4.1: A live, cyclic and bounded IPN.

Since there is no null columns and all of them are different from each other, by Proposition 3.12, the net is event-detectable. The following CML can be obtained from its p-invariants:

$$
\begin{array}{lrl}
1) & M(p_1)+M(p_2)+M(p_3)+M(p_4)+M(p_5)+M(p_6)+ & \\
 & M(p_7)+M(p_8)+M(p_9)+M(p_{10})+M(p_{11})+M(p_{12}) & =4\\
2) & M(p_2)+M(p_3)+M(p_4)+M(p_{14}) & =1\\
3) & M(p_2)+M(p_3)+M(p_4)+M(p_5)+M(p_6)+M(p_{15}) & =1\\
4) & M(p_6)+M(p_7)+M(p_{16}) & =3\\
5) & M(p_8)+M(p_9)+M(p_{10})+M(p_{11})+M(p_{12})+M(p_{18}) & =1\\
6) & M(p_8)+M(p_9)+M(p_{10})+M(p_{17}) & =2
\end{array}
$$

Thus, $\forall p_i \in P$ the lower and upper marking bounds are

$$
M^{UB}(p_i)=\begin{cases} 4, & i\in\{1\}\\ 3, & i\in\{7,16\}\\ 2, & i\in\{17\}\\ 1, & otherwise\end{cases}
\qquad
M^{LB}(p_i)=\begin{cases} 1, & i\in\{17\}\\ 0, & otherwise\end{cases}
$$

Let us select a set of places such that $\forall[\alpha^i_j M(p_j)=k_i]\in CML$, $\alpha^i_j\neq 0$, for example the set $\pi=\{p_3,p_{11},p_{16},p_{17}\}$. The synchronic distance between the input and output transitions of each place of π is

j	$^\bullet(p_j)$	$(p_j)^\bullet$	$SD((Q,M_0),^\bullet(p_j),(p_j)^\bullet)$	$\mathbb{D}(p_j)$
3	$\{t_2\}$	$\{t_3\}$	1	1
11	$\{t_{10}\}$	$\{t_{11}\}$	1	1
16	$\{t_7\}$	$\{t_5\}$	3	3
17	$\{t_{10}\}$	$\{t_7\}$	1	1

Observe that, for each place of π, the synchronic distance SD is equal to range of possible reachable markings $\mathbb{D}$. Hence, by Theorem 4.5, the net is w-marking-detectable. In fact, for example the marking $M_k = [1^2, 3, 11, 6^3, 17^2] \in \mathbf{R}(N, M_0)$ can be determined.

4.3.2 W-observable *IPN*

Theorem 4.12 *An IPN given by (Q, M_0) is W-observable in k steps if and only if it is event-detectable and w-marking-detectable in k steps.*

Proof. (Sufficiency) Since (Q, M_0) is w-marking-detectable in k steps, $\exists k < \infty$ such that M_k is known. Due to the event-detectability property, the firing sequence $\sigma_\omega = t_i t_j \ldots t_v$ yield by the input word ω and the output word $z = \varphi(M_0)\varphi(M_1)\ldots\varphi(M_k)$ are known, where $M_0 \xrightarrow{t_i} M_r \xrightarrow{t_j} \ldots \xrightarrow{t_v} M_k$. It means that, $|O_z \cap I_\omega| = \{M_0 M_r \ldots M_k\}$; otherwise, M_k were undistinguished yet. In this case, M_0 can be computed recursively solving the set of equations: $\mathbb{M} = \{M_{i-1} = M_i - C\overrightarrow{t_v}, \ldots, M_1 = M_2 - C\overrightarrow{t_j}, M_0 = M_1 - C\overrightarrow{t_i}\}$. Observe that the terms $C\overrightarrow{t_i}, C\overrightarrow{t_j}, \ldots, C\overrightarrow{t_k}$ are known by the event-detectability property. Thus, M_0 can be computed using the information provided by the system structure, input and output sequences.

Now, suppose that, a transition sequence $\sigma_{\omega'} = t_u t_w \ldots t_x$ fires from M_k. Thus, the marking sequence $M_k M_{k+1} \ldots M_{k+i}$ is yield, where $M_k \xrightarrow{t_u} M_{k+1} \xrightarrow{t_w} \ldots \xrightarrow{t_x} M_{k+i}$, and $i = |\sigma_{\omega'}|$. Since the net is event-detectable, every transition firing is be detected and uniquely determined; thus, $M_k M_{k+1} \ldots M_{k+i}$ is the only sequence that yield the output word $y = \varphi(M_k)\varphi(M_{k+1})\ldots\varphi(M_{k+i})$,.i.e. $O_y = \{M_k M_{k+1} \ldots M_{k+i}\}$. Hence, and $\forall I_{\omega_i}$, $|I_{\omega_i} \cap O_y| = 1$ and the marking M_{k+i} can be uniquely determined.

Therefore, (Q, M_0) is W-observable.

(Necessity) Suppose that, (Q, M_0) is not event-detectable. Although a marking M_k, such that $M_0 \xrightarrow{\sigma} M_k$, can be determined, the firing sequence $\sigma \in \mathcal{L}_\omega(Q, M_0)$ cannot be uniquely determined. Thus, it is not possible to compute the initial marking M_0. Hence, (Q, M_0) is not W-observable, a contradiction.

Now, suppose that, (Q, M_0) is not w-marking-detectable in k steps. It means that, for at least a marking $M_k \in \mathbf{R}(N, M_0)$, $\exists M_0 M_i \ldots M_j$ and $M_0 M_r \ldots M_k$ such that $M_0 M_i \ldots M_j$, $M_0 M_r \ldots M_k \in I_\omega$ and $M_0 M_i \ldots M_j$, $M_0 M_r \ldots M_k \in O_z$, where $\omega \in \mathcal{L}_{in}(Q, M_0)$ and $z \in \mathcal{L}_{out}(Q, M_0)$ Thus, by Theorem 3.8, (Q, M_0) is not W-observable in k steps, a contradiction. ∎

The proves of the following corollaries are straightforward from the previous Theorem and the results on W-marking-detectability of the previous subsection.

Corollary 4.13 *Let (Q, M_0) be a live, cyclic and bounded IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is W-observable in k steps if it is event-detectable and $\forall p_j \in P_{nm}$ either $SD((Q, M_0), {}^\bullet(p_j), (p_j)^\bullet) = \mathbb{D}(p_j)$ or $SD((Q, M_0), (p_j)^\bullet\ {}^\bullet(p_j)) = \mathbb{D}(p_j)$, where $\mathbb{D}(p_j) = M^{UB}(p_j) - M^{LB}(p_j)$.*

Corollary 4.14 *Let (Q, M_0) be a live, cyclic and conservative IPN, where M_0 is unknown. (Q, M_0) is W-observable in k steps if it is event-detectable and it holds that $\forall \xi_i : Y_i^T M = k_i \in CML(Q, M_0)$ $\exists p_j \in P$ such that $Y_i(p_j) \neq 0$ and $SD(((Q, M_0), {}^\bullet(p_j), (p_j)^\bullet) = k_i$.*

Corollary 4.15 *Let (Q, M_0) be a live, cyclic and 1-bounded IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is W-observable in k steps if it is event-detectable.*

Corollary 4.16 *Let (Q, M_0) be a live, cyclic and conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is observable in k steps if it is event-detectable and $\exists M \in \mathbf{R}(N, M_0)$ such that $\varphi^{-1}(\varphi(M)) = \{M\}$.*

Corollary 4.17 *Let (Q, M_0) be a live, cyclic and conservative $IFCN$, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained.. (Q, M_0) is W-observable in k steps if it is event-detectable*

4.4 Example

Consider the live, bounded and cyclic IPN of Figure 4.2. The measurable places are $P_m = \{p_1, p_2, p_4, p_6, p_8, p_{10}\}$ and the controllable transition are $T_c = \{t_1, t_2, t_4, t_6, t_9, t_{11}, t_{12}\}$. In this case, the output matrix is

$$\varphi C = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Thus, by Proposition 3.12, the net is event-detectable. Consider the reachable marking $M = [1, 2, 6]$ whose output is $\varphi(M) = [1\ 1\ 0\ 1\ 0\ 0]^T$ It can be shown that, $\forall M'\mathbf{R}(N, M_0)$ with $M' \neq M$, it holds that $\varphi(M') \neq \varphi(M)$. Hence, by Corollary 4.16, the net is W-observable in a finite number of steps.

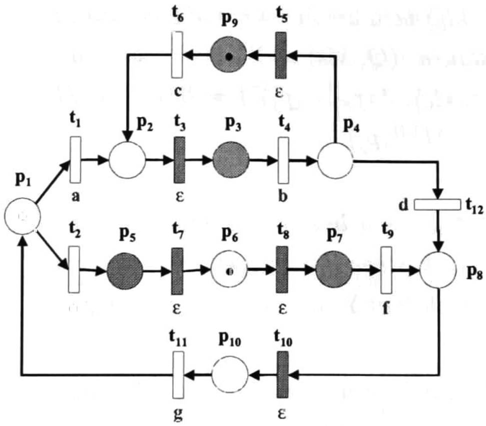

Figure 4.2: An live, bounded and cyclic IPN.

4.5 Discussion

In this chapter, the concepts of W-observability in k steps was introduced. In order to characterize W-observable IPN, the property of W-marking-detectability was introduced. The property of W-marking-detectability is a less restrictive notion than marking-detectability since the former allows a net to enter an infinite cyclic behavior in which the actual marking cannot be determined, only if there exists the possibility of exit the cyclic behavior and reach a identifiable marking.

It was shown that, event-detectability and W-marking-detectability represent a necessary and sufficient condition for W-observability. In this context, several sufficient conditions for W-marking-detectability were provided. In particular, it is shown that event-detectability is a necessary and sufficient condition for W-observability in live, cyclic and bounded interpreted free-choice nets. All these results are based on the concept of synchronic distance among the input and output transitions of the places of the net and the knowledge of a set of conservative marking laws. This allows to verify that a place has reached it upper or lower marking bound, so its marking becomes known.

The concept of W-observability is less restrictive than the concept of observability since the latter implies the possibility of determine a reachable marking in every cyclic behavior of the net and W-observability only implies the existence of a finite input word whose occurrence leads to determine the reached marking, so a net is allowed to enter infinite cycle behaviors where no reachable marking can be determined. In this context, note that observability implies W-observability but not the contrary.

Chapter 5

Observability with respect to a Language

SUMMARY. This chapter addresses the observability problem for IPN whose behavior is confined into a sublanguage of the firing language. Thus, concept of observability with respect to a prefix closed sublanguage is introduced, L-observability for short. A characterization of this kind of observability is derived.

5.1 Introduction

In the two previous chapters, the observability and w-observability problems were studied under the assumption that any system firing sequence can be fired. However, when the system behavior is confined into a prefix closed firing sublanguage $L(Q, M_0)$, only a part of the system is involucrate. In this case, the tests for observability performed in the whole net may return wrong results. Then, it is necessary to establish under which conditions the actual system state can be recovered in a finite-length word of the sublanguage $L(Q, M_0)$. Note that although, the system can be observable or w-observable, when it is confined into a prefix closed sublanguage, say $L(Q, M_0) \subseteq \pounds(Q, M_0)$, it could be the case of no firing sequence $\sigma \in L(Q, M_0)$ leads to a distinguishable marking, so the system is not marking-detectable and therefore unobservable with respect to $L(Q, M_0)$. It is assumed that, $L(Q, M_0)$ is realizable language, i.e. the IPN can be forced to execute only the sequences belonging to $L(Q, M_0)$. In particular, the case of an IPN confined to a realizable cyclic sublanguage is addressed in this chapter.

5.2 Basic definitions

Definition 5.1 *Let (Q, M_0) be an IPN and $G_{in}(Q, M_0)$ be a prefix closed sublanguage of $\pounds_{in}(Q, M_0)$. (Q, M_0) is observable in k steps with respect to $G_{in}(Q, M_0)$ if and only if $\forall \omega \in G_{in}(Q, M_0) \subseteq \pounds_{in}(Q, M_0)$, $\exists z$ with $k \leq |z| < \infty$ such that $\omega z \in G_{in}(Q, M_0)$ and the information provided by the knowledge of ωz, the output word generated by ωz the system structure and $CML(Q, M_0)$ suffices to uniquely determine M_0 andthe firing sequence generated by ωz, $\sigma = t_i t_j \ldots t_r$.*

Observe that, the input sublanguage $G_{in}(Q, M_0)$ generates a firing language; thus, in this chapter, the concept of *observable with respect to* a firing sublanguage is used instead of observable with respect to an input sublanguage.

In Chapter 3, it is shown that a live, conservative and cyclic IPN is observable if and only if it is both event-detectable and marking-detectable. In that chapter, it is concluded that, event-detectability depends on the net structure; while marking-detectability depends on the initial marking, i.e. it is a behavioral property.

Since a firing sublanguage $G(Q, M_0)$ involucrates only a subset of the transitions of the net, say $T_G \subseteq T$, then it is only necessary to test event-detectability for the transitions belonging to T_G. Similarly, since $G(Q, M_0) \subseteq \pounds(Q, M_0)$, for testing marking-detectability it is only necessary to verify if the synchronic distance conditions hold in the language $G(Q, M_0)$.

Definition 5.2 *Let (Q, M_0) be an IPN and $L(Q, M_0)$ be a prefix closed sublanguage of $\mathcal{L}(Q, M_0)$. Let $T_L = \{t \in Q | t$ is part of any $\sigma \in L(Q, M_0)\}$ be the set of transitions belonging to $L(Q, M_0)$. The net (Q, M_0) is event-detectable with respect to $L(Q, M_0)$, or L-event-detectable for short, if the firing of every transition $t \in T_L$ can be uniquely detected.*

In this case, the incidence matrix of an IPN can be split into two matrices: $C = [C^L \vdots C^{\bar{L}}]$, where C^L is formed by the columns of C corresponding to the transitions belonging to T_L, while $C^{\bar{L}}$ is formed by the columns of C corresponding to the transitions no belonging to T_L.

Definition 5.3 *Let (Q, M_0) be an IPN and $L(Q, M_0)$ be a prefix closed sublanguage of $\mathcal{L}(Q, M_0)$. The net (Q, M_0) is marking-detectable in k steps with respect to $L(Q, M_0)$, or L-marking-detectable for short, if and only if $\forall \sigma \in L(Q, M_0)$, $\exists \delta$ such that $|\delta| \leq k < \infty$, $\sigma\delta \in L(Q, M_0)$ and the information provided by the knowledge of $\sigma\delta$, the output word generated by ωz, the system structure and a CML suffices to uniquely determine $M_k \in \mathbf{R}(N, M_0)$, where $M_0 \xrightarrow{\sigma\delta} M_k$.*

5.3 Characterization of L-observable IPN

The following characterizations of L-observable IPN are particularizations of the results presented in Chapter 3, considering that if the behavior of an IPN is confined into a cyclic firing sublanguage $L(Q, M_0)$, then it means that every firing sequence of $L(Q, M_0)$ will be occur at least once.

5.3.1 L-event-detectable IPN

Proposition 5.4 *Let (Q, M_0) be an IPN, $L(Q, M_0)$ be a sublanguage of $\mathcal{L}(Q, M_0)$ and is L-event-detectable if and only if*

1. $\forall i \in [1, 2, \ldots, m]$, $\varphi C^L(\bullet, i) \neq 0$, *and*

2. $\forall j \neq k \in [1, 2, \ldots, m]$ *such that* $\varphi C^L(\bullet, j) = \varphi C^L(\bullet, k)$, $\lambda(t_j) \neq \lambda(t_k)$,

   where $C^L(\bullet, i)$ is the i-th column of the matrix C^L

Proof. Similar to the proof of Proposition 3.12 considering that only the transitions belonging to T_L are being analyzed. ■

5.3.2 L-marking-detectable IPN

Definition 5.5 *Let (Q, M_0) be an IPN and $L(Q, M_0)$ be a sublanguage of $\pounds(Q, M_0)$. The set of reachable markings by $L(Q, M_0)$ is*

$$\mathbf{R}_L(Q, M_0) = \{M \in \mathbf{R}(N, M_0) | M_0 \xrightarrow{\sigma} M \wedge \sigma \in L(Q, M_0)\}$$

Definition 5.6 *Let (Q, M_0) be an IPN. A language $L\ (Q, M_0) \subseteq \pounds(Q, M_0)$ is a cyclic language if $\forall M \in \mathbf{R}_L(Q, M_0)$ $\exists \sigma \in L(Q, M_0)$ such that $M \xrightarrow{\sigma} M$.*

Definition 5.7 *Let (Q, M_0) be an IPN and $L(Q, M_0)$ be a prefix closed sublanguage of $\pounds(Q, M_0)$. The synchronic distance of a set of transitions $R \subseteq T$ with respect to a set of transitions $S \subseteq T$ in $L(Q, M_0)$, denoted by $SD((Q, M_0)|_L\ R,\ S)$, is the maximum number of firings of the transitions of R that can occur in all possible firing sequences $\sigma \in L(Q, M_0)$ without firing any transition of S.*

Theorem 5.8 *Let (Q, M_0) be a conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is marking-detectable in k steps with respect to a cyclic prefix closed sublanguage $L(Q, M_0)$ of $\pounds(Q, M_0)$, if it is L-event-detectable and $\forall p_j \in P_{nm}$ either*

i) $SD((Q, M_0)|_L,\ ^\bullet(p_j)\ ,\ (p_j)^\bullet) = \mathbb{D}(p_j)$ *or*

ii) $SD((Q, M_0)|_L,\ (p_j)^\bullet\ \ ^\bullet(p_j)) = \mathbb{D}(p_j)$

where $\mathbb{D}(p_j) = M^{UB}(p_j) - M^{LB}(p_j)$.

Proof. Similar to the proof of Theorem 3.18, considering that the firing language of the net is confined into $L(Q, M_0)$. ∎

Theorem 5.9 *Let (Q, M_0) be a conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is marking-detectable in k steps with respect to a cyclic prefix closed sublanguage $L(Q, M_0)$ of $\pounds(Q, M_0)$, if it is L-event-detectable and $\forall \xi_i : Y_i^T M = k_i \in CML(Q, M_0)$ $\exists p_j \in P$ such that $Y_i(p_j) \neq 0$ and*

$$SD(Q, M_0)|_L,\ ^\bullet(p_j)\ ,\ (p_j)^\bullet) = k_i$$

Proof. Similar to the proof of Theorem 3.19, considering that the firing language of the net is confined into $L(Q, M_0)$. ∎

Observe that, the markings bounds M^{UB} and M^{LB} of each place are defined from the CML. However, when the system behavior is confined into a sublanguage $L(Q, M_0)$ of the firing language, a place could reach different marking bounds according to the set of reached markings $\mathbf{R}_L(Q, M_0)$ by the sequences of $L(Q, M_0)$.

Definition 5.10 *Let (Q, M_0) be an IPN and $L(Q, M_0)$ be a sublanguage of $\mathcal{L}(Q, M_0)$. The upper and lower marking bounds of a place $p_i \in P$ with respect to $L(Q, M_0)$ are*

$$\begin{aligned} M_L^{UB}(p_i) &= \max(M(p_i) \in \mathbf{R}_L(Q, M_0)) \\ M_L^{LB}(p_i) &= \min(M(p_i) \in \mathbf{R}_L(Q, M_0)) \end{aligned}$$

Example 5.11 *Consider the IPN shown in Figure 5.1 and the cyclic prefix closed sublanguage $L(Q, M_0) = \overline{(t_4 t_5 t_1)}^+$ The set of reachable markings by $L(Q, M_0)$ is*

$$\mathbf{R}_L(Q, M_0) = \{M_1,\ M_3,\ M_6\}$$

Thus, the lower and upper marking bounds with respect to $L(Q, M_0)$ are

i	$\mathbf{M}_L^{LB}(\mathbf{p}_i)$	$\mathbf{M}_L^{UB}(\mathbf{p}_i)$
1	0	1
2	0	1
3	1	1
4	0	1
5	0	1

In general, when the system behavior is confined into a cyclic prefix closed sublanguage $L(Q, M_0)$ of the firing language, $\mathbf{M}_L^{LB}(\mathbf{p}_i) \geq \mathbf{M}^{LB}(\mathbf{p}_i)$ and $\mathbf{M}_L^{UB}(\mathbf{p}_i) \leq \mathbf{M}^{UB}(\mathbf{p}_i)$. Hence, if the set $\mathbf{R}_L(Q, M_0)$ is available, then the marking bounds can be redefined and the following condition must be satisfied to achieve marking-detectability.

Theorem 5.12 *Let (Q, M_0) be a conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is marking-detectable in k steps with respect to a cyclic prefix closed sublanguage $L(Q, M_0)$ of $\mathcal{L}(Q, M_0)$, if it is L-event-detectable and $\forall p_j \in P_{nm}$ either*

i) $SD((Q, M_0)|_L,\ {}^\bullet(p_j)\quad (p_j)^\bullet) = \mathbb{D}_L(p_j)$ *or*

ii) $SD((Q, M_0)|_L,\ (p_j)^\bullet,\ {}^\bullet(p_j)) = \mathbb{D}_L(p_j)$

where $\mathbb{D}_L(p_j) = M_L^{UB}(p_j) - M_L^{LB}(p_j)$.

Proof. Similar to the proof of Theorem 3.18, considering that the firing language of the net is limited to $L(Q, M_0)$ which is cyclic and the marking of each place can reach a value within the integer interval $[M_L^{LB}(p_j) \ldots M_L^{UB}(p_j)]$. ■

Theorem 5.13 *Let (Q, M_0) be a conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is marking-detectable in k steps with respect to a cyclic prefix closed sublanguage $L(Q, M_0)$ of $\pounds(Q, M_0)$, $\exists M \in \mathbf{R}_L(Q, M_0)$ such that $\varphi^{-1}(\varphi(M)) = \{M\}$.*

Proof. Let $M \in \mathbf{R}_L(Q, M_0)$ such that $\varphi^{-1}(\varphi(M)) = \{M\}$. Thus, when the output $\varphi(M)$ is presented, the only solution for the CML is the marking M. Thus, the marking of every non-measurable place becomes known. Therefore, (Q, M_0) is w-marking-detectable in k steps. ■

5.3.3 L-observable IPN

Theorem 5.14 *Let (Q, M_0) be a conservative IPN, where M_0 is unknown. (Q, M_0) is observable in k steps with respect to a cyclic prefix closed sublanguage $L(Q, M_0)$ of $\pounds(Q, M_0)$ if and only if it is event-detectable and marking detectable with respect to $L(Q, M_0)$.*

Proof. (Sufficiency) Since (Q, M_0) is marking-detectable in k steps with respect to $L(Q, M_0)$, $\exists k < \infty$ such that the reached marking M_k is known. Due to the L-event-detectability property,when the input word ω is given, the firing sequence $\sigma_\omega = t_i t_j \ldots t_v \in L(Q, M_0)$and the output word $z = \varphi(M_0)\varphi(M_1)\ldots\varphi(M_k)$ are known, where $M_0 \xrightarrow{t_i} M_r \xrightarrow{t_j} \ldots \xrightarrow{t_v} M_k$. It means that, $|O_z \cap I_\omega| = \{M_0 M_r \ldots M_k\}$; otherwise, M_k were undistinguished yet. In this case, M_0 can be computed recursively solving the set of equations:

$$\begin{aligned} M_{i-1} &= M_i - C\vec{t_v} \\ &\ldots \\ M_1 &= M_2 - C\vec{t_j} \\ M_0 &= M_1 - C\vec{t_i} \end{aligned}$$

(observe that the terms $C\vec{t_i}, C\vec{t_j}, \ldots, C\vec{t_k}$ are known by the L-event-detectability property). Thus, M_0 can be computed using the information provided by the system structure, input and output sequences.

Now, suppose that, a transition sequence $\sigma_{\omega'} = t_u t_w \ldots t_x \in L(Q, M_0)$ fires from M_k. Thus, the marking sequence $M_k M_{k+1} \ldots M_{k+i}$ is yield, where $M_k \xrightarrow{t_u} M_{k+1} \xrightarrow{t_w} \ldots \xrightarrow{t_x} M_{k+i}$, and $i = |\sigma_{\omega'}|$. Since the net is L-event-detectable, every transition firing is be detected and uniquely determined; thus, $M_k M_{k+1} \dots M_{k+i}$ is the only sequence that yield the output word $y = \varphi(M_k)\varphi(M_{k+1}) \dots \varphi(M_{k+i})$,.i.e. $O_y = \{M_k M_{k+1} \dots M_{k+i}\}$. Hence, and $\forall I_{\omega_i}$, $|I_{\omega_i} \cap O_y| = 1$ and the marking M_{k+i} can be uniquely determined.

Therefore, (Q, M_0) is observable in k steps with respect to $L(Q, M_0)$.

(Necessity) Suppose that, (Q, M_0) is not L-event-detectable. Although a marking M_k, such that $M_0 \xrightarrow{\sigma} M_k$ where $\sigma \in L(Q, M_0)$ can be determined, the firing sequence $\sigma \in \pounds_\omega(Q, M_0)$ cannot be uniquely determined. Thus, it is not possible to compute the initial marking M_0. Hence, (Q, M_0) is not observable with respect to $L(Q, M_0)$, a contradiction.

Now, suppose that, (Q, M_0) is not marking-detectable in k steps with respect to $L(Q, M_0)$. It means that, for at least a marking $M_k \in \mathbf{R}(N, M_0)$, $\exists M_0 M_i \dots M_j$ and $M_0 M_r \dots M_k$ such that $M_0 M_i \dots M_j$, $M_0 M_r \dots M_k \in I_\omega$ and $M_0 M_i \dots M_j$, $M_0 M_r \dots M_k \in O_z$, where $\omega \in \pounds_{in}(Q, M_0)$ and $z \in \pounds_{out}(Q, M_0)$ Thus, $|I_\omega \cap O_z| > 1$ and, by Theorem 3.8, (Q, M_0) is unobservable in k steps with respect to $L(Q, M_0)$, a contradiction. ■

Corollary 5.15 *Let (Q, M_0) be a conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is observable with respect to a cyclic prefix closed sublanguage $L(Q, M_0)$ of $\pounds(Q, M_0)$, if it is L-event-detectable and $\forall p_j \in P_{nm}$ either*

1. $SD((Q, M_0)|_L, {}^\bullet(p_j), (p_j)^\bullet) = \mathbb{D}(p_j)$ *or*
2. $SD((Q, M_0)|_L, (p_j)^\bullet \ {}^\bullet(p_j)) = \mathbb{D}(p_j)$
   where $\mathbb{D}(p_j) = M^{UB}(p_j) - M^{LB}(p_j)$.

Corollary 5.16 *Let (Q, M_0) be a conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is observable in k steps with respect to a cyclic prefix closed sublanguage $L(Q, M_0)$ of $\pounds(Q, M_0)$, if it is L-event-detectable and $\forall \xi_i : Y_i^T M = k_i \in CML(Q, M_0)$, $\exists p_j \in P$ such that $Y_i(p_j) \neq 0$ and*

$$SD(Q, M_0)|_L, {}^\bullet(p_j), (p_j)^\bullet) = k_i$$

Corollary 5.17 *Let (Q, M_0) be a conservative IPN, where M_0 is unknown, , but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is observable in k steps with respect to a cyclic prefix closed sublanguage $L(Q, M_0)$ of $\pounds(Q, M_0)$, if it is L-event-detectable and $\forall p_j \in P_{nm}$ either*

1. $SD((Q, M_0)|_L, {}^\bullet(p_j), (p_j)^\bullet) = \mathbb{D}_L(p_j)$ *or*
2. $SD((Q, M_0)|_L, (p_j)^\bullet \ {}^\bullet(p_j)) = \mathbb{D}_L(p_j)$
   where $\mathbb{D}_L(p_j) = M_L^{UB}(p_j) - M_L^{LB}(p_j)$.

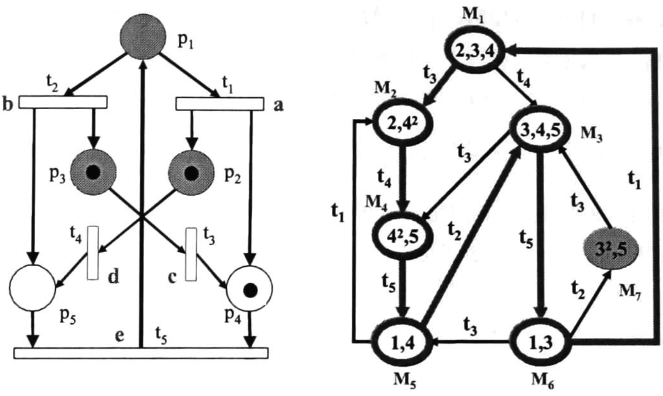

Figure 5.1: An observable IPN and its reachability set showing a sublanguage of $\pounds_{in}(Q, M_0)$.

Corollary 5.18 *Let (Q, M_0) be a conservative IPN, where M_0 is unknown, but the $CML(Q, M_0)$ can be obtained. (Q, M_0) is observable in k steps with respect to a cyclic prefix closed sublanguage $L(Q, M_0)$ of $\pounds(Q, M_0)$ if it is event-detectable and $\exists M \in \mathbf{R}_L(Q, M_0)$ such that $\varphi^{-1}(\varphi(M)) = \{M\}$.*

5.4 Example

Consider the live and bounded IPN and its reachability set shown in Figure 5.1. Suppose that the net is being confined into the cyclic prefix closed language $L(Q, M_0) = (\overline{t_3t_4t_5t_2t_5t_1})^+ \subset \pounds(Q, M_0)$. In this case, $T_L = T$.

The set of observable places is $P_m = \{p_4, p_5\}$, while all the transitions are controllable. i.e. $T_c = T$. Thus,

$$\varphi C^L = \varphi C = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 \end{bmatrix}$$

Although, $\varphi C(\bullet, 1) = \varphi C(\bullet, 3)$ and $\varphi C(\bullet, 2) = \varphi C(\bullet, 4)$, $a = \lambda(t_1) \neq \lambda(t_3) = c$ and $b = \lambda(t_1) \neq \lambda(t_3) = d$. Thus, by Proposition 5.4, the net is L-event-detectable.

A CML can be defined on the elementary p-invariants of the net

$$CML = \left\{ \begin{array}{c} \xi_1 : M(p_1) + M(p_3) + M(p_4) = 2, \\ \xi_2 : M(p_1) + M(p_2) + M(p_5) = 1 \end{array} \right\}$$

The upper and lower marking bounds of each place are the following

i	$\mathbf{M}^{LB}(\mathbf{p}_i)$	$\mathbf{M}^{UB}(\mathbf{p}_i)$
1	0	1
2	0	1
3	0	2
4	0	2
5	0	1

In $L(Q,M_0) = \overline{(t_3t_4t_5t_2t_5t_1)^+}$, the synchronic distances are the following

i	${}^\bullet(p_i)$	$(p_i)^\bullet$	$\mathbf{SD}(\mathbf{Q},\mathbf{M}_0)\vert_L, {}^\bullet(p_i), (p_i)^\bullet)$
1	$\{t_5\}$	$\{t_1,t_2\}$	1
2	$\{t_1\}$	$\{t_4\}$	1
3	$\{t_2\}$	$\{t_3\}$	1
4	$\{t_1,t_3\}$	$\{t_5\}$	2
5	$\{t_2,t_4\}$	$\{t_5\}$	1

Observe that, $\mathbb{D}(p_3) = 2 > 1 = \mathbf{SD}(\mathbf{Q},\mathbf{M}_0)|_L, {}^\bullet(p_3), (p_3)^\bullet)$. However, for ξ_1 it holds that $SD((Q,M_0)|_L, {}^\bullet(p_4), (p_4)^\bullet) = k_1 = 2$ and, for ξ_2, it holds that, $SD(Q,M_0)|_L, {}^\bullet(p_2), (p_2)^\bullet) = k_2 = 1$. Thus, by Corollary 5.16, (Q,M_0) is observable with respect to $L(Q,M_0)$.

5.5 Discussion

In this chapter, the problem of estimating the actual state of a system confined into a cyclic prefix closed sublanguage of the firing language $L(Q,M_0)$ was studied. It is assumed that, $L(Q,M_0)$ is a realizable language, i.e. the IPN can be forced to execute only the sequences belonging to $L(Q,M_0)$.

The concept of observability with respect a firing sublanguage was introduced. Also, the concepts of event-detectability and marking-detectability with respect a firing sublanguage were introduced. If an IPN exhibits this property, then it is possible to distinguish a reachable marking M in a finite number of event occurrences of the sequences of $L(Q,M_0)$. In this context, the characterizations of marking-detectable IPN presented in the previous chapters were generalized for marking-detectable IPN with respect to a firing sublanguage. Moreover, it was shown that a event-detectability and marking-detectability with respect to a firing sublanguage are necessary and sufficient conditions for observability with respect to a firing sublanguage.

The concept of l-observability seems to be more general than the concept of observability, since in the latter the firing language of the system is not constrained, i.e. $L(Q,M_0) =$

$\mathcal{L}(Q, M_0)$. However, l-observability does not implies observability, but observability implies l-observability.

In the herein presented approach, to test the observability with respect a firing language, it is required the knowledge of the reached markings by the sequences of $L(Q, M_0)$.

Chapter 6

Sensor Choice for Observability

SUMMARY. This chapter studies the minimal sensor choice problem such that the observability of an IPN model is preserved. The main result is a polynomial algorithm for computing a minimal initial cost sensor configuration for observability that can be applied to any class of live, conservative and cyclic IPN.

6.1 Introduction

Reducing costs is a common practice in all engineering areas, such is the case in designing and operating Discrete-Event Systems (DES). An interesting problem on this topic is concerned with the selection of the lowest cost sensor configuration such that certain system properties like observability were preserved.

This problem has been addressed in DES modeled by Finite State Machines (FSM). For example, in [15] and [44], authors provide algorithms to choose a minimal set of measurable events such that the resulting FSM is observable in the sense of definition presented in [22]. In such approach, the state space is partitioned into equivalence classes according to a controllable language. In [4], the state space of a FSM is also partitioned, but in this case, the partition is performed according with several a priori known possible failure scenarios. Based on this knowledge, the authors provide an algorithm to determine the minimal number of event sensors needed to distinguish failure events belonging to different failure scenarios. In none of these works, however, the complete computation of the non-measurable events occurrences or the exact reconstruction of the system state is not required. An extension to these works is presented in [25], where a DES is modeled by timed FSM. In that work, the authors provide an algorithm to compute a minimal cost event and state sensor configuration such that the event sequence can be completely reconstructed.

The approach herein presented provides an algorithm for computing a minimal initial cost sensor configuration for observability that can be applied to any class of live, conservative and cyclic IPN. This algorithm considers that the actuator signals are attached to the transitions of the net. Additionally, the algorithm considers the unavailability of sensors for certain places of the net (e.g. it is difficult to implement a sensor for measuring the working state of a machine) and that a state sensor may be attached to several places of the net (e.g. a continuous level sensor can emit a signal for every discretized level in a water tank). The algorithm exploits the structural characterization of event-detectability and the characterization of marking-detectability presented in Chapter 3.

6.2 Basic definitions

The observability of an IPN depends on the way of assigning the signals of the available sensors to its nodes. To solve the minimal sensor choice problem for observability, it is assumed that there exists non-measured IPN to which a set of sensor signals will be assigned.

Definition 6.1 *A non-measured IPN is an IPN given by*

$$Q_{nm} = (N', \Sigma, \Phi = \{\}, \lambda, \varphi = \mathbf{0})$$

where $\mathbf{0}$ *is a* $n \times n$ *zero matrix.*

Observe that, the actuator signals are already assigned to the transitions of the net and every place is non-measurable. Thus, the minimal sensor choice problem for observability consists of assigning the signals of a subset of the available sensors $r \subseteq S$ to the places of the net, such that the resulting IPN is observable with the minimal sensor cost. The sensors herein considered are not necessarily binary sensors, i.e. a sensor can emit more than two signals.

This subset of sensors together with the corresponding function to attach their signals to the places of a given net is called a sensor configuration, which is formally defined as follows.

Definition 6.2 *Let* $Q_{nm} = (N', \Sigma, \Phi = \{\}, \lambda, \varphi = 0)$ *be a non-measured IPN and* S *be its set of available state sensors for the places of the net. A sensor configuration of* N' *is a 4-tuple* $r_c = (r, \Psi_S, f_S, G_S)$, *where*

- $r \subseteq S$ *is a subset of the available sensors*
- $\Psi_S = \{v_1, v_2, \ldots, v_z\}$ *is the set of signals emitted by the state sensors,*
- $f_S : \Psi_S \rightarrow S$ *is a function that indicates the sensor that emits a given signal, where* ε *is the null signal,* $\emptyset$ *represents the null sensor and* $f_S(\varepsilon) = \emptyset$.
- $G_S : \mathbf{R}(Q, M_0)|_p \rightarrow \Psi_S$ *is a function that indicates the sensor signal attached to a reachable marking of a given place, where* $\mathbf{R}(Q, M_0)|_p$ *is the set of reachable markings of a given place and* $\forall p \in P$, *if* $M(p) = 0$ *then* $G_S(M(p)) = \varepsilon$, *where* ε *is the null signal. This function has the following restrictions:*

  a) $\forall p_i, p_j \in P$ *and* $\forall M_u(p_i) \in \mathbf{R}(Q, M_0)|_{Pi}$, $M_v(p_j) \in \mathbf{R}(Q, M_0)|_{Pj}$, *if* $G_S(M_u(p_i)) \in \Psi_S$ *and* $G_S(M_v(p_j)) \in \Psi_S$, *then* $G_S(M_u(p_i)) \neq G_S(M_v(p_j))$. *It means that, a sensor signal can be only assigned to a marking.*

  b) $\forall p \in P$ *and* $\forall M_j(p), M_k(p) \in \mathbf{R}(Q, M_0)|_p$, *if* $G_S(M_j(p)) \in \Psi_S$ *and* $G_S(M_k(p)) \in \Psi_S$, *then* $f_S(G_S(M_j(p_i))) = f_S(G_S(M_k(p_i)))$. *It means that, the sensor signals of the markings of a given place are emitted by the same sensor.*

Figure 6.1 shows a scheme of functions G_S and f_S.

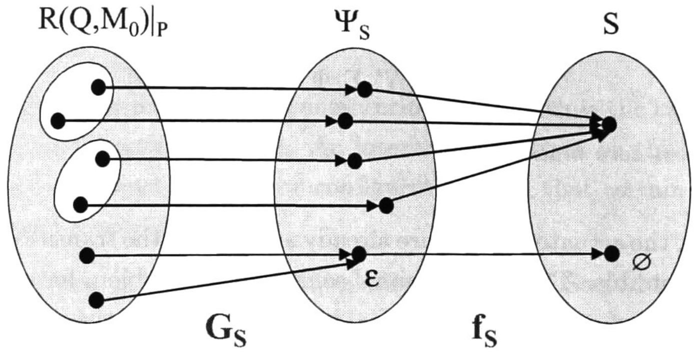

Figure 6.1: The domains and codomains of functions G_S and f_S.

Since all the sensor signals assigned to the reachable markings of a given place are emitted by the same sensor, it can be thought that a state sensor is attached to the place; thus,

$$\begin{aligned} \varkappa_S &: P \to S \\ \varkappa_S(p) &= \begin{cases} f_S(G_S(M_i(p))), & G_S(M_i(p)) \in \Psi_S \\ \emptyset, & \text{otherwise} \end{cases} \end{aligned}$$

where $\varkappa_S(p) = \emptyset$ means that no sensor is attached to the place p. Function $\varkappa_S$ can be extended to a set of places in the following way:

$$\begin{aligned} \varkappa'_S &: 2^P \to 2^S \\ \varkappa'_S(z) &= \{\varkappa_S(p_i), \varkappa_S(p_j), \ldots, \varkappa_S(p_k)\} = \bigcup_{p_r \in z} \varkappa_S(p_r) \end{aligned}$$

where $z = \{p_i, p_j, \ldots, p_k\} \in 2^P$

Similarly, $\varkappa'_S$ can be extended to a family of sets of places as follows:

$$\begin{aligned} \varkappa''_S &: 2^{2^P} \to 2^S \\ \varkappa''_S(X) &= \{\varkappa'_S(z_i), \varkappa'_S(z_j), \ldots, \varkappa'_S(z_k)\} = \bigcup_{z_r \in X} \varkappa'_S(z_r) \end{aligned}$$

where $X = \{z_i, z_j, \ldots, z_k\}$ is a family of set of places.

Observe that, an IPN can be thought as conformed by a non-measured IPN together with a sensor configuration, if the sensor configuration is attached to the nodes of the net as follows.

Definition 6.3 *A non-measured IPN given by $Q_{nm} = (N', \Sigma', \Phi = \{\}, \lambda', \varphi = 0)$ and a sensor configuration $r_c = (r, \Psi_S, f_S, G_S)$ form the IPN given by $Q = (N, \Sigma, \Phi, \lambda, \varphi)$ where*

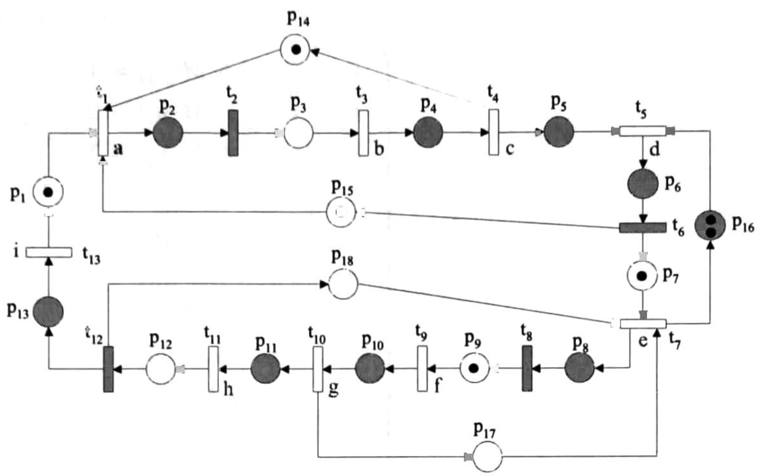

Figure 6.2: An *IPN* model of an automated manufacturing cell.

1. $N = N'$,
2. $\Sigma = \Sigma'$,
3. $\Phi = \Psi_S$.
4. $\lambda = \lambda'$,
5. Assume that the set of places is ordered as $\{p_1 \ldots p_v | p_{r+1} \ldots p_n\}$ such that $\forall p \in \{p_1 \ldots p_v\}$ if $M(p_i) \neq 0$ then $G_S(M(p_i)) \in \Psi_S$. Thus, $\forall i \in 1, 2, \ldots, r$

$$\varphi = [e_1\, e_2\, \ldots e_v]^T$$

Example 6.4 *Consider the IPN of Figure 6.2. The sensor configuration is the following:*

- $r = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9\}$
- $\Psi_S = \{k_1, k_2, k_3, l, n_1, n_2, n_3, q, r, s, u, v, w\}$
-

ψ	k_1	k_2	k_3	l	n_1	n_2	n_3	q	r	s	u	v	w	ε
$f_S(\psi)$	s_1	s_1	s_1	s_2	s_3	s_3	s_3	s_4	s_5	s_6	s_7	s_8	s_9	$\emptyset$

$$\mathbf{G}_S(\mathbf{M}(\mathbf{p}_i)) = \begin{cases} k_1, & i=1 \wedge M(p_i)=1 \\ k_2, & i=1 \wedge M(p_i)=2 \\ k_3, & i=1 \wedge M(p_i)=3 \\ l, & i=3 \wedge M(p_i)=1 \\ n_1, & i=7 \wedge M(p_i)=1 \\ n_2, & i=7 \wedge M(p_i)=2 \\ n_3, & i=7 \wedge M(p_i)=3 \\ q, & i=9 \wedge M(p_i)=1 \\ r, & i=12 \wedge M(p_i)=1 \\ s, & i=14 \wedge M(p_i)=1 \\ u, & i=15 \wedge M(p_i)=1 \\ v, & i=17 \wedge M(p_i)=1 \\ w, & i=18 \wedge M(p_i)=1 \\ \varepsilon, & otherwise \end{cases}$$

$\mathbf{p}_i$	$\mathbf{p}_1$	$\mathbf{p}_3$	$\mathbf{p}_7$	$\mathbf{p}_9$	$\mathbf{p}_{12}$	$\mathbf{p}_{14}$	$\mathbf{p}_{15}$	$\mathbf{p}_{17}$	$\mathbf{p}_{18}$
$\varkappa_S(\mathbf{p}_i)$	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9

$\mathbf{p}_i$	$\mathbf{P}\backslash\{\mathbf{p}_1,\mathbf{p}_3,\mathbf{p}_7,\mathbf{p}_9,\mathbf{p}_{12},\mathbf{p}_{14},\mathbf{p}_{15},\mathbf{p}_{17},\mathbf{p}_{18}\}$
$\varkappa_S(\mathbf{p}_i)$	$\emptyset$

For simplicity, hereafter, a sensor configuration will be referred as a set of sensors, under the assumption that its respective functions are well defined. Also, the notation $s = p_k$ is used instead of $s = \varkappa_S(p_k) \in S$. In the same way, the notation $^\bullet(x)^\bullet$ is used instead of $(x)^\bullet \cup^\bullet (x)$, where $x \in T \cup P$.

Since the selection of a particular sensor configuration is defined in cost terms, the following sensor cost function is established: $W : 2^S \rightarrow \mathbb{R}^+$. where 2^S is the power set of S.

Definition 6.5 *Let $Q_{nm} = (N', \Sigma, \Phi = \{\}, \lambda, \varphi = 0)$ be an non-measured IPN, S be the set of available sensors of N' and $r_c = (r, \Psi_S, f_S, G_S)$ be a sensor configuration of N' The sensor configuration r_c satisfies observability if the IPN, resulting of assigning the sensor signals of r_c to the nodes of N, is observable.*

Remark 6.6 *The set of all sensor configurations of N' that satisfies observability of the resulting (Q, M_0) is denoted by $\hat{S}(Q, M_0)$.*

Now, the problem of finding out a minimal cost sensor configuration can be defined as follows.

Definition 6.7 *Let $Q_{nm} = (N', \Sigma, \Phi = \{\}, \lambda, \varphi = 0)$ be an non-measured IPN, S be the set of available sensors of N' Let $w : S \to \mathbb{R}^+$ be a sensor cost function that assigns a cost to a each sensor of S, $w(s \in S) = k$, and $W : 2^S \to \mathbb{R}^+$ be a cost function that assigns a cost to a set of elements of S and $W(\{s_i, s_j, \ldots, s_k\}) = w(s_i) + w(s_j) + \ldots + w(s_k)$. be a sensor cost function defined over S. The minimal sensor choice problem for observability is defined as to compute*

$$w^* = r \in \hat{S}(Q, M_0) min\ W(r)$$

and then selecting $r^* \in \hat{S}(Q, M_0)$, such that $W(r^*) = w^*$.

Therefore, to solve the minimal cost sensor choice problem for observability, first, it is necessary to compute the set of all sensor configurations that satisfy observability, then to obtain the cost of those configurations and finally to select a minimal cost one. The following sections discuss the computation of. the set $\hat{S}(Q, M_0)$ and the solution of the minimal sensor choice problem.

6.3 Minimal sensor choice

In the previous three chapters, it was stated that to guarantee the observability, W-observability or L-observability of an IPN, it is necessary to satisfy event-detectability. Moreover, in live, cyclic and conservative $IFCN$ and in live, cyclic and 1-bounded IPN, the w-observability problem is reduced to the event-detectability problem. Thus, in those chapters, a test for event-detectability is provided. For the contrary, this section is devoted to obtain a method of selecting a sensor configuration for event-detectability an then to provide an algorithm to determine a minimal cost sensor configuration for observability and, since observability implies W-observability and L-observability, the algorithm is also useful to compute a minimal cost sensor configuration preserving W-observability or L-observability.

The following theorem establishes a characterization of the set of measurable nodes of an IPN exhibiting event-detectability.

Theorem 6.8 *An IPN given by (Q, M_0) is event-detectable if and only if*

1.$\forall t_k \in T$, $\exists p_r \in P_m$ such that $p_r \in^\bullet (t_k)^\bullet \setminus^\bullet (t_k) \cap (t_k)^\bullet$ and

2. $\forall t_i \neq t_j \in T$ with $\lambda(t_i) = \lambda(t_j)$, $\exists p_r \in P_m$, such that $p_r \in^\bullet (t_i) \setminus^\bullet (t_i) \cap^\bullet (t_j)$ or $p_r \in (t_i)^\bullet \setminus (t_i)^\bullet \cap (t_j)^\bullet$

Proof. (Necessary) Here, we prove that, event-detectability implies points (1) and (2).

Since (Q, M_0) is event-detectable, by Proposition 3.10 the columns in φC are no null and different from each other:

Let $\varphi C(\bullet, k)$ be the k-th column of φC corresponding to a transition, say t_k. Since there are no null columns in φC, $\exists r$ such that $\varphi C(r, k) \neq 0$, which corresponds to a measurable input or output place of t_k that is not in self-loop with t_k, i.e. $p_r \in^\bullet (t_k)^\bullet \setminus^\bullet (t_k) \cap (t_k)^\bullet$

Now, let $\varphi C(\bullet, i)$ and $\varphi C(\bullet, j)$ be two columns of φC, which correspond to two transitions, say t_i and t_j, respectively, such that $\lambda(t_i) = \lambda(t_j)$. Thus, by Proposition 3.10, $\varphi C(\bullet, i) \neq \varphi C(\bullet, j)$. It means that, they differ at least in one entry, say $\varphi C(r, i) \neq \varphi C(r, j)$. It is clear that, the r^{th} row of φC corresponds to a non-common input or output measurable place of both transitions, say p_r, i.e. $p_r \in^\bullet (t_i) \setminus^\bullet (t_i) \cap^\bullet (t_j)$ or $p_r \in (t_i)^\bullet \setminus (t_i)^\bullet \cap (t_j)^\bullet$

(Sufficiency) Here, we prove that, if points (1) and (2) hold then (Q, M_0) is event-detectable.

Let t_k be a transition such that $\exists p_r \in P_m$ and $p_r \in^\bullet (t_k)^\bullet \setminus^\bullet (t_k) \cap (t_k)^\bullet$ Thus, by Proposition 3.10, $\varphi C(r, k) \neq 0$ and, therefore, $\varphi C(\bullet, k)$ is a no null column.

Now, let t_i, t_j be two arbitrary different transitions $(i \neq j)$ such that $\lambda(t_i) = \lambda(t_j)$, and let p_r be a measurable place, such that $p_r \in^\bullet (t_i) \setminus^\bullet (t_i) \cap^\bullet (t_j)$ or $p_r \in (t_i)^\bullet \setminus (t_i)^\bullet \cap (t_j)^\bullet$ Thus, $\varphi C(r, i) \neq \varphi C(r, j)$ and, therefore, $\varphi C(\bullet, i)$ and $\varphi C(\bullet, j)$ are different from each other.

Hence, (Q, M_0) is event-detectable. ■

From Theorem 3.17, it is known that if a live, cyclic and conservative IPN is event-detectable and the SD conditions for marking-detectability are satisfied then the net is observable.

Proposition 6.9 *Let (Q_{nm}, M_0) be a non-measured IPN and $r_c = (r_1 \cup r_{SD}, \Psi_S, f_S, G_S)$ be a sensor configuration of the net, where $r_1 \subset S$ and r_{SD} are the sensors attached to the places that do not satisfy the SD conditions of Theorem 3.17. If r_c satisfies event-detectability, then the IPN resulting by attaching r_c to (Q_{nm}, M_0) is observable.*

Proof. Let P_{SD} be the set of places that do not satisfy the SD conditions of Theorem 3.17 and r_{SD} the sensors attached to them. Since $r_{SD} \subseteq r$, then $\forall p \in P_{SD},\ p \in P_m$. Thus, $P_{nm} \cap P_{SD} = \varnothing$. Hence, every measurable place satisfies the SD conditions, so by Theorem 3.17, the net is marking-detectable. Since r also satisfies event-detectability, by Theorem 3.21, the net is observable. ■

Thus, given a non-measured IPN system and the set of places not satisfying the SD conditions, if those places are set as measurable and some of the remaining places are selected as measurable such that the resulting IPN is event-detectable, then the net is observable. The following paragraphs are devoted to obtain an algorithm that exploits this fact and the result of Theorem 6.8 to compute a minimal cost sensor configuration for observability in IPN.

Firstly, let us introduce some definitions and lemmas.

Definition 6.10 *The f-union of two families of sets A and B is $A \sqcup B = \{a \cup b | a \in A \wedge b \in B\}$.*

Lemma 6.11 *Let (Q, M_0) be a non-measured IPN and $t_k \in T$ be a transition.*

1. *The set of measurable nodes making detectable the transition t_k is*

$$S(t_k) = \{\{p_r\} | p_r \in^\bullet (t_k)^\bullet \setminus^\bullet (t_k) \cap (t_k)^\bullet\}$$

2. *The set of measurable nodes making distinguishable t_j and t_k from each other is*

$$S_{j,k} = \begin{cases} S(t_j) \sqcup S(t_k), & \lambda(t_j) \neq \lambda(t_k) \\ S(t_j) \sqcup S(t_k) \setminus \{\{a, b\} \in S(t_j) \sqcup S(t_k) \mid & \\ [a \in^\bullet (t_i) \cap^\bullet (t_j) \vee a \in (t_i)^\bullet \cap (t_j)^\bullet] \wedge & \textit{otherwise} \\ [b \in^\bullet (t_i) \cap^\bullet (t_j) \vee b \in (t_i)^\bullet \cap (t_j)^\bullet] \; \} & \end{cases}$$

Proof. Obviously from Theorem 6.8. ■

It means that, given a transition, if there is a sensor signal attached to any of its input/output places then its firing can be detected. However, to make it distinguishable from another transition, then its sets of measurable nodes must be f-joined and, if necessary, the sets containing only common input or output places must be removed.

Example 6.12 *In the IPN of Figure 6.2, $^\bullet(t_1)^\bullet = \{p_1, p_2, p_{14}, p_{15}\}$ and $^\bullet(t_4)^\bullet = \{p_4, p_5, p_{14}\}$. Thus, $S(t_1) = \{\{p_1\}, \{p_2\}, \{p_{14}\}, \{p_{15}\}\}$ and $S(t_4) = \{\{p_4\}, \{p_5\}, \{p_{14}\}\}$. In this case, $S(t_1) \sqcup S(t_4) = \{\{p_1, p_4\}, \{p_1, p_5\}, \{p_1, p_{14}\}, \{p_2, p_4\}, \{p_2, p_5\}, \{p_2, p_{14}\}, \{p_{14}, p_4\}, \{p_{14}, p_5\}, \{p_{14}\}, \{p_{15}, p_4\}, \{p_{15}, p_5\}, \{p_{15}, p_{14}\}\}$.*

Since $a = \lambda(t_1) \neq \lambda(t_4) = c$, the set of measurable nodes making transitions t_1 and t_4 distinguishable from each other is $S_{1,4} = S(t_1) \sqcup S(t_4)$.

Unfortunately, in most of cases, there is not an available sensor for each node of the net; thus, the set $S(t_k)$ is reduced to the set of measurable nodes that, make detectable the transition t_k and have attached available sensors or simply, the set of available measurable nodes.

Corollary 6.13 *Let (Q, M_0) be a non-measured IPN, S be the set of available sensors of the net, and $t_j, t_k \in T$ be two transitions.*

1. The set of available measurable nodes making detectable the transition t_k is

$$\tilde{S}(t_k) = \{\{p_r\} | (p_r \in^\bullet (t_k)^\bullet \setminus^\bullet (t_k) \cap (t_k)^\bullet) \wedge \varkappa_S(p_r) \in S\}$$

2. The set of available measurable nodes making distinguishable t_j and t_k from each other is

$$\tilde{S}_{j,k} = \begin{cases} \tilde{S}(t_j) \sqcup \tilde{S}(t_k), & \lambda(t_j) \neq \lambda(t_k) \\ \tilde{S}(t_j) \sqcup \tilde{S}(t_k) \setminus \{\{a,b\} \in \tilde{S}(t_j) \sqcup \tilde{S}(t_k) \mid & \\ [a \in^\bullet (t_i) \cap^\bullet (t_j) \vee a \in (t_i)^\bullet \cap (t_j)^\bullet] \wedge & \text{otherwise} \\ [b \in^\bullet (t_i) \cap^\bullet (t_j) \vee b \in (t_i)^\bullet \cap (t_j)^\bullet]\ \} & \end{cases}$$

Observe that, if $|\tilde{S}(t_k)| = 0$ or $|\tilde{S}_{j,k}| = 0$ then the transition t_k will be undetectable or indistinguishable from t_j, respectively.

The second point of the previous definition suggests that, to make distinguishable a transition t_i from the others, the sets of available measurable nodes that make distinguishable transition t_i from each t_j where $j \in \{1, 2, \ldots, |T|\} \setminus \{i\}$ must be computed; and then, the f-union of all this sets also must be computed.

Lemma 6.14 *Let (Q, M_0) be a non-measured IPN. The family set of measurable nodes making transition t_i distinguishable from each $t_j \in T \setminus \{t_i\}$ is*

$$Z_i = \begin{cases} \tilde{S}(t_i), & |T| = 1 \\ \bigsqcup_{t_j \in T \setminus \{t_i\}} \tilde{S}_{i,j}, & \textit{otherwise} \end{cases}$$

Proof. If $|T| = 1$, then $Z_i = \tilde{S}(t_i)$. Thus, by definition of $\tilde{S}(t_i)$,every element of Z_i makes t_i detectable.

If $|T| \geq 2$, by induction on the number of transitions r.

Let $|T| = 2$. Thus, $Z_1 = \tilde{S}_{12}$ and, by Lemma 6.11, every element of Z_i is a set of nodes that make t_i distinguishable from t_j.

Suppose that for $2 \leq |T| = k$, $\forall c \in Z_i^k = \bigsqcup_{j=1}^{k} \tilde{S}_{i,j}$, c is a set of nodes that make distinguishable t_i from every $t_j \in \{t_2, t_3, \ldots, t_k\}$.

Let $|T| = k+1$. We proof that, $\forall c \in Z_i^{k+1} = Z_i^k \sqcup \tilde{S}_{i,k+1}$, c is a set of nodes making distinguishable t_i from every $t_j \in \{t_2, t_3, \ldots, t_{k+1}\}$. Let $a = \{p_r, p_s, \ldots p_v\} \in Z_i^k$ and $b = \{p_x, p_z\} \in \tilde{S}_{i,k+1}$. Thus, the set $c = a \cup b = \{p_r, p_s, \ldots, p_v, p_x, p_z\}$ is an element of Z_i^{k+1}. Hence, the places of c make distinguishable t_i from every $t_j \in \{t_2, t_3, \ldots, t_{k+1}\}$.

Therefore, Z_i make distinguishable t_i from every $t_j \in T \backslash \{t_i\}$. ■

Hence, in order to obtain the sets of measurable nodes making distinguishable every transition from each other, the sets Z_i of every transition must be joined through the f-union.

Lemma 6.15 *Let (Q, M_0) be a non-measured IPN. The family set of available measurable nodes making distinguishable the transitions of the net from each other is*

$$R = \begin{cases} \tilde{S}(t_i), & |T| = 1 \\ \bigsqcup_{i=1}^{|T|} Z_i, & \textit{otherwise} \end{cases}$$

Proof. Suppose that, $|T| = 1$. Thus, by definition $R = \tilde{S}(t_i)$ containing all the sets of nodes that make t_i detectable.

Let $|T| = 2$. Hence $R = Z_1 \sqcup Z_2 = \tilde{S}_{12} \sqcup \tilde{S}_{21}$. Thus, $\forall a \in R$, a makes t_1 and t_2 distinguishable from each other.

Suppose that for $|T| = k$ with $2 \leq k < |T|$, $\forall c \in R$, c makes distinguishable from each other every transition $t \in \{t_1, t_2, \ldots t_k\}$.

Let $|T| = k+1$. Thus, $R = Z_1 \sqcup Z_2 \sqcup \ldots \sqcup Z_{k+1}$. Let $a_1 = \{p_r, \ldots p_s\} \in Z_1$, $a_2 = \{p_u, \ldots, p_v\} \in Z_2$, $a_i = \{p_x, \ldots, p_y\} \in Z_i$, and so on. By lemma 6.14, the set of measurable nodes $a_1, a_2, \ldots, a_i, \ldots, a_{k+1}$ make distinguishable every transition t_i from the others, respectively. Hence, $d = a_1 \cup a_2 \cup \ldots \cup a_i \cup \ldots \cup a_{k+1} = \{p_r, \ldots p_s, p_u, \ldots, p_v, \ldots p_x, \ldots, p_y, \ldots, p_w, \ldots, p_z\} \in R$ contains all the necessary measurable nodes to make every transition distinguishable from the others. ■

Once the family set R is computed, in order to obtain the sets of available measurable nodes that make the net observable, it must be added to each set of R the set of places no-satisfying the SD conditions.

Lemma 6.16 *Let (Q, M_0) be a non-measured IPN and $P_{SD} = \{\{p_w, p_x, \cdots, p_y\}\}$ be the set of places no-satisfying the SD conditions in (Q, M_0). The family set of available measurable nodes making (Q, M_0) observable is*

$$\tilde{R} = R \sqcup P_{SD}$$

Proof. Let $r = \{p_r, p_s, \ldots p_u\} \in R$ be a set of available measurable nodes that make distinguishable the transitions of the net. Thus, if the corresponding sensors of these nodes are attached to the places of the net as indicated in Definition 6.3 then, by Theorem 6.8, the resulting IPN is event-detectable.

Now, let $a = r \cup \{p_w, p_x, \cdots, p_y\} = \{p_r, p_s, \cdots p_u, p_w, p_x, \cdots, p_y\} \in \tilde{R}$. Since the places of P_{SD} are set as measurable, every non-measurable place satisfies the SD conditions of Theorem 3.17, so the resulting net of attaching the corresponding sensors of the set a to the net, make the resulting net marking-detectable. Hence, the net is observable.

Therefore, $\tilde{R}$ is the family set of available measurable nodes that make (Q, M_0) observable. ∎

Since we are searching a sensor configuration, the sets of available measurable nodes must be converted to sets of available sensors.

Proposition 6.17 *Let (Q, M_0) be a non-measured IPN, S be the set of available sensors and t_j, t_k be two transitions of the net.*

1. The family set of available sensors that make detectable the transition t_k is

$$\begin{aligned} \chi(t_k) &= \varkappa_S(\tilde{S}(t_k)) \\ &= \{\{s_r\} | (p_r \in^{\bullet} (t_k)^{\bullet} \setminus^{\bullet} (t_k) \cap (t_k)^{\bullet}) \wedge s_r = \varkappa(p_r) \in S\} \end{aligned}$$

2. The family set of available sensors that make transitions t_j and t_k distinguishable from each other is

$$\chi_{j,k} = \varkappa''_S(\tilde{S}_{j,k})$$

Proof. Straightforward from Lemma 6.16 and the definition of function $\varkappa''$ ∎

Observe that, if for any $t_k \in T$, $\tilde{S}(t_k) = \{\}$ then t_k is not detectable and the whole net is not event-detectable. On the other hand, notice that

$$\varkappa''_S(A) \sqcup \varkappa''_S(B) = \varkappa''_S(A \sqcup B)$$

where A, B are sets of measurable nodes. In this case, it is easy to prove that,

$$\hat{S} = \varkappa''_S(\tilde{R})$$

is the family set of available sensors that make the net observable.

Using the previous results the following algorithm to choose a minimal cost sensor configuration for observability is derived.

Algorithm 6.18 ***Minimal sensor choice for observability in live, cyclic and conservative*** IPN ***(first approximation)***

INPUTS:

C - incidence matrix, *λ the labeling function of transitions*

S set of available sensors, *$\varkappa_E$, $\varkappa_S$ - sensor assignment functions*

W - sensor cost function *P_{SD} places not satisfying the SD conditions*

OUTPUTS:

r^* a minimal cost sensor configuration, *w^* the minimal cost*

---

1. *Initialization*

$$R \leftarrow \{\{\}\}, Z \leftarrow \{\{\}\}$$

2. *Compute the set of available measurable nodes of each transition $t_k \in T$*

$$\tilde{S}(t_k) \leftarrow \{\{x\} | (x = p_r \in^{\bullet} (t_k)^{\bullet} \setminus^{\bullet} (t_k) \cap (t_k)^{\bullet}) \wedge \varkappa(x) \in S\}$$

3. ***IF*** $|T| = 1$ ***THEN*** $\{$

Assign the set of available measurable nodes of t to R: $R \leftarrow \tilde{S}(t)$

$\}$

4. ***IF*** $|T| \geq 2$ ***THEN*** $\{$

FOR i ***FROM*** 1 ***TO*** $|T| - 1$ ***DO*** $\{$

$Z \leftarrow \{\{\}\}$

FOR j ***FROM*** $i + 1$ ***TO*** $|T|$ ***DO*** $\{$

a) *Compute the f-union of the set of available measurable nodes of t_i and t_j:*

$$\tilde{S}_{i,j} = \begin{cases} \tilde{S}(t_i) \sqcup \tilde{S}(t_j), & \lambda(t_i) \neq \lambda(t_j) \\ \tilde{S}(t_i) \sqcup \tilde{S}(t_j) \setminus \{\{a, b\} \in \tilde{S}(t_i) \sqcup \tilde{S}(t_j) \mid & \\ [a \in^{\bullet} (t_i) \cap^{\bullet} (t_j) \vee a \in (t_i)^{\bullet} \cap (t_j)^{\bullet}] \wedge & , \quad \textit{otherwise} \\ [b \in^{\bullet} (t_i) \cap^{\bullet} (t_j) \vee b \in (t_i)^{\bullet} \cap (t_j)^{\bullet}] \; \} & \end{cases}$$

b) Join the sets of $\tilde{S}_{i,j}$ with the previous sets:

$$Z \leftarrow Z \sqcup \tilde{S}_{i,j}$$

}

Join the sets of Z with the previous sets:

$$R \leftarrow R \sqcup Z$$

}

}

5. *Join the sets of R to the set of places no-satisfying the SD conditions and eliminate redundances:*

$$\tilde{R} \leftarrow R \sqcup P_{SD}$$

6. *Convert the sets of available measurable nodes to available sensor configurations:*

$$\hat{S} \leftarrow \varkappa_S''(\tilde{R})$$

7. *Compute the minimal sensor cost:*

$$w^* \leftarrow r \in \hat{S} \min W(r)$$

8. *Select a sensor configuration r^* whose cost is equal to w^* :*

$$r^* \in \hat{S}$$
$$\textit{such that } W(r^*) = w^*$$

***9. RETURN** r^* and w^**

Observe that the algorithm contemplates the unavailability of sensors for certain nodes of the net, since the set $\tilde{S}(t)$ is used instead of the set $S(t)$. Moreover, also algorithm contemplates the fact that a state sensor may be attached to several places, since it does not makes any restriction respect to functions $\varkappa_E$, $\varkappa_S$.

Unfortunately, the cardinality of $\hat{S}$ can be too large. In fact the space complexity of the algorithm in the worst case is $(n+m)^2$ where n is the number of places and m is the number of transitions. However, in order to reduce the number of final sets in $\hat{S}$, some redundant sets can be removed just after a f-union operation is realized.

Definition 6.19 *The operation reduction of a family set A is*

$$\mathfrak{red}(A) = A \backslash \{a \in A | a \supsetneq b \in A\}$$

Some properties of the operation $\mathfrak{red}$ with respect to the f-union are the following:

1. $\mathfrak{red}(A \sqcup B) = \mathfrak{red}(\mathfrak{red}(A) \sqcup \mathfrak{red}(B))$,
2. $\mathfrak{red}(A \sqcup A) = \mathfrak{red}(A)$,
3. $\mathfrak{red}((A \sqcup B) \sqcup (A \sqcup C)) = \mathfrak{red}(A \sqcup B \sqcup C)$

The following lemma states that, the application of the reduction operation does not affect the selection of the minimal cost sensor configuration.

Lemma 6.20 *Let A and B be two families of sets of available measurable nodes.*

$$min(\{r = W(c_i) | c_i \in \varkappa_S(\mathfrak{red}(A \sqcup B))\}) = \min(\{r = W(d_i) | d_i \in \varkappa_S(A \sqcup B)\})$$

where W is the sensor cost function.

Proof. *Let $r = \{a_i, a_j, \dots, a_k\}$, $s = \{a_r, a_s, \dots, a_v\} \in A \sqcup B$, such that $r \subset s$, so r has redundant elements. By translating these sets into sensors, the sets $\hat{r} = \{\varkappa(a_i), \varkappa(a_j), \dots, \varkappa(a_k)\}$ and $\hat{s} = \{\varkappa(a_r), \varkappa(a_s), \dots, \varkappa(a_v)\}$ are obtained. Since $r \subset s$, s has more nodes than r. Thus, $\hat{r} \subseteq \hat{s}$ and $w(\hat{r}) \leq w(\hat{s})$. Hence, s can be removed from $A \sqcup B$ without affecting the minimal value selection.* ■

A way of limiting the number of sets in $\hat{S}$ is to avoid certain f-union operations in sets Z_i of R that result in redundant sets.

Lemma 6.21 *Let (Q, M_0) be a non-measured IPN such that $|T| \geq 2$ and $R = \bigsqcup_{i=1}^{|T|} Z_i$ be the family set that makes distinguishable every transition of the net.*

$$\mathfrak{red}(R) = \mathfrak{red}\left(\bigsqcup_{i=1}^{|T|-1} \bigsqcup_{j=i+1}^{|T|} \tilde{S}_{i,j}\right)$$

Proof. By definition, $R = \bigsqcup_{i=1}^{|T|} Z_i = Z_1 \sqcup Z_2 \sqcup \dots \sqcup Z_{|T|} = \tilde{S}_{1,2} \sqcup \tilde{S}_{1,3} \sqcup \dots \sqcup \tilde{S}_{2,1} \sqcup \tilde{S}_{2,3} \sqcup \dots \sqcup \tilde{S}_{|T|,1} \sqcup \tilde{S}_{|T|,2} \sqcup \dots \sqcup \tilde{S}_{|T|,|T|-1}$. By commutativity, $\tilde{S}_{i,j} = \tilde{S}_{j,i}$; thus, $R = \tilde{S}_{1,2} \sqcup \tilde{S}_{1,2} \sqcup$

$\tilde{S}_{1,3} \sqcup \tilde{S}_{1,3} \sqcup \ldots \sqcup \tilde{S}_{|T|-1,|T|} \sqcup \tilde{S}_{|T|-1,|T|}$. By applying the $\mathfrak{red}$ operation to R, $\mathfrak{red}(R) = \mathfrak{red}(\tilde{S}_{1,2} \sqcup \tilde{S}_{1,2} \sqcup \tilde{S}_{1,3} \sqcup \tilde{S}_{1,3} \sqcup \ldots \sqcup \tilde{S}_{|T|-1,|T|} \sqcup \tilde{S}_{|T|-1,|T|})$. Since $\mathfrak{red}((A \sqcup B) \sqcup (A \sqcup C)) = \mathfrak{red}(A \sqcup B \sqcup C)$, $R = \mathfrak{red}(\tilde{S}_{1,2} \sqcup \tilde{S}_{1,3} \sqcup \ldots \sqcup \tilde{S}_{1,|T|} \sqcup \tilde{S}_{2,3} \sqcup \tilde{S}_{2,4} \sqcup \ldots \sqcup \tilde{S}_{|T-1|,|T|})$. Hence, $\mathfrak{red}(R) = \mathfrak{red}(\bigsqcup_{i=1}^{|T|-1} \bigsqcup_{j=i+1}^{|T|} \tilde{S}_{i,j})$.
■

Now, we prove that eliminating some $\tilde{S}_{i,j}$ does not affect the final selection of the minimal cost sensor configuration. In the following lemma, the set U contains all the sets $\tilde{S}_{i,j}$ of transitions t_i and t_j that have the same input symbol and at least one common input or output place; while, the set V consists of sets $\tilde{S}(t_k)$ of transitions no included in any set $\tilde{S}_{i,j} \in U$. It means that, if the places of a transition t_k are already considered in a set $\tilde{S}_{i,j} \in U$, then they do not need to be considered again since they generate redundant sets; thus, $\tilde{S}(t_k)$ can be removed from the f-union.

Lemma 6.22 *Let (Q, M_0) be a non-measured IPN such that $|T| \geq 3$ and $R = \bigsqcup_{i=1}^{|T|} Z_i$ be the family set that makes distinguishable every transition of the net.*

$$\mathfrak{red}(R) = \mathfrak{red}\left(\left(\bigsqcup_{w=1}^{|U|} \tilde{S}^w_{i,j}\right) \sqcup \left(\bigsqcup_{h=1}^{|V|} \tilde{S}^h(t_k)\right)\right)$$

where $\tilde{S}^w_{i,j} \in U = \{\tilde{S}_{i,j} | S_{i,j} \subset S(t_j) \sqcup S(t_k)\}$ (see Lemma 6.11) and $\tilde{S}_h(t_k) \in V = \{\tilde{S}(t_k) | \nexists \tilde{S}^w_{k,j} \in U\}$.

Proof. By Lemma 6.21, $\mathfrak{red}(R) = \mathfrak{red}\left(\bigsqcup_{i=1}^{|T|-1} \bigsqcup_{j=i+1}^{|T|} \tilde{S}_{i,j}\right) = \mathfrak{red}(\tilde{S}_{1,2} \sqcup \tilde{S}_{1,3} \sqcup \ldots \sqcup \tilde{S}_{1,|T|} \sqcup \tilde{S}_{2,3} \sqcup \tilde{S}_{2,4} \sqcup \ldots \sqcup \tilde{S}_{|T-1|,|T|})$. Let us order the sets $\tilde{S}_{i,j}$ of R in the following way: $R = \tilde{S}^1_{i,j} \sqcup \tilde{S}^2_{i,k} \sqcup \ldots \sqcup \tilde{S}^q_{j,k} \sqcup \tilde{S}^{q+1}_{s,k} \sqcup \ldots \sqcup \tilde{S}^r_{i,s}$, such that $\forall_i \in \{1, 2, \ldots, q\}$ $\tilde{S}^i_{j,k} = \tilde{S}(t_j) \sqcup \tilde{S}(t_k)$; and $\forall_i \in \{q+1, q+2, \ldots r\}$ $\tilde{S}^i_{s,k} \subset \tilde{S}(t_s) \sqcup \tilde{S}(t_k)$.

Thus, for $i \in \{1, 2, \ldots, q\}$ every set $\tilde{S}^i_{j,k}$ can be split into $\tilde{S}(t_j) \sqcup \tilde{S}(t_k)$. Hence,

$$\begin{aligned}
\mathfrak{red}(R) &= \mathfrak{red}(\tilde{S}^1_{i,j} \sqcup \tilde{S}^2_{i,k} \sqcup \ldots \sqcup \tilde{S}^q_{j,k} \sqcup \tilde{S}^{q+1}_{s,k} \sqcup \ldots \sqcup \tilde{S}^r_{i,s}) \\
&= \mathfrak{red}(\tilde{S}(t_i) \sqcup \tilde{S}(t_j) \sqcup \tilde{S}(t_i) \sqcup \tilde{S}(t_k) \sqcup \ldots \sqcup \tilde{S}(t_j) \sqcup \tilde{S}(t_k) \sqcup \tilde{S}^{q+1}_{s,k} \sqcup \ldots \sqcup \tilde{S}^r_{i,s}) \\
&= \mathfrak{red}(\mathfrak{red}(\tilde{S}(t_i) \sqcup \tilde{S}(t_j) \sqcup \tilde{S}(t_i) \sqcup \tilde{S}(t_k) \sqcup \ldots \sqcup \tilde{S}(t_j) \sqcup \tilde{S}(t_k)) \sqcup \mathfrak{red}(\tilde{S}^{q+1}_{s,k} \sqcup \ldots \sqcup \tilde{S}^r_{i,s})) \\
&= \mathfrak{red}(\mathfrak{red}(\tilde{S}(t_i) \sqcup \tilde{S}(t_j) \sqcup \tilde{S}(t_k) \sqcup \ldots \sqcup \tilde{S}(t_m)) \sqcup \mathfrak{red}(\tilde{S}^{q+1}_{s,k} \sqcup \ldots \sqcup \tilde{S}^r_{i,s})) \\
&= \mathfrak{red}(\tilde{S}(t_i) \sqcup \tilde{S}(t_j) \sqcup \tilde{S}(t_k) \sqcup \ldots \sqcup \tilde{S}(t_m) \sqcup \tilde{S}^{q+1}_{s,k} \sqcup \ldots \sqcup \tilde{S}^r_{i,s})
\end{aligned}$$

Let $T' = \{t_i, t_j, t_k \ldots, t_m\}$ be the set of transitions corresponding to the sets $\tilde{S}^i_{j,k}$, where $i \in \{1, 2, \ldots, q\}$. Suppose that, for any $t_x \in T'$ $\exists \tilde{S}^w_{x,k}$ for any $w \in \{q+1, q+2, \ldots r\}$.

We prove that,

$$\mathfrak{red}(\tilde{S}(t_x) \sqcup \tilde{S}_{x,k}) = \mathfrak{red}(\tilde{S}_{x,k})$$

Let $\{p_r\} = a \in \tilde{S}(t_x)$ and $\{p_s, p_v\} = b \in \tilde{S}_{x,k}$. Two cases are possible: $a \subset b$ or $a \not\subseteq b$. In the first case, $p_r = p_s$ or $p_r = p_v$, then $c = \{p_r, p_v\} = b \in \tilde{S}_{x,k}$. In the second case, $d = \{p_r, p_s, p_v\} \in \tilde{S}_{x,k}$. Since the elements of the set $\tilde{S}(t_x)$ are included in $\tilde{S}_{x,k}$, both cases occur. Thus, by applying the Red operation the set d is eliminated, since $d \supset c$. Hence, $\mathfrak{red}(\tilde{S}(t_x) \sqcup \tilde{S}_{x,k}) = \mathfrak{red}(\tilde{S}_{x,k})$.

Therefore, some $\tilde{S}(t_j)$ sets can be omitted. Let $T" = \{t_a, t_b, \dots, t_c\} = T \setminus \{t_j \mid \exists \tilde{S}^i_{j,k}$, where $i \in \{1, 2, \dots, q\} \wedge \exists \tilde{S}^w_{j,k}$ where $w \in \{q+1, q+2, \dots r\}\}$.

$$\begin{aligned}\mathfrak{red}(R) &= \mathfrak{red}(\tilde{S}(t_a) \sqcup \tilde{S}(t_b) \sqcup \dots \sqcup \tilde{S}(t_c) \sqcup \tilde{S}_{q+1} \sqcup \dots \sqcup \tilde{S}_r) \\ &= \mathfrak{red}\left(\left(\bigsqcup_{h=1}^{|V|} \tilde{S}^h(t_k)\right) \sqcup \left(\bigsqcup_{w=1}^{|U|} \tilde{S}^w_{i,j}\right)\right)\end{aligned}$$

where $\tilde{S}^w_{i,j} \in U = \{\tilde{S}_{i,j} | S_{i,j} \subset S(t_j) \sqcup S(t_k)\}$ and $\tilde{S}_h(t_k) \in V = \{\tilde{S}(t_k) | \nexists \tilde{S}^w_{k,j} \in U\}$. ■

The following algorithm for selecting of a minimal cost sensor configuration for observability in live, cyclic and conservative IPN resumes the above lemmas.

Algorithm 6.23 ***Minimal sensor choice for observability in live, cyclic and conservative IPN (improved version)***

INPUTS:

C	- *incidence matrix,*	λ	*the labeling function of transitions,*
S	*set of available sensors,*	$\varkappa_E$, $\varkappa_S$	*sensor assignment functions,*
W	- *sensor cost function,*	P_{SD}	*places not satisfying the SD conditions*

OUTPUTS:

r^* *a minimal cost sensor configuration,* w^* *the minimal cost*

---

1. *Initialization:*

$$\begin{aligned} R &\leftarrow \{\{\}\}, \\ B &\leftarrow \{\{\}\}, \\ \hat{S} &= \{\}, \\ T_{Analyzed} &\leftarrow \{\} \end{aligned}$$

2. *Compute the set of available measurable nodes of each transition $t_k \in T$*

$$\tilde{S}(t_k) \leftarrow \{\{x\}|(x = p_r \in^{\bullet} (t_k)^{\bullet} \setminus^{\bullet} (t_k) \cap (t_k)^{\bullet}) \wedge \varkappa(x) \in S\}$$

3. IF $|T| = 1$ ***THEN*** {

Assign to R the set of available measurable nodes of t:

$$R \leftarrow \tilde{S}(t)$$

}

4. IF $|T| \geq 2$ ***THEN*** {

4.1 FOR i ***FROM*** 1 ***TO*** $|T|-1$ ***DO*** {

FOR j ***FROM*** $i+1$ ***TO*** $|T|$ ***DO*** {

IF $\lambda(t_i) = \lambda(t_j)$ ***and*** $({}^{\bullet}(t_i) \cap^{\bullet} (t_j) \neq \emptyset$ ***or*** $(t_i)^{\bullet} \cap (t_j)^{\bullet} \neq \emptyset)$ ***THEN*** {

a) *Compute the f-union of the set of available measurable nodes of* t_i *and* t_j*:*

$$\begin{aligned}&\tilde{S}_{i,j} \leftarrow \tilde{S}(t_i) \sqcup \tilde{S}(t_j) \setminus \{\{s_1, s_2\} \in S_{i,j} \mid \\ &[s_1 \in^{\bullet} (t_i) \cap^{\bullet} (t_j) \vee s_1 \in (t_i)^{\bullet} \cap (t_j)^{\bullet}] \wedge \\ &[s_2 \in^{\bullet} (t_i) \cap^{\bullet} (t_j) \vee s_2 \in (t_i)^{\bullet} \cap (t_j)^{\bullet}]\ \}\end{aligned}$$

b) *Eliminate the redundant sets of* $\tilde{S}_{i,j}$*:*

$$\tilde{S}_{i,j} \leftarrow \mathfrak{red}(\tilde{S}_{i,j})$$

c) *Join the sets of* $\tilde{S}_{i,j}$ *with the previous sets:*

$$R \leftarrow R \sqcup \tilde{S}_{i,j}$$

d) *Eliminate the redundant sets of R:*

$$R \leftarrow \mathfrak{red}(R)$$

e) *Set* t_i *and* t_j *as analyzed:*

$$T_{Analyzed} \leftarrow T_{Analyzed} \cup \{t_i, t_j\}$$

}

}

}

4.2 Compute the set of non-analyzed transitions:

$$T_{non-analyzed} = T \backslash T_{Analyzed}$$

4.3 FOR i ***FROM*** 1 ***TO*** $|T_{non-analyzed}|$ ***DO*** {

Compute the f-union of the sets $\tilde{S}(t)$ of the non-analyzed transitions, eliminating redundant sets:

$$B \leftarrow \mathfrak{red}(B \sqcup \tilde{S}(t_i))$$

}

4.4 Join the sets of R and B, eliminating redundant sets:

$$R \leftarrow \mathfrak{red}(R \sqcup B)$$

}

5. *Join the sets of R to the set of places no-satisfying the SD conditions and eliminate redundances:*

$$\tilde{R} \leftarrow R \sqcup P_{SD}$$

6. *Convert the sets of available measurable nodes to available sensor configurations:*

$$\hat{S} \leftarrow \mathcal{H}_S''(\tilde{R})$$

7. *Compute the minimal sensor cost:*

$$w^* \leftarrow r \in \hat{S} \min W(r)$$

8. *Select a sensor configuration r^* whose cost is equal to w^* :*

$$r^* \in \hat{S}$$
$$\textit{such that } W(r^*) = w^*$$

9. RETURN r^* *and* w^*

Theorem 6.24 *Let (Q, M_0) be a live, cyclic and conservative non-measured IPN, S be the set of available sensors of the net, $\varkappa_E$, $\varkappa_S$ be sensor assignment functions, W be a sensor cost function defined over 2^S and $P_{SD} = \{p_1^{SD}, \ldots, p_w^{SD}\}$ be the set of places not satisfying the SD conditions. The Algorithm 6.23 solves the minimal sensor choice problem for observability in (Q, M_0).*

Proof. By Lemma 6.22, the family set R, computed at step 3 or 4 contains the non-redundant sets of available measurable nodes that make distinguishable every transition of the net. By Lemma 6.16, the family set $\tilde{R}$, computed at step 5 contains all the non-redundant sets of available measurable nodes that make the net observable. Thus, the set $\hat{S} = \varkappa_S''(\tilde{R})$, computed at step 6, consists of all non-redundant sets of available sensors that make observable the net. At step 7, these sets are evaluated and w^* is computed as the minimal sensor cost, by Lemma 6.20. Thus, at step 8, $r^* \in \hat{S}$, such that $W(r^*) = w^*$ is selected as a minimal cost sensor configuration satisfying observability in the resulting IPN.

Therefore the algorithm 6.23 solves the minimal sensor choice problem for observability. ■

6.4 Example

The following example, adopted from [45], uses the Algorithm 6.23 to compute a minimal cost sensor configuration that preserves the observability property of an automated manufacturing system. The algorithm was implemented in a personal computer using Maple V 5.0.

Example 6.25 *Consider the automated manufacturing cell shown in Figure 6.3. The cell is devoted to produce two types of parts: $P1$ and $P2$. It consists of three machines: $M1$, $M2$ and $M3$, one robot: R, an incoming conveyor: CI and an outgoing conveyor: CO. The robot R handles the parts between machines, loading and unloading from the conveyors. To produce a $P1$ product, an available $P1$ part is processed in the machines in the following order: first in $M1$, then in $M2$ and, finally in $M3$. Similarly, to produce a $P2$ product, a $P2$ part is processed in $M2$, then in $M1$ and finally in $M3$.A PN model for this cell and the physical meaning of its nodes are shown in Figure 6.4.*

Suppose that there is an available sensor for each node of the net, which are attached to them as follows

$$\varkappa_S(p_i) = s_i$$

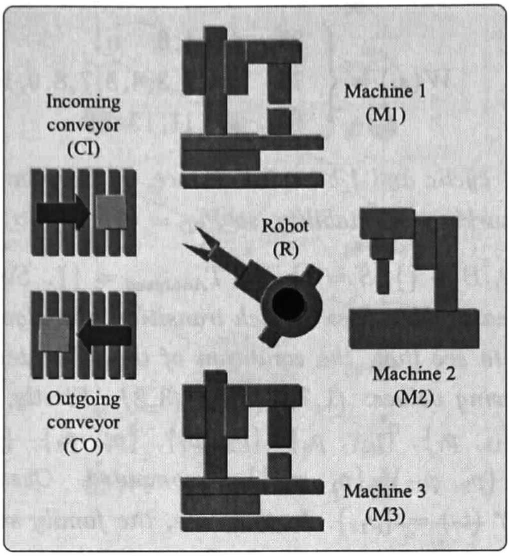

Figure 6.3: A typical automated manufacturing cell.

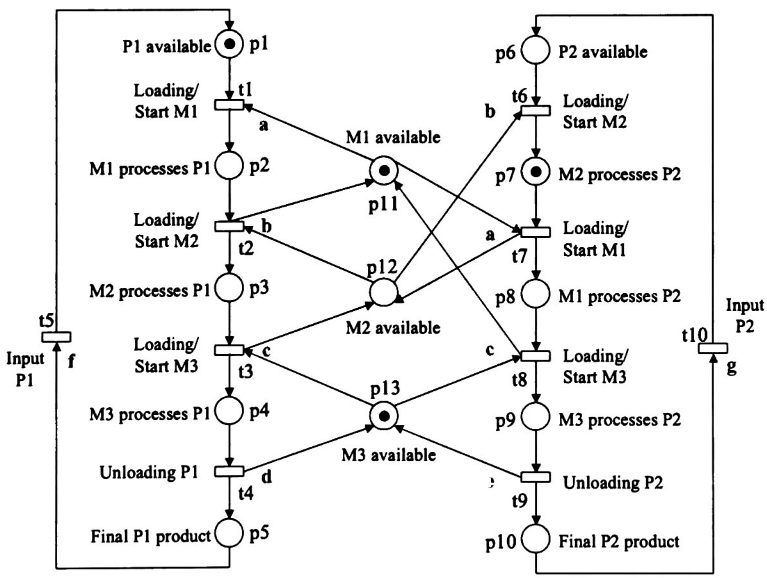

Figure 6.4: A PN model of the automated production cell.

The sensor cost function is the following:

$$W(s_i) = \begin{cases} 25, & i = 1,6 \\ 75, & i = 2,3,4,5,7,8,9,10 \\ 60, & i = 11,12,13 \end{cases}$$

Clearly, the net is live, cyclic and 1-bounded. Hence, by Corollary 4.7, every place satisfies the SD conditions for w-marking-detectability, so $P_{SD} = \{\}$.

Initially, $R = \{\{\}\}$, $B = \{\}$, $\hat{S} = \{\}$ *and* $T_{Analyzed} = \{\}$. *Since* $|T| = 10$, *after computing the sets of available measurable nodes of each transition, the algorithm enters the step 4. From Figure 6.4, it is easy to see that, the condition of the IF statement becomes true only when* i *and* j *take the following values:* $(1,7)$, $(2,6)$, $(3,8)$. *Firstly, when* $i = 1$ *and* $j = 7$. *The family set* $\tilde{S}_{1,7} = \{\{p_{11}, p_7\}, \{p_{11}, p_8\}, \{p_2, p_7\}, \{p_2, p_8\}, \{p_1, p_7\}, \{p_1, p_8\}, \{p_2, p_{12}\}, \{p_1, p_{11}\}, \{p_{11}, p_{12}\}, \{p_2, p_{11}\}, \{p_1, p_{12}\}\}$ *is computed. Observe that the set* $\{p_{11}\}$ *is not included, since* ${}^\bullet(t_1) \cap {}^\bullet(t_7) = \{p_{11}\}$. *In this case, the family set* $\tilde{S}_{1,7}$ *has no redundant sets; thus,* $\tilde{S}_{1,7} \leftarrow \mathfrak{red}(\tilde{S}_{1,7}) = \tilde{S}_{1,7}$. *Since this is the first family set computed at step 4,* $R = \tilde{S}_{1,7}$ *and* $T_{Analyzed} = \{t_1, t_7\}$.

Similarly, when, $i = 2$ *and* $j = 6$, *the family set* $\tilde{S}_{2,6}$ *is computed, and after removing the redundances,* $\tilde{S}_{2,6} \leftarrow \mathfrak{red}(\tilde{S}_{2,6}) = \{\{p_{11}, p_7\}, \{p_2, p_7\}, \{p_{11}, p_6\}, \{p_2, p_6\}, \{p_3, p_6\}, \{p_3, p_7\}, \{p_3, p_{12}\}, \{p_2, p_{12}\}, \{p_{12}, p_6\}, \{p_{12}, p_7\}, \{p_{11}, p_{12}\}\}$ *is obtained. This family set is f-joined to* R *and, after removing the redundances, the set* $R = \{\{p_{11}, p_7\}, \{p_2, p_7\}, \{p_1, p_3, p_7\}, \{p_1, p_{12}, p_7\}, \{p_1, p_3, p_6, p_8\}, \{p_2, p_6, p_8\}, \{p_{11}, p_6, p_8\}, \{p_1, p_{12}, p_6\}, \{p_2, p_{11}, p_6\}, \{p_1, p_3, p_{12}\}, \{p_1, p_{11}, p_6\}, \{p_2, p_{12}\}, \{p_{11}, p_{12}\}\}$ *is obtained. Now,* $T_{Analyzed} = \{t_1, t_2, t_6, t_7\}$.

In the same way, when, $i = 3$ *and* $j = 8$, *the family set* $\tilde{S}_{3,8}$ *is f-joined to the current* R *and then eliminated the redundances. Hence, after step 4:* $R = \{\{p_1, p_{11}, p_4, p_6\}, \{p_1, p_{11}, p_3, p_6\}, \{p_1, p_3, p_7, p_8\}, \{p_1, p_3, p_6, p_8\}, \{p_2, p_3, p_6, p_8\}, \{p_2, p_3, p_7, p_8\}, \{p_{11}, p_3, p_7\}, \{p_{11}, p_3, p_6, p_8\}, \{p_2, p_{12}, p_8\}, \{p_{11}, p_4, p_6, p_8\}, \{p_2, p_{11}, p_3, p_6\}, \{p_2, p_{12}, p_{13}\}, \{p_2, p_{12}, p_9\}, \{p_1, p_{12}, p_6, p_8\}, \{p_2, p_4, p_7, p_{13}\}, \{p_2, p_4, p_7, p_8\}, \{p_2, p_4, p_7, p_9\}, \{p_2, p_7, p_9, p_{13}\}, \{p_2, p_7, p_8, p_{13}\}, \{p_1, p_3, p_7, p_{13}\}, \{p_1, p_3, p_7, p_9\}, \{p_{11}, p_7, p_{13}\}, \{p_{11}, p_4, p_7\}, \{p_2, p_{11}, p_4, p_6\}, \{p_1, p_3, p_{12}, p_{13}\}, \{p_1, p_3, p_{12}, p_9\}, \{p_2, p_{11}, p_6, p_{13}\}, \{p_{11}, p_6, p_8, p_{13}\}, \{p_1, p_{11}, p_6, p_{13}\}, \{p_1, p_{12}, p_7, p_9\}, \{p_1, p_3, p_{12}, p_8\}, \{p_1, p_{12}, p_6, p_{13}\}, \{p_1, p_{12}, p_6, p_9\}, \{p_1, p_{12}, p_7, p_{13}\}, \{p_2, p_3, p_7, p_9\}, \{p_1, p_{12}, p_7, p_8\}, \{p_2, p_4, p_6, p_8\}, \{p_2, p_3, p_7, p_{13}\}, \{p_{11}, p_{12}\}, \{p_2, p_6, p_8, p_{13}\}\}$ *and* $T_{Analyzed} = \{t_1, t_2, t_3, t_6, t_7, t_8\}$.

Thus, $T_{non-analyzed} = \{t_4, t_5, t_9, t_{10}\}$. *In this case,* $B = \mathfrak{red}(\tilde{S}(t_4) \sqcup \tilde{S}(t_5) \sqcup \tilde{S}(t_9) \sqcup \tilde{S}(t_{10})) = \{\{p_5, p_6, p_{13}\}, \{p_5, p_6, p_9\}, \{p_1, p_6, p_{13}\}, \{p_1, p_{10}, p_4\}, \{p_1, p_4, p_6, p_9\}, \{p_1, p_{10}, p_{13}\}, \{p_5, p_{10}\}\}$.

Afterwards, at step 5, the family sets R *and* B *are f-joined an reduced the redundances,*

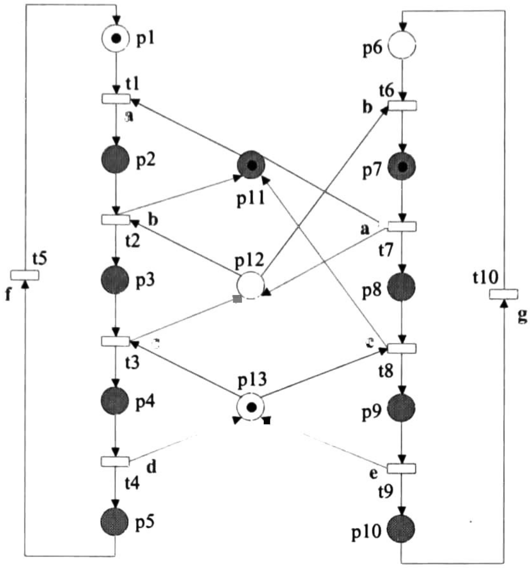

Figure 6.5: The resulting IPN of assigning the sensor configuration $a = \{s_1, s_6, s_{12}, s_{13}\}$.

$R = \mathfrak{red}(R \sqcup B)$, *resulting in 93 no redundant sets of available measurable nodes that make the net event-detectable. Since the net is live, cyclic and 1-bounded, these sets make also the net observable, i.e.* $P_{SD} = \{\{\}\}$ *and* $\tilde{R} = R$.

At step 6, the sets of $\tilde{R}$ *are translated into sets of available nodes. In this case, every place has an unshared available sensor.*

From these sensor configurations, at step 7, the minimal cost is computed. Thus, $w^* = 170$*, and the sensor configurations that have this cost are* $a=\{s_1, s_6, s_{12}, s_{13}\}$ *and* $b=\{s_1, s_6, s_{11}, s_{13}\}$*. In this case,* $P_m = \{p_1, p_6, p_{12}, p_{13}\}$ *or* $P_m = \{p_1, p_6, p_{11}, p_{13}\}$*. Figure 6.5 shows the resulting* IPN *of assigning the sensor configuration a.*

6.5 Discussion

In this chapter, a simple algorithm to choose a minimal cost sensor configuration for preserving observability of a live, conservative and cyclic Interpreted Petri Nets was derived. However it is also useful for w-observability or l-observability by providing the set of places that do not satisfy the SD conditions for w-observability or l-observability instead of the set of places that do not satisfy the SD condition for observability.

This algorithm exploits a characterization of the set of measurable nodes preserving the event-detectability property and certain synchronic distance conditions on the transitions of the net, which represent a sufficient condition for observability. The algorithm only considers state sensors. Additionally, the algorithm contemplates the unavailability of sensors for certain nodes of the net and the fact that a state sensor may be attached to several places.

Besides the sensor cost reduction, the algorithm allows to significantly reduce the system design time due to its simplicity. Finally, the algorithm was successfully applied to an illustrative example, where it was obtained a significant reduction in the number of sensors and in the acquisition cost.

Chapter 7

Asymptotic Observer Design

SUMMARY. This chapter is devoted to solve the asymptotic observer design problem in IPN. A procedure to design an asymptotic observer is presented. The observer convergency analysis is based on the observer estimation error, which is also represented as an IPN. This observer can be used to estimate the actual marking of an observable, W-Observable or L-observable IPN.

7.1 Introduction

In several real-time discrete event system applications such as state feed-back control [12] and fault tolerant systems [18], there exists the necessity of having a complete knowledge of the system state. However, in actual systems, it is not possible to associate a sensor to each state due to the technical limitations, economic reasons or avoiding complicated communication systems. In these cases, it is needed the use of an external entity called observer to estimate those states that cannot be directly measured. Moreover, in fault tolerant systems, when the system state becomes totally or partially unknown due to communication failures, an asymptotic observer can be used to estimate the system state.

This chapter presents a methodology to build asymptotic observers for systems modeled by observable, w-observable or l-observable IPN. This procedure is derived from a convergency analysis realized on the observer estimation error model, which is also represented as an IPN. This analysis allows to find out conditions that must be satisfied to achieve an asymptotic convergency of the observer state to the actual system state.

7.2 Observer issues

In Chapters 3, 4 and 5, it was presented a test to decide when a given IPN exhibits the observability, w-observability or l-observability properties. Also, in Chapter 6, it was proposed an algorithm to select a minimal cost sensor configuration for observability given a non-measured IPN. Once it is determined that an IPN satisfies observability, w-observability or l-observability, the next step is to provide a mechanism to compute the actual marking of the net. This goal can be achieved using the following procedure derived from the definition of sequence invariants.

1. Compute all possible initial markings that agrees the output y_0 :

$$R_0 = \{M \in \mathbf{R}(N, M_0) | \varphi(M) = y_0\}$$

2. Each time an output change $\Delta y_k = \varphi(M_k) - \varphi(M_{k-1})$ is detected:

   (a) Compute the transition that was fired by selecting the column of φC such that:

$$\begin{aligned} \varphi C(\bullet, k) &= \varphi(M_k) - \varphi(M_{k-1}) \text{ and} \\ \lambda(t_k) &= \omega_k \end{aligned}$$

   where ω_k is the input symbol accepted.

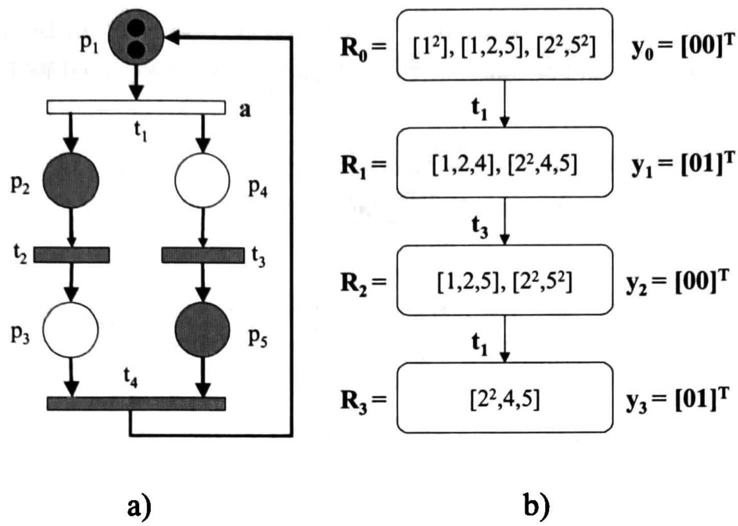

Figure 7.1: An observable IPN and its sets of reachable markings due to a given firing sequence.

(b) Compute the set of possible reached markings by firing t_k that agree the output $y_k = \varphi(M_k)$

$$R_k = \{M \in \mathbf{R}(N, M_0) | M_{k-1} \xrightarrow{t_k} M \wedge \varphi(M) = y_k\}$$

3. Goto Step 2.

Example 7.1 *Consider the observable IPN shown in Figure 7.1.a). Suppose that, the initial marking of this net is unknown. Since the initial output is* $y_0 = [00]^T$ *the set of possible markings is* $R_0 = \{[1^2], [1,2,5], [2^2,5^2]\}$*, i.e. all the reachable markings* M *such that* $\varphi M = [00]^T$

Now, suppose that, the transition t_1 *is fired at* M_0 *and the output* $y_1 = [01]^T$ *is generated by the net. In this case,* $R_1 = \{[1,2,4], [2^2,4,5]\}$ *where* $\varphi([1,2,4],) = \varphi([2^2.4,5]) = y_1$ *and these markings could be reached from a marking of* R_0 *by firing* t_1*. It means that, the actual marking is* $[1,2,4]$ *or* $[2^2,4,5]$.

Figure 7.1b) shows the estimation tree for the firing sequence $\sigma = t_1t_3t_1$*. Observe that,* $|R_3| = 1$*, so the marking* $M = [2^2,4,5]$ *is the actual marking of the net after firing* σ.

Although this approach is quite simple to understand and to implement, it implies the computation of the whole reachability set of the net in order to compute every set R_i, i.e. a NP problem. In order to avoid problems like this, an asymptotic observer can be used to estimate the system state. In particular, an asymptotic observer can be used to estimate the system state as it evolves. Formally an asymptotic observer is defined as follows.

Definition 7.2 *Let S be an observable discrete event system and O be a model of S. The model O is an asymptotic observer of S if and only if for every input word ω accepted by S and O,*

$$\begin{array}{ll} \mathit{1)} & \|x_0 - \hat{x}_0\| \geq \|x_1 - \hat{x}_1\| \geq ... \geq \|x_k - \hat{x}_k\| \text{ and} \\ \mathit{2)} & \lim\limits_{|\omega| \to \infty} \|x_k - \hat{x}_k\| = 0. \end{array}$$

where x_i and $\hat{x}_i$ are the system and observer states at the i-th event, respectively; $|\omega|$ is the length of the input word and $\|x_i - \hat{x}_i\| = e_i$ is the norm of the estimation error.

In other words, if the same input word is accepted by the system and the observer, then the estimation error must tend asymptotically to zero while the length of the input word increases. This goal can be achieved by selecting an appropriate observer initial condition and an effective observer dynamic.

7.3 Observer design

In this section, a methodology to design an asymptotic observer is presented. This methodology is derived from a convergency analysis realized on the observer estimation error model, which is also represented as an IPN.

In this work, the scheme of the asymptotic observer shown in Figure 7.2 is used. The asymptotic observer consists of

- An IPN system model extended with a set of output transitions (block F), whose firings are used to correct the estimation error.
- A block called *System Firing Detector*, which determines the transitions that have been firing in the system from the knowledge of the output difference, $y_{k+1} - y_k = \varphi(M_{k+1}) - \varphi(M_k)$, and the input word ω given to the system.
- A block called *Marking Corrector*, which generates firing sequences for the transitions of the block F.

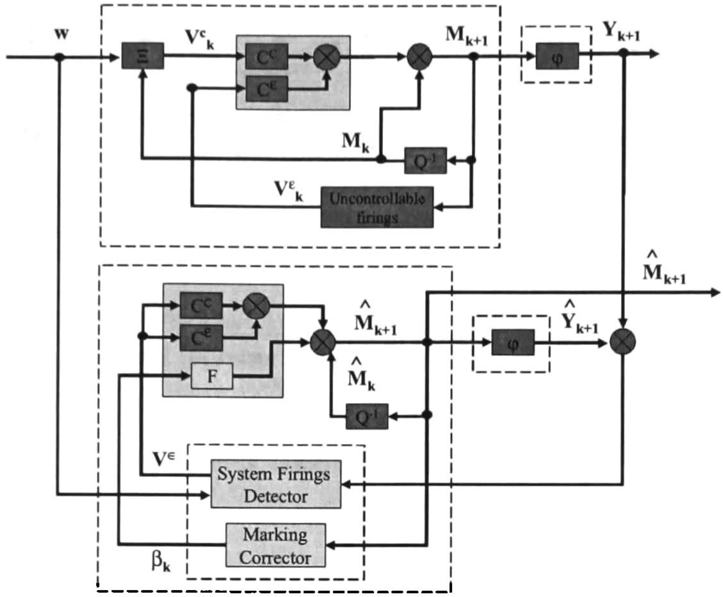

Figure 7.2: The block diagram of the system-asymptotic observer pair.

Definition 7.3 *Let S be a DES and $N_S = (N, \Sigma, \Phi, \lambda, \varphi)$ be an IPN model for S, where its state equation is:*

$$M_{k+1} = M_k + C^e \vec{v}^{\epsilon}_k + C^c \Xi(M_k, \vec{v}^c_k) \tag{7.1}$$
$$y_{k+1} = \varphi M_{k+1}$$

Then the IPN given by $N_O = (N, \Sigma, \Phi, \lambda = Id, \varphi = Id)$ is an observer for S if the state equation of N_O is:

$$\hat{M}_{k+1} = \hat{M}_k + C^e \vec{v}^{\epsilon}_k + C^c \Xi(M_k, \vec{v}^c_k) + \Gamma(\hat{M}_k - M_k) \tag{7.2}$$
$$\hat{y}_{k+1} = \varphi \hat{M}_{k+1}$$

where Id is the identity matrix.

Note that, the observer has the same basic structure of the system. However, in the observer, all the places are measurable and all transitions are controllable. Moreover, the terms $C^e \vec{v}^{\epsilon}_k$ and $C^c \Xi(M_k, \vec{v}^c_k)$ are the same of the system since the *System Firing Detector* block computes them and the incidence matrix C is known; while the term $\Gamma(\varphi \hat{M}_k - \varphi M_k)$ represents the *Marking Corrector* block.

Based on the above definition, the observer estimation error is given by the equation

$$e_{k+1} = \hat{M}_{k+1} - M_{k+1} \tag{7.3}$$

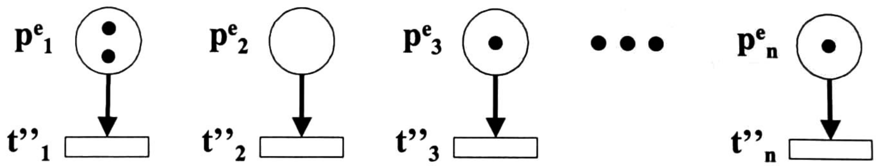

Figure 7.3: The equation of the observer estimation error represented as an *IPN*.

By substituting equations 7.1 and 7.2 in equation 7.3,

$$\begin{aligned} e_{k+1} &= \hat{M}_{k+1} - M_{k+1} \\ &= \hat{M}_k + C^e \vec{v}_k^e + C^c \Xi(M_k, \vec{v}_k^c) + \Gamma(\varphi \hat{M}_k - \varphi M_k) - M_k + C^e \vec{v}_k^e + C^c \Xi(M_k, \vec{v}_k^c) \\ &= \hat{M}_k - M_k + \Gamma(\varphi \hat{M}_k - \varphi M_k) \\ &= e_k + \Gamma(\varphi e_k) \end{aligned}$$

where $e_k = \hat{M}_k - M_k$.

Thus, the dynamics of the observer estimation error is

$$e_{k+1} = e_k + \Gamma(\varphi e_k) \tag{7.4}$$

Since Γ is a design parameter, the following value is proposed for Γ:

$$\Gamma(\varphi e_k) = F\vec{\beta}_k = -Id(\vec{\beta}_k) \tag{7.5}$$

where Id is an identity matrix of appropriated dimensions and $\vec{\beta}_k$ is a firing vector. In this case, the error equation becomes:

$$e_{k+1} = e_k - Id(\vec{\beta}_k) \tag{7.6}$$

Observe that equation (7.6) represents an *IPN*, like the one depicted in Figure 7.3 where all transitions are controllable. In the figure, the places of the error net are labeled as $p_1^e, p_2^e, \ldots, p_n^e$. Thus, $M_k(p_i^e) = e_k(p_i) = |M_k(p_i) - \hat{M}_k(\hat{p}_i)|$.

Using the above result, the observer equation can be stated as follows:

$$\hat{M}_{k+1} = \hat{M}_k + [C \mid -Id] \begin{bmatrix} \vec{v}_k \\ \vec{\beta}_k \end{bmatrix} \tag{7.7}$$

$$\hat{y}_{k+1} = \hat{M}_{k+1}$$

where $F = -Id$ and $\vec{v}_k$ is the transition sequence fired in the system.

Now, if the observer initial marking is greater or equal than the system initial marking, it could be concluded that

$$e_0 = \hat{M}_0 - M_0 \geq 0$$

i.e. the initial marking in the error equation is:positive; thus, it is a valid IPN marking. In order to fullfil this requirement, the observer initial marking can be defined as follows:

Definition 7.4 *Given an observable IPN, (Q, M_0), where a CML is defined, and its observer net, N_O. The observer initial marking $\hat{M}_0$ is:*

$$\hat{M}_0(\hat{p}_i) = \begin{cases} M_0^e(\hat{p}_i) + M_0^a(\hat{p}_i), & p_i \in P_{nm} \\ M_0(p_i), & otherwise \end{cases}$$

where

$$M_0^e(\hat{p}_i) \left\{ \begin{array}{ll} \min(\{M(\hat{p}_i) \mid M(\hat{p}_i) = k_j - A_j M_0^a\} \cup \{M_L^{UB}(p_i)\}), & p_i \in P_{nm} \\ 0, & otherwise \end{array} \right\} \tag{7.8}$$

where k_j is the total amount of tokens in the $j-th$ CML, $A_j = \begin{bmatrix} \alpha_1^j & \ldots & \alpha_n^j \end{bmatrix}$ are the coefficients of the $j-th$ CML such that $\alpha_i^j \geq 1$, and

$$M_0^a(\hat{p}_k) = \begin{cases} M_0(p_k), & p_k \in P_m \\ \hat{M}_0(\hat{p}_k), & p_k \in ||\xi_r|| \text{ for any solved } \xi_r \in CML \\ 0, & otherwise \end{cases}$$

is the marking of the measurable places belonging to the $j-th$ CML

For a place $\hat{p}_i$ the marking $M_0^a(\hat{p}_i)$ represents the actual number tokens in $\hat{p}_i$, while $M_0^e(\hat{p}_i)$ is an estimation of the number of tokens that may be in $\hat{p}_i$. Observe that, for the computation M_0^a, initially only the marking of the measurable places is known; however, if this information is enough to solve some of the conservative marking laws, then the marking of some non-measurable places becomes known and this knowledge is used to solve another conservative marking law. This procedure is repeated until it is not possible to solve another conservative marking law or the complete CML has been solved.

Note that, in the case of observable or W-observable IPN, the term$\{M_L^{UB}(p_i)\}$ has no effect in the equation 7.8 since $M_L^{UB}(p_i) = M^{UB}(p_i)$. This term is used only in the case of L-observable nets where $M_L^{UB}(p_i) \leq M^{UB}(p_i)$.

In general, every observer reachable marking can be represented as

$$\hat{M}_k(\hat{p}_i) = M_k^a(\hat{p}_i) + M_k^e(\hat{p}_i)$$

where

$$M^e_{k+1}(\hat{p}_i) = \begin{cases} \min(& \{M(\hat{p}_i)\,|M(\hat{p}_i) = k_j - A_j M^a_{k+1}\} \cup \\ & \{M(\hat{p}_i)|M(\hat{p}_i) = M^{UB}_L(p_i) - A_j M^a_{k+1}\}), \quad p_i \in P_{nm} \\ 0, & \text{otherwise} \end{cases}$$

such that $\hat{p}_i \in ||\xi_j|| \in CML$ (i.e. $\alpha^j_i \geq 1$), k_j is the total amount of tokens in ξ_j, $A_j = \left[\begin{array}{ccc} \alpha^j_1 & \ldots & \alpha^j_n \end{array} \right]$ are the coefficients of ξ_j, and $\forall \hat{p}_r \in \hat{P}$

$$M^a_{k+1}(\hat{p}_r) = \max(0, M^a_k(\hat{p}_r) + C(r,k))$$

Thus, $\hat{M}_k(\hat{p}_i)$ is. the sum of the number of known tokens in p_i and the estimated number of tokens that may be in p_i. Note that, if p_i is a measurable place $\forall_k$ $M^e_k(\hat{p}_i) = 0$. Otherwise, if p_i is a non-measurable place then $M^e_k(\hat{p}_i) \geq 0$. On the other hand, M^a_{k+1} is computed from the evolution of the system.

As in the definition of the observer initial marking, in the case of observable or w-observable IPN, the term $\{M(\hat{p}_i)|M(\hat{p}_i) = M^{UB}_L(p_i) - A_j M'_{k+1}\})$ has no effect in the equation 7.2, since $M^{UB}_L(p_i) = M^{UB}(p_i)$. This term has effect in the case of L-observable nets since $M^{UB}_L(p_i) \leq M^{UB}(p_i)$.

The above representation of the observer marking has the advantage of distinguishing between the actual marking and the estimated marking of each non-measurable place. This fact can be exploited by a state feedback controller to compute the control law.

In order to reach the zero marking in the estimation error net, for every $p_i \in P$, since $e_0(p_i) > 0$, the transition t''_i must be fired $e_0(p_i)$ times, and then $e_k(p_i) = 0$.

However, if p_i is a non-measurable place, then the value $e_0(p_i) = \hat{M}_0(\hat{p}_i) - M_0(p_i)$ is unknown, because the marking $M_0(p_i)$ is unknown. Thus, the number of times that the transitions of $\mathcal{F}$ must be fired in order to reduce the error to zero is also unknown. To cope this problem, an estimate of the observer error and a firing policy for the transitions of $\mathcal{F}$ are proposed as follows.

Definition 7.5 *Let $N_S = (Q, M_0)$ be the IPN of a system S and N_O be its observer with an initial marking as in Definition 7.4. Suppose that the system transition t_j is fired at the marking M_k. The i-th element of the firing vector $\vec{\beta}_k$ of N_O is computed as:*

$$\beta_k(i) = \hat{M}_k(\hat{p}_i) + C(i,j) - [M^a_{k+1}(\hat{p}_i) + M^e_{k+1}(\hat{p}_i)] \qquad (7.9)$$

In the equation 7.9, the value $\hat{M}_k(\hat{p}_i) + C(i,j)$ represents the current observer marking reached after firing the transition $\hat{t}_j$; and, since the value, $M^a_{k+1}(\hat{p}_i)$ represents the actual tokens in $\hat{p}_i$ and $M^e_{k+1}(\hat{p}_i)$ the tokens that may be in $\hat{p}_i$, the value $M^a_{k+1}(\hat{p}_i) + M^e_{k+1}(\hat{p}_i)$ represents the estimated marking that must have the observer. Thus, the value $\beta_k(i)$ represents the number of exceeding tokens in the reached observer marking.

Lemma 7.6 *Let (Q, M_0) be the IPN model for a system S and O be its observer with an initial marking as in Definition 7.4. If the firing vector $\vec{\beta}_k$ is computed like in definition 7.5, then the observer estimation error is not increasing.*

Proof. By definition of $\hat{M}_0$, for a place $p_r \in P$, $e_0(p_r) = \hat{M}_0(p_r) - M_0(p_r) \geq 0$. Suppose that the system transition t_k is fired at M_k. Then, the corresponding observer transition $\hat{t}_k$ is enabled and also fired at $\hat{M}_k$. Let $\beta_{k+1}(r) = \hat{M}_{k+1}(\hat{p}_r) - M^a_{k+1}(\hat{p}_r) + M^e_{k+1}(\hat{p}_r))$. The following cases are possible:

1. Suppose that for a place $\hat{p}_r \in^\bullet (\hat{t}_k)$, $M^a_k = 0$:

   (a) For $\hat{p}_r \in^\bullet (\hat{t}_k)$,

   $$\begin{aligned}\hat{M}_{k+1}(\hat{p}_r) &= \hat{M}_k(\hat{p}_r) - 1\\ &= M^a_k(\hat{p}_r) + M^e_k(\hat{p}_r) - 1\end{aligned}$$

   On the other hand, $M^a_{k+1}(\hat{p}_r) = M^a_k$ and $M^e_{k+1}(\hat{p}_r) = M^e_k(\hat{p}_r) - 1$. Thus,

   $$\begin{aligned}\beta_{k+1}(r) &= M^a_k(\hat{p}_r) + M^e_k(\hat{p}_r) - 1 - [M^a_k(\hat{p}_r) + M^e_k(\hat{p}_r) - 1]\\ &= 0\end{aligned}$$

   and $e_{k+1}(p_r) = e_k(p_r)$.

   (b) Let $\hat{p}_v \in (\hat{t}_k)^\bullet$ such that $\hat{p}_v, \hat{p}_r \in ||\xi_i||$, where $\xi_i \in CML$. Thus,

   $$\begin{aligned}\hat{M}_{k+1}(\hat{p}_v) &= \hat{M}_k(\hat{p}_v) + 1\\ &= M^a_k(\hat{p}_v) + M^e_k(\hat{p}_v) + 1\end{aligned}$$

   While $M^e_{k+1}(\hat{p}_v) = M^e_k(\hat{p}_v) - 1$ and $M^a_{k+1}(\hat{p}_v) = M^a_k(\hat{p}_v) + 1$. Hence,

   $$\begin{aligned}\beta_{k+1}(v) &= M^a_k(\hat{p}_v) + M^e_k(\hat{p}_v) + 1 - [M^a_k(\hat{p}_v) + 1 + M^e_k(\hat{p}_v) - 1]\\ &= 1\end{aligned}$$

   and $e_{k+1}(p_v) = e_k(p_v) - 1$.

   (c) Let $\hat{p}_u \notin^\bullet (\hat{t}_k)^\bullet$ such that $\hat{p}_u, \hat{p}_r \in ||\xi_i||$, where $\xi_i \in CML$ such that $M^e_{k+1}(\hat{p}_u) = M^e_k(\hat{p}_u) - 1$. Thus,

   $$\hat{M}_{k+1}(\hat{p}_u) = \hat{M}_k(\hat{p}_u) = M^a_k(\hat{p}_u) + M^e_k(\hat{p}_u)$$

and $M^a_{k+1}(\hat{p}_u) = M^a_k(\hat{p}_u)$. Hence,

$$\begin{aligned}\beta_{k+1}(u) &= M^a_k(\hat{p}_u) + M^e_k(\hat{p}_u) - [M^a_k(\hat{p}_u) + M^e_k(\hat{p}_u) - 1] \\ &= 1\end{aligned}$$

and $e_{k+1}(p_u) = e_k(p_u) - 1$.

(d) For any other place $\hat{p}_z$, $M^e_{k+1}(\hat{p}_z) = M^e_k(\hat{p}_z)$ and $M^a_{k+1}(\hat{p}_z) = M^a_k(\hat{p}_z)$. Thus,

$$\beta_{k+1}(z) = 0$$

and $e_{k+1}(p_z) = e_k(p_z)$.

2. Suppose that for a place $\hat{p}_r \in^{\bullet} (\hat{t}_k)$, $M^a_k(\hat{p}_r) > 0$.

(a) Let $\xi_i \in CML$ such that $\hat{p}_r \in ||\xi_i||$. Since $M^a_k(\hat{p}_r) > 0$, then $\forall \hat{p}_h \in ||\xi_i||$, $M^a_{k+1}(\hat{p}_h) = M^a_k(\hat{p}_h)$. Thus, it is easy to see that

$$\beta_{k+1}(h) = 0$$

and $e_{k+1}(p_h) = e_k(p_h)$.

(b) For any other place, $\hat{p}_z$, $M^e_{k+1}(\hat{p}_z) = M^e_k(\hat{p}_z)$ and $M^a_{k+1}(\hat{p}_z) = M^a_k(\hat{p}_z)$. Thus,

$$\beta_{k+1}(z) = 0$$

and $e_{k+1}(p_z) = e_k(p_z)$.

Observe that in any case, $e_{k+1}(p_i) \leq e_k(p_i)$. Therefore, the error is not increasing. ∎

It means that, when an unknown token is moved from one place to another, it becomes known, so the number of unknown tokens of each place belonging to the same conservative marking law must be adjusted and the error decreases. Note that, it is not necessary that a place have reached its upper marking bound to fire its G-transition. On the other hand, when a known token is moved, no new information is obtained, so the error does not change.

Theorem 7.7 *Let (Q, M_0) be a cyclic, live, conservative and event-detectable IPN model for a system S, where M_0 is unknown. Let O be the observer of (Q, M_0) with an initial marking as in definition 7.4. If it is fired a sequence that satisfies the synchronic distance conditions of Corollary 3.22, 3.23 or 3.24 and the firing vector $\vec{\beta}_k$ is computed like in definition 7.5 then the observer estimation error asymptotically tends to zero.*

Proof. Let $\mathbb{C}_i$ a marking cycle of (Q, M_0) and σ_{C_i} its firing sequence. Let p_r be a non-measurable place. Suppose that it is fired a firing sequence $\sigma \in \bar{\sigma}_{C_i}$ that satisfies the condition $SD(\sigma_{C_i}, {}^\bullet(p_r), (p_r)^\bullet) = \mathbb{D}(p_r)$ of Corollary 3.22. Thus, in the sequence σ the difference in the number of firings between ${}^\bullet(p_r)$ and $(p_r)^\bullet$ is $\mathbb{D}(p_j) = M^{UB}(p_r) - M^{LB}(p_r)$. Suppose that σ is fired in S and, since the net is event-detectable then it is detected and fired also in O. The firing of σ in S leads to a marking M_k where $M_k(p_r) = M^{UB}(p_r)$. It means that all the tokens of the places belonging to the support of a $\xi_i \in CML$ where moved to p_r. In particular, the tokens whose localization was unknown at M_0 were also moved to p_r. Thus, the cases 1.b) and 1.c) of Lemma 7.6 have occurred $e_0(p_r)$ times. Hence, $\beta(j) = e_0(p_r)$ and therefore, $e_k(p_r) = 0$. Since this holds for every non-measurable place or at least one place of the support of every $\xi \in CML$, eventually the marking estimation error of every non-measurable place becomes equal to zero.

In a similar way, it can be also proved that the observer estimation is equal to zero after the firing of a sequence that satisfies the synchronic distance conditions of Corollaries 3.22, 3.23 or 3.24. ■

Theorem 7.8 *Let (Q, M_0) be a live, cyclic, conservative and event-detectable IPN, where M_0 is unknown. Let O be the observer of (Q, M_0) with an initial marking as in definition 7.4. If it is fired a sequence that satisfies the synchronic distance conditions of Corollaries 4.10, 4.11, 4.12 or 4.14 and the firing vector $\vec{\beta}_k$ is computed like in definition 7.5 then the observer estimation error asymptotically tends to zero.*

Proof. Similar to the proof of Theorem 7.7. ■

Theorem 7.9 *Let (Q, M_0) be a conservative and event-detectable IPN, where M_0 is unknown. Let O be the observer of (Q, M_0) with an initial marking as in definition 7.4. If it is fired a sequence that satisfies the synchronic distance conditions of Corollaries 5.15, 5.16, 5.17 or 5.18 and the firing vector $\vec{\beta}_k$ is computed like in definition 7.5 then the observer estimation error asymptotically tends to zero.*

Proof. Similar to the proof of Theorem 7.7. ■

7.4 Examples

The following example illustrates the use of the asymptotic observer proposed in the previous section to estimate the state of a typical manufacturing cell.

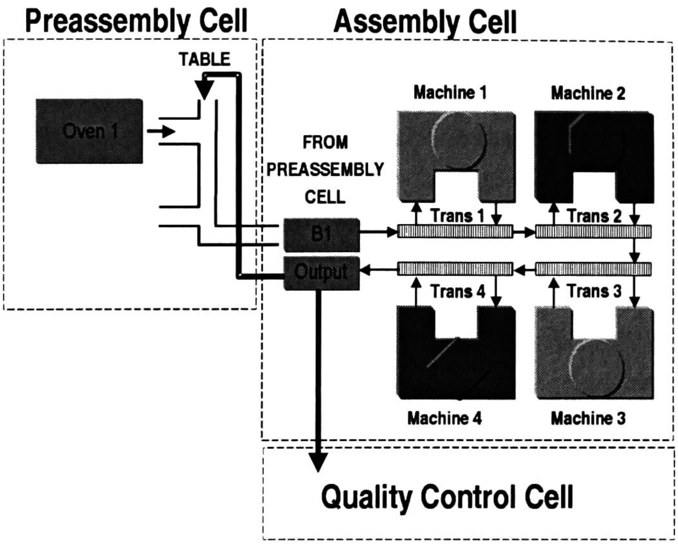

Figure 7.4: Scheme of an automated assembly cell dedicated to produce ovens.

Example 7.10 *Consider the automated cell for assembling microwave ovens depicted in Figure 7.4. The cell consists of four machines (M_1, M_2, M_3, M_4) and four transporting conveyors (T_1, T_2, T_3, T_4) to carry preprocessed parts from a machine to another. Once a part is on the i-th conveyor, it can by loaded into the i-th machine or transported to the next conveyor. A part visits the machines in a specific order depending on a pre-established production plan. A conveyor and a machine can hold only a part and, for security, only three machines can be working at once.*

An IPN model for a machine and its transporting conveyor is shown in Figure 7.5. Notice

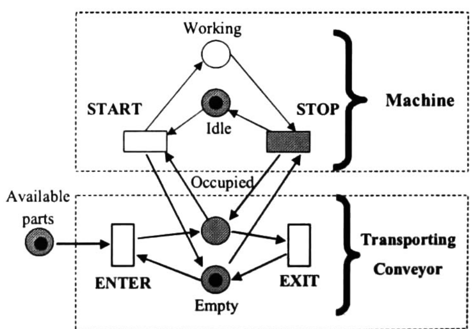

Figure 7.5: An *IPN* model for a machine and a conveyor of the assembly cell.

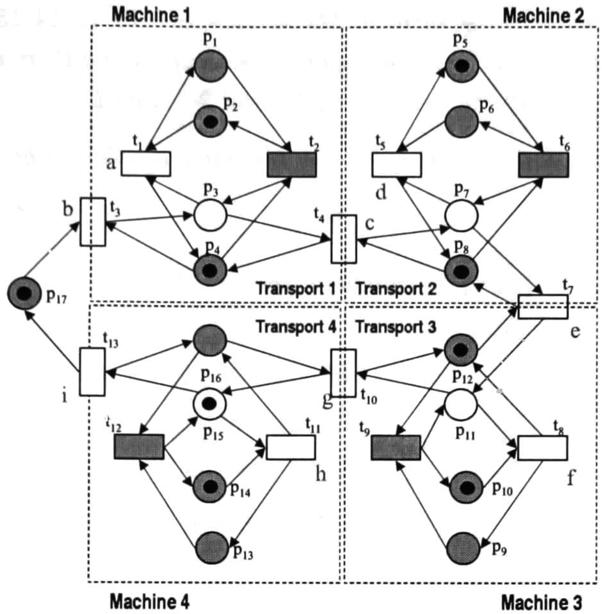

Figure 7.6: The complete IPN model of the automated assembly cell.

that, the place representing that the machine is working is measurable, while the transitions representing the events of entering and leaving the conveyor are controllable. Figure 7.6 shows the complete IPN model of the assembly cell. The place p_{17} represents available parts.

The functions λ and φ of the net are the following:

t_i	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}
$\lambda(t_i)$	a	ε	b	c	d	ε	e	f	ε	g	h	ε	i

$$\varphi = [e_3\ e_7\ e_{11}\ e_{15}]^T$$

The output matrix is

$$\varphi C = \begin{Bmatrix} -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \end{Bmatrix}$$

Observe that, $\varphi C(\bullet,2) = \varphi C(\bullet,3)$, but $\varepsilon = \lambda(t_2) \neq \lambda(t_3) = b$. Similarly, $\varphi C(\bullet,11) = \varphi C(\bullet,13)$, but $h = \lambda(t_{11}) \neq \lambda(t_{13}) = i$. Thus, by Proposition 3.10, the net is event-detectable.

The initial marking in list form is $M_0 = \{2,4,5,8,10,12,14,15,17\}$, *indicating that all machines, except* M_2 *are idle, only conveyor* T_4 *is occupied and there is one pre-assembled part. This initial marking makes the net live, cyclic and 3-bounded.*

According to the cell description, for any reachable marking it holds that:

1. *A machine can be idle or processing a part:*

$$\begin{aligned} M(p_1) + M(p_2) &= 1 \\ M(p_5) + M(p_6) &= 1 \\ M(p_9) + M(p_{10}) &= 1 \\ M(p_{13}) + M(p_{14}) &= 1 \end{aligned}$$

2. *A conveyor can be empty or occupied:*

$$\begin{aligned} M(p_3) + M(p_4) &= 1 \\ M(p_7) + M(p_8) &= 1 \\ M(p_{11}) + M(p_{12}) &= 1 \\ M(p_{15}) + M(p_{16}) &= 1 \end{aligned}$$

3. *Only three parts can be in the cell:*

$$\begin{gathered} M(p_1) + M(p_3) + M(p_5) + M(p_7) + \\ M(p_9) + M(p_{11}) + M(p_{13}) + M(p_{15}) + M(p_{17}) = 3 \end{gathered}$$

These equations form a CML. *From them, it can be established that*

$$M^{UB}(p_i) = \begin{cases} 3, & i = 17 \\ 1, & otherwise \end{cases}$$

and

$$\forall p_i \in P,\ M^{LB}(p_i) = 0$$

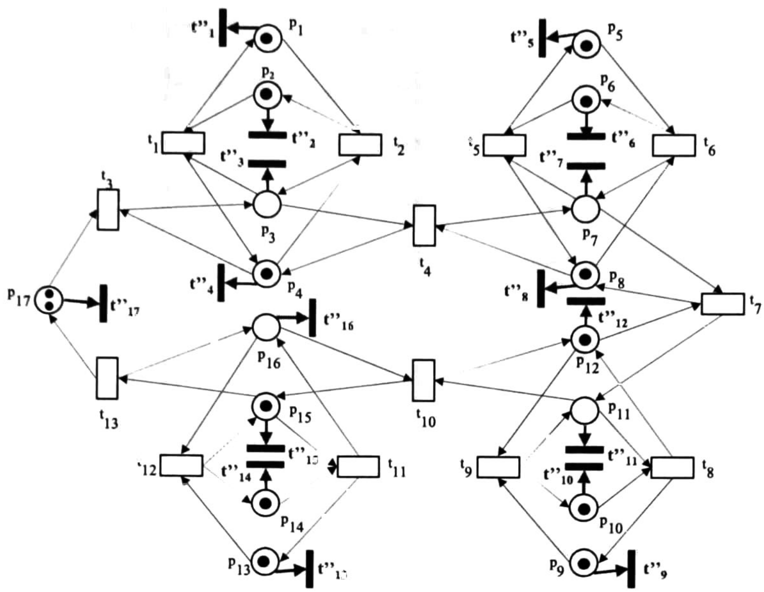

Figure 7.7: The observer net for the assembly cell.

It can be shown that,

cml i	p_k	$SD((Q, M_0), ^\bullet(p_k), (p_k)^\bullet)$	k_i
1	p_1	1	1
2	p_6	1	1
3	p_9	1	1
4	p_{13}	1	1
5	p_3	1	1
6	p_7	1	1
7	p_{11}	1	1
8	p_{16}	1	1
9	p_{17}	3	3

Thus, by Theorem 4.10, the net is w-observable and, by Theorem 7.7, an asymptotic observer can be constructed. Figure 7.7 shows the IPN observer net for the assembly cell with an initial marking computed as in Definition 7.4.

$$\hat{M}_0 = [1\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1\ 0\ 2]^T$$

Thus, the initial estimation error is

$$e_0 = [1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]^T$$

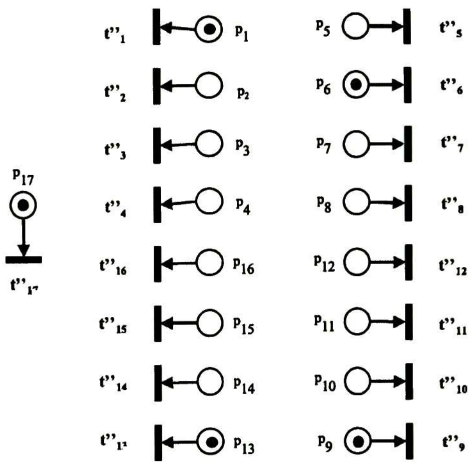

Figure 7.8: The observer error IPN which is marked with the initial error.

Figure 7.8 shows the observer error net with the initial error.

Initially,

$$\begin{aligned} M_0^a &= [0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,1\,0\,0]^T \\ M_0^e &= [1\,1\,0\,0\,1\,1\,0\,0\,1\,1\,0\,0\,1\,1\,0\,0\,2]^T \end{aligned}$$

Observe that, $y_0 = \hat{y}_0 = [0\ 0\ 0\ 1]^T$ *Suppose that, the uncontrollable transition* t_6 *fires, so the following marking is reached:*

$$M_1 = [0\,1\,0\,1\,0\,1\,1\,0\,0\,1\,0\,1\,0\,1\,1\,0\,1]^T$$

and the output $y_1 = [0\ 1\ 0\ 1]^T$ *is generated. Thus,* $\Delta y = y_1 - y_0 = [0\ 1\ 0\ 0]^T = \varphi C(\bullet, 4) = \varphi C(\bullet, 6)$*; however, since* $\omega = \varepsilon$*, it is determined that* t_6 *was fired. Since the transition* $\hat{t}_6$ *is enabled at* $\hat{M}_0$*, it is fired and the observer net reaches the marking*

$$\hat{M}_1^* = [1\,1\,0\,1\,0\,2\,1\,0\,1\,1\,0\,1\,1\,1\,1\,0\,2]^T$$

and,

$$\begin{aligned} M_1^a &= [0\,0\,0\,1\,0\,1\,1\,0\,0\,0\,0\,1\,0\,0\,1\,0\,0]^T \\ M_1^e &= [1\,1\,0\,0\,0\,0\,0\,0\,1\,1\,0\,0\,1\,1\,0\,0\,1]^T \end{aligned}$$

so

$$\begin{aligned} \vec{\beta}_1 &= \hat{M}_1^* - (M_1^a + M_1^e) \\ &= [0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1]^T \end{aligned}$$

or

$$\beta_1 = t''_6 t''_{17}$$

By firing $\vec{\beta}_1$, the observer marking is

$$\hat{M}_1 = [1\,1\,0\,1\,0\,1\,1\,0\,1\,1\,0\,1\,1\,1\,1\,0\,1]^T$$

while the current error is

$$e_1 = [1\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,0]^T$$

Now, suppose that the input word $\omega = b$ is given to the system, so t_3 is fired and the system reaches the marking

$$M_2 = [0\,1\,1\,0\,0\,1\,1\,0\,0\,1\,0\,1\,0\,1\,1\,0\,0]^T$$

Then, using a similar reasoning as in the previous firing, t_3 is identified by the observer, so $\hat{t}_3$ is fired and the new observer marking is

$$\hat{M}_2^* = [1\,1\,1\,0\,0\,1\,1\,0\,1\,1\,0\,1\,1\,1\,1\,0\,0]^T$$

and

$$\begin{aligned} M_2^a &= [0\,0\,1\,0\,0\,1\,1\,0\,0\,0\,0\,1\,0\,0\,1\,0\,0]^T \\ M_2^e &= [0\,1\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0]^T \end{aligned}$$

so

$$\vec{\beta}_2 = [1\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,0]^T$$

or

$$\beta_2 = t''_1 t''_9 t''_{13}$$

By firing $\vec{\beta}_2$, the observer reaches the marking

$$\hat{M}_2 = [0\,1\,1\,0\,0\,1\,1\,0\,0\,1\,0\,1\,0\,1\,1\,0\,0]^T$$

Thus,

$$e_1 = [0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0]^T$$

i.e. $\hat{M}_2 = M_2$. From this moment the observer marking will be equal to the system marking for any evolution of the system. ∎

The following example illustrates the design and the convergency of an asymptotic observer of a manufacturing cell constrained to a firing sublanguage.

Example 7.11 *Consider again the manufacturing cell of the previous example. Suppose that, its behavior is constrained to the cyclic language* $L(Q, M_0) = (t_3\ t_1\ t_2\ t_4\ t_3\ t_7\ t_1\ t_8\ t_2\ t_4\ t_9\ t_7\ t_{10}\ t_8\ t_{13}\ t_9\ t_{10}\ t_{13})^+$ *and its current marking is* $M_0 = [2\ 3\ 6\ 8\ 9\ 12\ 14\ 16\ 17]$*; however, due to a communication error this marking becomes unknown, so it is necessary to recover it through an asymptotic observer.*

From the previous example it is known that the net is event-detectable. By setting the CML *as the one of example 7.10, the lower and upper marking bounds with respect to* $L(Q, M_0)$ *are*

$$M_L^{LB}(p_i) = \begin{cases} 1, & i = 6, 14, 17 \\ 0, & otherwise \end{cases}$$

$$M_L^{UB}(p_i) = \begin{cases} 0, & i = 5, 13 \\ 3, & i = 17 \\ 1, & otherwise \end{cases}$$

while the synchronic distance between the input and output transitions of the non-measurable places with respect to $L(Q, M_0)$ *are the following:*

k	$\mathbf{SD((Q,M_0)\vert_L, {}^\bullet(p_k), (p_k)^\bullet)}$	$\mathbb{D}_L(p_k)$		**k**	$\mathbf{SD((Q,M_0)\vert_L, {}^\bullet(p_k), (p_k)^\bullet)}$	$\mathbb{D}_L(p_k)$
1	1	1		**10**	1	1
2	1	1		**12**	1	1
4	1	1		**13**	0	0
5	0	0		**14**	0	0
6	0	0		**16**	1	1
8	1	1		**17**	2	2
9	1	1				

Hence, by Theorem 5.12 (Q, M_0) *is marking-detectable with respect to* $L(Q, M_0)$*. In this case, by Theorem 5.14,* (Q, M_0) *is observable with respect to* $L(Q, M_0)$*, so an asymptotic observer can be constructed.*

Thus, the asymptotic observer initial marking is

$$\hat{M}_0 = [1\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 2]^T$$

computed as stated in Definition **??**. *Notice that, the markings of* $\hat{p}_5$, $\hat{p}_6$, $\hat{p}_{13}$ *and* $\hat{p}_{14}$ *are forced to their upper marking bound with respect to* $L(Q, M_0)$*. Thus, the initial estimation error is*

$$e_0 = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1]^T$$

|*Suppose that, transition t_4 fires. Thus,*

$$M_1 = [0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 1]^T$$

Since t_4 is detectable, $\hat{t}_4$ fires in the observer, reaching the marking

$$\hat{M}_1^* = [1\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 2]^T$$

Thus,

$$M_1^a = [0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 0]^T$$
$$M_1^e = [1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 2]^T$$

and

$$\vec{\beta}_1 = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$$

So $\hat{M}_1^* = \hat{M}_1 = [1\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 2]^T$ and $e_1 = e_0$.

Now, the uncontrollable transition t_9 fires. In this case,

$$M_2 = [0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 1]^T$$

Also, $\hat{t}_9$ is fired; thus,

$$\hat{M}_2^* = [1\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 2\ 1\ 0\ 0\ 1\ 0\ 1\ 2]^T$$

and

$$M_2^a = [0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 0]^T$$
$$M_2^e = [1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]^T$$

and

$$\vec{\beta}_2 = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]^T$$

i.e. $t"_{10}$ and t''_{17} are fired. The observer reaches the marking

$$\hat{M}_2 = [1\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 1]^T$$

and the estimation error is

$$e_2 = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$$

Following in the same way, after the firing of $\sigma = t_7\ t_{10}\ t_8\ t_{13}\ t_9\ t_{10}\ t_{13}\ t_3\ t_1 \in L(Q, M_0)$, the estimation error becomes equal to zero.

7.5 Discussion

This chapter presents a methodology to design asymptotic observers for Discrete Event Systems modeled by cyclic, live and conservative IPN. The proposed methodology is useful to estimate the actual marking of observable, w-observable or l-observable IPN.

The observer design parameters were derived from an analysis on the estimation error model, which is also represented as an IPN. Using the stability concepts presented in [30], it was shown that for the kind of proposed asymptotic observers, there exists a firing sequence, such that the estimation error becomes equal to zero.

The proposed asymptotic observer uses the information provided by the CML of the net to set an observer initial marking closer to the actual system state than other approaches. Also, the CML is used to compute the corrector firing vector $\vec{\beta}$ as the system-observer pair evolves. Moreover, by using the information provided by the evolution of the system, the observer convergency ratio is reduced.

The observer marking is represented as a sum of the actual number of tokens and the estimated number of tokens that could be in every place. The first value is computed from the system evolution, while the second value is computed using the CML. This fact can be exploited by a state feedback controller to compute the control law, since there is known the tokens which location is correct.

Chapter 8

Conclusions and Further Work

8.1 Conclusions

This work has addressed the observability, the minimal cost sensor choice for observability and asymptotic observer design problems in Discrete Event Systems (DES) modeled by Interpreted Petri Nets (IPN). The original contributions are the following:

For observability:

1. A definition of DES observability in IPN terms, which is independent of the method used to estimate the system state.

2. The concepts of input and output sequence invariants of an IPN were introduced. These invariants lead to establish a necessary and sufficient condition for observability. This powerful characterization of observability is similar to the one given for linear continuous systems using geometric approach. Although, the resulting test is quite simple, in the general case, the computation of the sequence invariants has a high computational complexity.

3. For live, cyclic and conservative IPN, the characterization using sequence invariants can be reduced to a structural characterization, which consists of the verification of the event-detectability and marking-detectability properties. Necessary and sufficient conditions for event-detectability, and sufficient conditions for marking-detectability are provided. Event-detectability can be tested in polynomial time, while for certain classes of nets marking-detectability also can be tested in polynomial time.

4. The previous characterizations of observability require that the synchronic distance be satisfied by every input word of length equal or greater than k. This condition seems to be too restrictive for a lot of nets. Then the notion of W-observability was introduced. The W-observability property is related to the existence of at least an input word that satisfies the synchronic distance. It has been shown, that event-detectability and W-marking-detectability are necessary and sufficient conditions for W-observability. Several sufficient conditions for W-marking-detectability have been provided, leading to several characterizations of W-observability. In particular, it has been shown that W-marking-detectability holds in every live, cyclic and conservative Free-Choice Petri net; hence, event-detectability is a necessary and sufficient condition for W-observability in this class of IPN.

5. The provided notions of observability and W-observability consider all the possible firing sequences of the net under study. However, the firing language of a SED is usually confined into a realizable and cyclic desired behavior. Thus, only the firing sequences belonging to the desired language occur during the system operation. In this context, the concept of observability with respect to a language, L-observability, has been introduced. In order to check this property, it has been introduced the concepts of event-detectability and marking-detectability with respect to a language, L-marking-detectability and L-event-detectability, respectively, resulting on several sufficient conditions for L-observability that can be tested in a polynomial time.

For sensor choice:

1. A structural characterization of the set of measurable places preserving the event-detectability property of a IPN is given.

2. A simple algorithm to choose a minimal cost sensor configuration for preserving observability of a live, bounded and cyclic IPN is provided This algorithm was derived from the characterization of measurable places preserving event-detectability and knowledge of the places no satisfying the synchronic distance conditions of the characterizations of marking-detectability. If the set of synchronic distance conditions for observability is changed by the set of synchronic distance conditions for W-observability or L-observability, the algorithm can be used to choose a minimal cost sensor configuration for preserving these two properties. The proposed algorithm only considers state sensors and assumes that the actuator signals are already attached to the transition of the net. Additionally, the algorithm contemplates the unavailability of sensors for certain nodes of the net and the fact

that a state sensor may be attached to several places. Besides the sensor cost reduction, the algorithm allows to significantly reduce the system design time due to its simplicity.

For the asymptotic observer design:

1. A methodology to design asymptotic observers for observable, L-observable or W-observable IPN is given. The observer consists of an extended IPN model where an output transition has been added to each place of the observer. These transitions allows to reduce the observer estimation error while the system evolves. Since the observer is also represented as an IPN, further analysis of the properties of the pair $DES - Observer$ such as stability and liveness can be performed using the well-known PN techniques. The observer uses a decreasing convergency method: the observer initial marking is equal or greater than the system initial marking; as the system evolves, a firing policy, derived from an analysis realized on the estimation error model, allows to reduce the error in a faster ratio than the previous approaches. This king of observers requires the knowledge of the total among of resources contained in the system (the CML) and, in the case of L-observable nets, the knowledge of the states visited by the firing sequences of the desired behavior is also required. Thus, the proposed observer can be used to estimate in a easy way the actual marking of observable, W-observable and L-observable IPN.

8.2 Recommendations for further work

It is recommended that further work be carried out in the following areas:

1. In this research, it has been seen that a necessary and sufficient condition for marking-detectability is the existence of a sequence that solves the CML. Although some characterizations of such sequences have been provided in this work, they represent only sufficient conditions. Hence, it is necessary to deeply study such sequences in order to obtain also necessary conditions.

2. The study of L-observability can be extended to non-cyclic firing sublanguages.

3. The proposed definitions of the observer initial marking and the firing policy of the additional output transitions can be extended to the observer proposed in [30], where there is also an input transition to each observer place. This will allow to have a wider range of observer initial markings and, may be, a faster convergency ratio.

4. The use of the state estimates of the presented asymptotic observers for state feedback control is also another open research area.
5. The presented results can be extended to nets with several home-spaces, where it is necessary to identify the home-space to which the current system marking belongs.
6. Also, the presented results can be extended to Timed Interpreted Petri Nets. In this case, it is necessary to study how the time information can help to reduce the convergency ratio.
7. Finally, it is desirable to implement the proposed asymptotic observer in a real system.

Bibliography

[1] L. Aguirre-Salas "Asymptotic observers for discrete event systems modeled by Petri nets. *MS Dissertation*, Guadalajara, Mexico: CINVESTAV, 1999, [in Spanish].

[2] L. Aguirre, A. Ramírez and O. Begovich. "Design of Asymptotic Observers for Discrete Event Systems." *Proc. of the IASTED International Conference on Intelligent Systems and Control.* Santa Barbara, CA. 1999, pp. 188-193.

[3] L. Aguirre, O. Begovich, A. Ramirez. Optimal Sensor Choice for Observability in Free-choice Petri Nets. To appear in: Proc. CCA/ISIC Conf., Mexico, September 2001.

[4] S. Bavichi and E. Chong. "Automated fault diagnosis using a discrete event systems framework." *Proc. of the IEEE Int. Symp. on Intelligent Control.* Columbus, Ohio. *1994.*

[5] R. D. Brandt, V. K. Garg and R. Kumar. "Formulae for calculating supremal controllable and normal sublanguages." *Systems and Control Letters*, vol. 15, pp. 1101-1117, 1990.

[6] Chi-Tsong Chen. "Linear system theory and design.' Harcourt Brace Jovanovich Inc. Sanders College Publishing. 1970.

[7] R. Cieslak, C. Desclaux, A. S. Fawaz, and P. Varaiya. "Supervisory control of Discrete-Event processes with partial observations." *IEEE Transactions on Automatic Control*, vol. 33, no. 3, pp. 249-260, 1988.

[8] C. M. Özveren and A. S. Willsky. "Observability of Discrete Event Dynamic Systems." *IEEE TAC*, vol. 35, no. 7, pp. 797-806, 1990.

[9] J. Desel and J. Esparza "Free choice Petri Nets." Cambridge University Press, 1995.

[10] J. Esparza. J. Esparza. "Model checking using net unfoldings." J. Science of Computer Programming, no. 23, pp.151-195, 1994.

[11] F. DiCesare, G. Harhalakis, J. M.Proth. M. Silva, F. B. Vernadat. Practice of Petri Nets in Manufacturing. Chapman & Hall Ed. Great Britain, 1993.

[12] A. Fanni, A. Giua and N. Sanna. "Control and error recovery of petri net models with event observers." *Proc. 2nd Int. Work. on Manufacturing and Petri Nets*, France, pp. 53-68, 1997.

[13] A. Fanni and A. Giua. "Discre Event representation of qualitative models using Petri nets." IEEE Trans on SMC, vol. 28, no.6, pp. 770-780, 1998.

[14] A. Giua. "Petri net state estimators based on event observation." *Proc. of the IEEE 36th CDC*, Dec. 1997.

[15] A. Haji-Valizadeh, K. A. Loparo. Minimizing the cardinality of an events set for supervisors of Discrete-Event Dynamical Systems. J IEEE TAC 1996; 41(11):1579-1593.

[16] J. Hopcroft and J. Ullman. *Introduction to Automata Theory, Languages and Computation*, Addison-Wesley Publishing Company, EEUU, 1979.

[17] A. Ichikawa and K. Hiraishi. "Analysis and Control of Discrete Event Systems represented by Petri nets." *Lecture notes in control and information sciences*, vol. 103, pp. 115-134, 1989.

[18] R. Isermann and P. Ballé. "Trends in the application of model-based fault detection and diagnosis of technical process." *Control Eng. Practice, Ed. Pergamon Press*, vol. 5, no. 5, pp. 709-719, 1997.

[19] R. Kumar and M. A. Shayman. "Formulae relating controllability, observability and co-observability." *Automatica*, vol. 34, no. 2, pp. 211-215.

[20] R. Kumar, V. K. Garg and S. I. Marcus. "On supervisory control of sequential behaviors." *IEEE Transactions on Automatic Control*, vol. 37, no. 12, pp. 1978-1985, 1992.

[21] M. Lawford, W. M. Wonham and J. Ostroff. "State-event observers for labeled transition systems." *Proc. of the 33rd CDC*, pp. 3642-3648.

[22] F. Lin and W. M. Wonham. "Decentralized Control and Coordination of Discrete-Event Systems with Partial Observation." *IEEE TAC*, vol. 35, no. 12, pp. 1330-1337, 1990.

[23] M. E. Meda and A. Ramírez. "Identification in discrete event systems." *Proc. of the IEEE International Conference Systems, Man and Cybernetics*, October 1998.

[24] T. Murata. "Petri nets: Properties, analysis and applications." *Proc. IEEE*, Vol. 77, no 4 pp. 541-580, Apr. 1989.

[25] Y. Park and K. P. Chong. "Sensor assignment for invertibility in interruptive timed discrete event systems." *Proc. of the IEEE International Symposium on Intelligent Control*. Columbus, Ohio, USA. 1994.

[26] J. H. Prosser, M. Kam and H. G. Kwatny. "On-line supervisor synthesis for partially observed Discrete-event systems." *IEEE Transactions on Automatic Control*, vol. 43, no. 11, pp. 1630-1634, 1998.

[27] F. DiCesare, J. M. Proth et at. "Practice of Petri nets in manufacturing." London, UK: Chapman & Hall Ed., 1993.

[28] P. J. Ramadge. "Observability of Discrete Event Systems." *Proc. of the IEEE 25th CDC*, pp. 1108-1112, 1986.

[29] A. Ramirez-Treviño, I. Rivera-Rangel and E. Lopez-Mellado. "Observer Design for Discrete Event Systems Modeled by Interpreted Petri Nets." *Proc. of the IEEE International Conference on Robotics and Automation*, San Francisco, USA. 2000.

[30] A. Ramírez-Treviño, L.Aguirre-Salas and O. Begovich. "Diseño de Observadores Asintóticos para Sistemas de Eventos Discretos basado en el Modelo del Error de Estimación" To appear in journal Computación y Systemas.

[31] I. Rivera-Rangel, L. Aguirre-Salas, A. Ramirez-Treviño and E. Lopez-Mellado. "Observer Design for Discrete Event Systems Modeled by Interpreted Petri Nets" *In 39th IEEE Conference on Decision and Control*, Sydney, Australia, 2000, pp 2260-2265.

[32] M. Sampath, R. Sengupta and S. Lafortune. "Diagnosability of discrete-event systems." *IEEE Transactions on Automatic Control*, vol. 40, no. 9, pp. 1555-1575. 1995.

[33] A. Giua and C. Seatzu. "Observability Properties of Petri Nets." I*n 39th IEEE Conference on Decision and Control*, Sydney, Australia, 2000.

[34] M. Silva. *Petri nets on automatica and data processing.* Madrid, Spain: AC Ed., 1985, [in Spanish].

[35] S. Takai, T. Ushio and S. Kodama. "Descentralized supervisory control using partial event and state observations." Proc. of the 34th CDC, pp. 723-724. 1995.

[36] S. Takai. "Optimal state-space partition for control of discrete-event systems with static specifications." *IEEE Transactions on Automatic Control*, vol. 43, no. 7, pp. 1013-1016, 1998.

[37] T. Ushio. "On the existence of finite-state supervisors under partial observations." *IEEE Transactions on Automatic Control*, vol. 42, no. 11, pp. 1577-1581, 1997.

[38] M. Valcher and J. Willems. "Observer Synthesis in the behavioral approach." *IEEE Trans. on Automatic Control*, vol. 44, no. 12, pp. 2297-2307. 1999.

[39] C.Wang and M. Schwartz. "Fault detection with multiple observers." *IEEE/ACM Trans on networking*, vol. 1, no. 1, pp. 48-55. 1993.

[40] J. Watson and A. Desrochers. "State-Space estimation of conservative Petri nets." Prc. of the 1992 IEEE International Conference on Robotics and Automation. Nice, France, 1992, pp.1037-1042.

[41] W. M. Wonham. *Linear Multivariable Control. A geometric approach.* New York, USA: Sprinver-Verlag, 1985.

[42] W. M. Wonham. *Notes on Control of Discrete-Event Systems.* Toronto University.1996.

[43] Y. Li and W. M. Wonham. "Controllability and Observability in the state-feedback Control of Discrete-Event Systems." *Proc. IEEE 27th CDC*, pp. 203-208, 1988.

[44] S. D. Young and V. K. Garg. "Optimal sensor and actuator choices for discrete event systems." *Proc. of 31st Allerton Conference on Communication Control and Computing.* Princeton, NJ. 1993.

[45] M. Zhou and K. Venkatesh. "Modelling, simulation and control of flexible manufacturing systems -a Petri net approach.' New York: World Scientific Ed., 1998.

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