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CINVESTAV

Centro de Investigación y de Estudios Avanzados del IPN
Unidad Guadalajara

Esquemas de observadores para SED modelados con RPI

Tesis que presenta
José Israel Rivera Rangel

Para obtener el grado de
Maestro en Ciencias

En la especialidad de
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Por:

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Esquemas de observadores para SED modelados con RPI

RESUMEN:

La observabilidad se define como la posibilidad de determinar de manera única y en un tiempo finito, el estado inicial de un sistema, a partir del conocimiento de sus entradas, salidas y estructura.

En este trabajo se extiende esa definición de observabilidad al ámbito de los Sistemas de Eventos Discretos (SED) modelados con Redes de Petri Interpretadas (RPI).

El problema de observabilidad se divide en dos partes: la posibilidad de detectar la ocurrencia de todos los eventos que se suceden en el sistema y la de determinar en algún momento su estado actual.

Se prueba que el primer problema puede resolverse en base al conocimiento de la estructura del modelo del sistema y de la salida. Para resolver el segundo problema se introduce la necesidad de conocer la cantidad de recursos con que cuenta el SED.

Se presentan condiciones suficientes para que el modelo de un SED en términos de RPI sea observable y finalmente, se presentan dos esquemas diferentes para diseñar observadores asintóticos en términos de RPI y que logran determinar de manera única el marcado actual del sistema cuando la secuencia de eventos que se han sucedido en él, cumple con cierta condición.

Observer schemes for DES modeled by IPN

SUMMARY:

Observability is defined as the possibility of uniquely determining in a finite time, the initial state of a system, from the knowledge of its inputs, outputs and structure.

In this work that definition of observability is extended to the scope of Discrete Event Systems (DES) modeled with Interpreted Petri Nets (IPN).

The observability problem is divided in two parts: the possibility of detecting the occurrence of all the events in the system and the one of determining at some moment its current state.

It is shown that the first problem can be solved on the basis of the knowledge of the structure of the model of the system and the outputs. In order to solve the second problem it is necessary to know the amount of resources contained in the DES.

Sufficient conditions for the observability of IPN modeling DES are presented. Finally, two different asympthotic observer schemes are given. These observers uniquely determine the current system marking when the event sequence occurring in it fulfills a certain condition.

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Chapter 1

General Introduction

Summary: At the end of this chapter it should be clear that discrete event systems are those where the state space is numerable and state value changes abruptly by the occurrence of events; also that observability is a property of dynamic systems that guaranties that the initial state of the system can be computed in a finite time from the knowledge of its inputs, outputs and structure; and that this property is important for control purposes, for fault tolerance and recovery and to determine the minimum number of sensors needed in a system.

1.1 Introduction

Consider a system devoted to maintain the temperature of a room within a certain rank of values. It is clear that temperature is a continuous value, but if significant thresholds are defined for it -in this case those could be "*bigger*", "*less*" and "*within*" the specified temperature rank- then it can be considered discrete. In the same way it can be established that the actuator -in this case the temperature regulator- can be "*increasing*", "*diminishing*" or "*maintaining*" the temperature. Under these assumptions it is possible to establish that the **system state** is a combination of the temperature in the room and the action that the actuator is making, i.e.: "*bigger-increasing*", "*bigger-diminishing*", "*bigger-maintaining*", etc. Notice that the set of values that the system state can take is numerable.

It can also be assumed that the "*heating*" and "*cooling*" of the room and the changes in the action that the actuator is making are instantaneous **events** that abruptly change the system state. Those events can be classified as controllable and no-controllable in the following way: since the changes in the action that the actuator is making depend only on the commands given to it, it is possible to say that those events are controllable. The "*heating*" and "*cooling*" of the room depend on factors like the temperature outside the room or the number of persons in it; that is why those events can be classified as no-controllable.

It is important to note that, because of malfunctions or communication failures, not all commands given to actuators are necessarily executed. Therefore, events can also be classified as measurable if there is a way to determine their occurrence and no-measurable in any other case. For the temperature regulator system example, only if it is assumed that the actuator emits a signal each time that a command is executed, then the changes in the action that it is making are measurable events.

Notice now that in order to determine the entire state of the temperature regulator system, besides the signal emitted by the actuator when a command is executed, a sensor devoted to measure the temperature in the room is needed. In that way, the temperature can be read from the sensor and the action that the actuator is making can be determined form the last command that the actuator has executed.

Systems that exhibit -by their own nature or because assumptions like those described above were made- a numerable state space and abrupt changes in the value of the state in consequence to the occurrence of events, are said to be **Discrete Event Systems** (DES). Under the assumptions described above, the temperature regulator system can be considered a DES.

Since the number of vehicles in a road, the data transmitted from a terminal to another one and in some cases the components of a product are numerable sets, traffic, computerized and manufacturing systems are usually considered as DES. The increasing importance of this kind of systems has made necessary to extend the results that exist for continuous system so that they can be applied to DES. Among those results are those that address the **observability** problem, and the design of **observers**. Observability deals with establishing the conditions that need to be held in a system so that its initial state can be computed in a finite time by the knowledge of its inputs, outputs and structure[4]; and the design of observers deals with establishing devices or algorithms to compute the system state.

1.2 Literature review

There are many works that deal with DES where the value of all states cannot be directly obtained by the system output, some of them [16][17][15][22] try to establish necessary conditions to guarantee that the system behavior can be restricted to stay within a set of desired behaviors. It will be shown in chapter 3 that it is not necessary to compute the system state to impose that restriction on the system behavior. For that reason most of the results presented in those works do not precisely deal with the observability problem.

Even when Özveren and Willsky in [20] present an observer that determines the system state at event intervals, no conditions to determine when the initial state can be computed, are given. An algorithm that uniquely determines the current state of the system is presented by Giua in [8]. The observability problem is presented as a necessary condition for controllability but it is addressed in a separated way. In that work it is assumed that all events that affect the state of the system are measurable and that the minimum value that each state can take is known. That value is assigned to the observer as an initial estimation of the system state and the estimation is improved each time that an event occurs in the system. Actually, since all events in the system are assumed to be known, the initial state can be computed once the current one has been determined. However, the assumption on the accessibility of all events is too restrictive, because -like in the temperature regulator system described above- it is common that external factors that are not controllable, affect the state of the system.

Aguirre *et al.* in [1] present a similar work. Interpreted Petri Nets (IPN), an extension to Petri Nets (PN), are used as DES modeling tool. Just like Giua, they assume that all events are known and reduce the estimation error as the system evolves. However, they take the maximum value that each state in the system can reach as the initial state estimation and present the algorithm to compute the state of the system as an observer in IPN terms.

The problem that arrises when not all events are known is addressed in [2]. It is shown that determining the occurrence of unknown events can be solved with a linear programing algorithm. Ichikawa and Hiraishi in [11] determine that any sequence of events can be computed if all the columns in the incidence matrix modeling the DES are *"additively independent"* Even when they present an easier solution to the problem, in that work the information provided by the events that are known is not used and the algorithm presented does only give an estimation of the system state and not a unique value.

1.3 Motivation

None of the works resumed above present a formal definition on observability for DES that captures the meaning of this property as it is understood in system theory. Besides none of them present an algorithm that -taking into account the existence of unknown events- uniquely determines the state of the system.

In order to optimize the instrumentation necessary to determine the state of a system, a simple characterization of observable DES is necessary. The design of observers -besides of computing the system state- can introduce in the system useful redundancy to provide security and fault detection, tolerance and recovery.

The following table shows some of the most important results included in works addressing the observability problem and the extension made herein to those works.

Year	Author	Results	Extensions in this work
1987	Ichikawa & Hiraishi	sufficient conditions to determine any transition firing sequence in a PN	
1988	Li & Wonham	supervisory control under partial observation	
1990	Özveren & Willsky	resilent observer for DES under partial observation	
1997	A. Giua	observer for PN models of DES were all events are accessible	observer for IPN models of DES were no accessible events are allowed
1999	Aguirre *et al.*	observer in IPN terms for DES were all events are accessible	observer that accepts different initial markings and no accessible events

1.4 Objectives and organization of this work

The objectives of this work are:

- To extend the existing definition on observability to a definition for IPN that accurately captures the meaning of this property as it is understood in system theory.
- To characterize observable IPN models.
- Designing an observer for IPN modeling DES where uncontrollable events are allowed.

This work is organized as follows:

In chapter 2 IPN and some of the properties that will be useful in the remaining of this work are introduced. Chapter 3 addresses the observability problem. An observability definition for IPN, that is an extension of the one used in system theory for continuous systems, is proposed. Also the concept of event-detectable system, which is a necessary condition for observability, is presented. Chapter 4 presents an asymptotic observer for IPN models. That observer is improved in chapter 5 to make it more general. Finally, some conclusions and future work are presented.

The results included herein have been presented or submitted to international conferences or magazines: [23] contains the results on the observability problem and observer design for DES modeled by binary IPN. [24] contains all the results presented herein for no-binary IPN, except for the observer scheme presented in chapter 5 which is included in [25].

Chapter 2

Interpreted Petri Nets

Summary: This chapter presents a PN extension called IPN that allows to assign a physical meaning to the models. It is accomplished by adding input and output alphabets and functions to PN. IPN are preferred among other modeling tools because they capture, in compact models, important characteristics of DES such as synchronism, concurrence, parallelism and mutual exclusion. IPN also provide qualitative and quantitative methods to analyze DES properties.

2.1 Introduction

There are several formalisms to model DES, one of the most popular is Finite Automata (FA) [10]. This tool describes systems as graphs where every different system state is represented as a node, and events by arcs going from a state to another. Models obtained with FA provide a clear description of small systems, however in complex systems the number of nodes in the model increases exponentially.

Another modeling tool for DES is PERT/CPM graphs. Even when there are fine analysis methods for these graphs, important DES characteristics such as cycles and decisions, cannot be captured by this tool. GERT graphs[21], an extension of the PERT/CPM graphs, is a more powerful modelling tool in the sense that it captures more DES characteristics, unfortunately not all GERT models can be analyzed.

PN is another formal tool for DES modeling. Models obtained in PN terms capture DES characteristics such as concurrence, parallelism, asynchronism, causal relationships and mutual exclusion. PN also provide a fine visual representation and a simple mathematical background to work with. For systems that exhibit parallelism, the size of the models obtained with this tool is considerably less than the size of those obtained in FA terms.

IPN is an extension to PN where input and output alphabets and labeling functions are added to give a physical meaning to models[18]. Being an extension to PN, IPN exhibit all modeling and analysis capabilities of PN.

In this chapter some basic concepts and properties of PN are presented. Also IPN, the extension to PN that is used herein as DES modeling tool is introduced.

2.2 Petri Nets

The formal definition of PN is given as follows:

Definition 2.1 *A **Petri Net structure** G is the 4-tuple $G = (P, T, I, O)$ where $P = \{p_1, p_2, ..., p_n\}$ and $T = \{t_1, t_2, ..., t_m\}$ are finite sets of elements respectively called places and transitions which are graphically represented as circles and bars. $I : P \times T \longrightarrow \{0, 1\}$ is a function representing the arcs going from places to transitions, and $O : P \times T \longrightarrow \{0, 1\}$ is a function representing the arcs going from transitions to places.*

The incidence matrix of G is $C = [c_{ij}]$, where $c_{ij} = O(p_i, t_j) - I(p_i, t_j)$. The function $M : P \longrightarrow \{\mathbb{Z}^{\geqslant 0}\}^n$ assigns a nonnegative integer to each place of the net, representing the number of tokens (depicted as dots into the places) residing in them.

Definition 2.2 *A **Petri Net** (PN) is the pair $N = (G, M_0)$, where G is the PN structure and M_0 is an initial token distribution.*

A transition t_j is enabled at a marking M if and only if $\forall p_i \in P$, $M(p_i) \geqslant I(p_i, t_j)$. An enabled transition t_j can be fired reaching a new marking M' which can be computed by the PN state equation: $M' = M + Cv$,

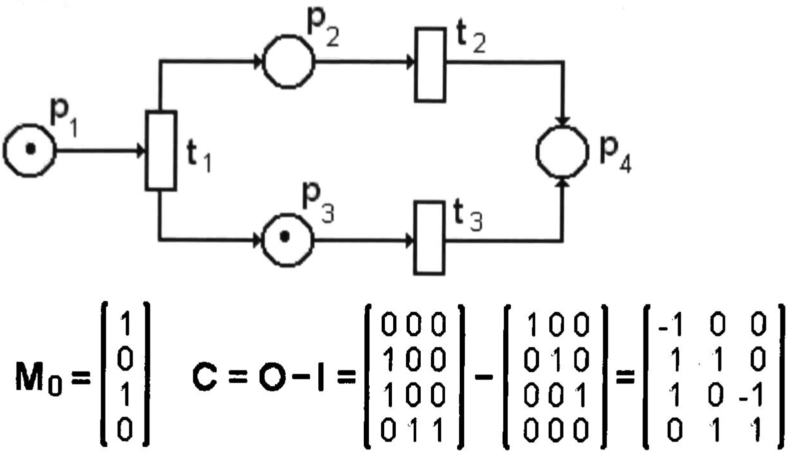


Figure 2.1: PN graphic representation, initial marking and incidence matrix.

where $v(i) = 0$ for $i \neq j$ and $v(j) = 1$ (that firing vector v can also be represented as $\overrightarrow{t_j}$). This fact is also represented as: $M \xrightarrow{t_j} M'$.

Example 2.1 *Figure 2.1 shows the pictorial representation of a PN, its initial marking and incidence matrix. According to that, the state equation for that PN is*

$$M_k = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} v_k$$

Notice that transition t_1 is enabled at marking M_0 because $\begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}^T \geqslant \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$. If transition t_1 is fired, then the new marking reached can be computed using the state equation in the following way:

$$M_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

Definition 2.3 *For a given transition $t_j \in T$, $\bullet(t_j)$ and $(t_j)\bullet$ denote the sets of all places p_i such that $I(p_i, t_j) \neq 0$ and $O(p_i, t_j) \neq 0$, respectively. Similarly, $\bullet(p_i)$ denotes the set of all transitions t_j such that $O(p_i, t_j) \neq 0$ and $(p_i)\bullet$ the set of all transitions t_j such that $I(p_i, t_j) \neq 0$.*

For the PN described in the previous example and depicted in figure 2.1, $\bullet(t_1) = \{p_1\}$, $(t_1)\bullet = \{p_2, p_3\}$, $\bullet(p_4) = \{t_2, t_3\}$ and $(p_4)\bullet$ is an empty set.

Definition 2.4 *A **firing sequence** of a PN (G, M_0) is a sequence $\sigma = t_i t_j ... t_k$ such that $M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} ... M_w \xrightarrow{t_k} ...$.*

Definition 2.5 *The* ***firing language*** *of* (G, M_0) *is:* $\pounds(G, M_0) = \{\sigma | \sigma = t_i t_j ... t_k$ *and* $M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} ... M_w \xrightarrow{t_k} ...\}$.

For instance, $t_3t_1t_2t_3$, $t_1t_2t_3t_3$, $t_1t_3t_2t_3$, $t_3t_1t_3t_2$, $t_1t_3t_3t_2$ and any prefix of those *words[10]* are firing sequences for the PN described in example 2.1 and depicted in figure 2.1. The set of all those firing sequences is the firing language of that PN.

A pair of well known concepts are the repetitive and conservative components of a PN[5], their formal definition is the following:

Definition 2.6 *Let C be the incidence matrix of the PN (G, M_0). A* ***t-semiflow*** *is a semi-positive rational-valued solution of equation $CY = 0$ and a* ***p-semiflow*** *is a semi-positive rational-valued solution of equation $XC = 0$.*

P-semiflows indicate that the addition of the markings of a set of places remains equal for any reached marking, t-semiflows indicate a transition firing sequence that when executed returns to the initial marking.

In the following example, t and p-semiflows are obtained for a PN.

Example 2.2 *Consider the PN depicted in figure 2.2. For this PN the incidence matrix is*

$$C = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \end{bmatrix}$$

and the initial marking is $M_0 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \end{bmatrix}^T$ *Vectors* $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \end{bmatrix}^T$ *and* $\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \end{bmatrix}^T$ *are semi-positive rational solutions for Y in the equation $CY = 0$ and vectors* $\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \end{bmatrix}^T$ *and* $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \end{bmatrix}^T$ *for X in $XC = 0$.*

If the transition sequence $\sigma_1 = t_1t_3t_5$, corresponding to $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \end{bmatrix}^T$ *is fired in that PN, the new marking M_j reached is equal to the initial marking M_0. The same holds for the transition firing sequence $\sigma_2 = t_2t_4t_5$, corresponding to* $\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \end{bmatrix}^T$ *That is the reason why those vectors are said to be repetitive components or t-semiflows in the PN.*

Notice that for any reachable marking M_k it holds that $M_k(p_1) + M_k(p_2) + M_k(p_4) = 2$ and $M_k(p_1) + M_k(p_3) + M_k(p_5) = 2$. Each equation respectively contains the places corresponding to $\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \end{bmatrix}^T$ *and* $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \end{bmatrix}^T$ *Since those vectors represent sets of places that conserve the same number of tokens for any reachable marking, it is said that they are conservative components or p-semiflows in the PN.*

Qualitative analysis methods available for PN can be classified as enumerative, structural, based on transformations and based on simulation.

Enumerative methods are based on the knowledge of the following set:

Definition 2.7 *The* ***reachability set*** *of a PN, $\mathbf{R}(G, M_0)$, is the set of all possible reachable markings from M_0 by firing enabled transitions.*

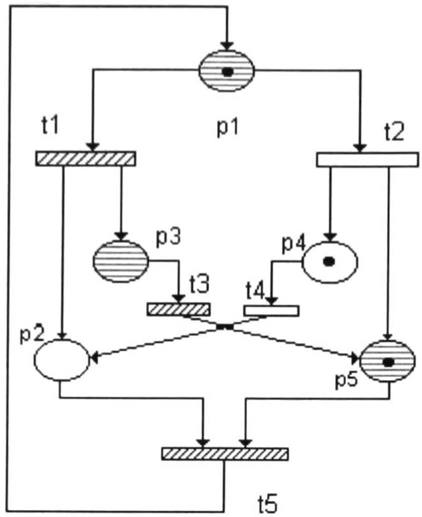


Figure 2.2: P and t-semiflows in a PN.

If the reachability set is a finite one, the following graph can be used to study qualitative properties of the PN.

Definition 2.8 *Given a PN (G, M_0) the* ***reachability graph*** *is the graph consisting of a node for each different marking contained in $\mathbf{R}(G, M_0)$ and arcs joining them, labeled with the transitions that, when firing, make the state change from a marking to another.*

Example 2.3 *The reachability graph of the PN depicted in figure 2.3.a) is shown in figure 2.3.b). Notice that*

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

is the set of all reachable states and that the firing sequences that lead to each of those states are represented in the reachability graph.

In the case when the reachability set is not finite, the coverability graph is used[7]. That case is out of the scope of this work.

Structural analysis methods are based in studying the incidence matrix of the net using linear algebra, convex geometry or linear programing.

Analysis methods based on transformations (simplification of G) provide algorithms to reduce a PN model to a simpler one where it is easier to decide if the model exhibits a certain property. These methods and those based on simulation are expensive in time and computational effort.

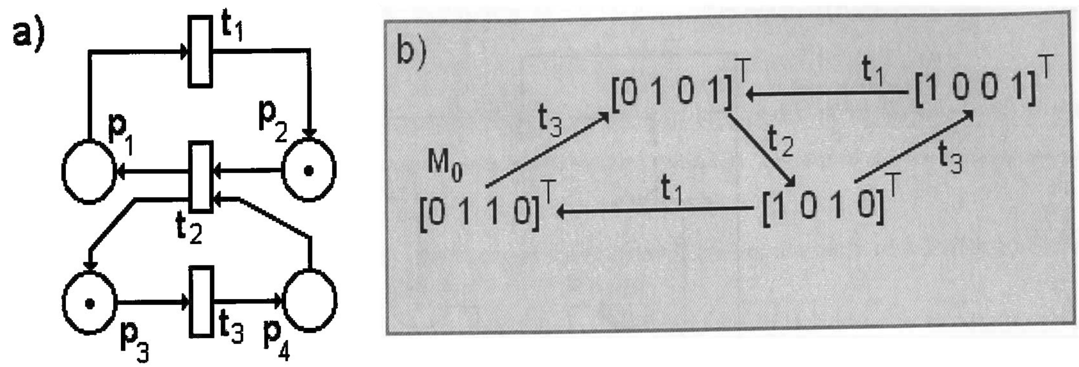


Figure 2.3: PN and reachability graph.

Herein, the reachability graph is used in order to clarify the properties and concepts presented. The interested reader can find more details about PN properties and analysis methods in [26] and [5].

Consider now the following concept:

Definition 2.9 *Given a PN (G, M_0), the* ***synchronic distance*** *of a transition t_i with respect to a transition t_j is the maximum number of firings of t_i without firing t_j in all transition firing sequences. This value is denoted as $SD(G, M_0; t_i, t_j)$.*

This concept will result specially useful in the observability study presented herein, because it allows to determine if a certain marking can be reached.

The following example illustrates this concept.

Example 2.4 *Consider the PN (G, M_0) depicted in figure 2.4.a). The initial marking of that PN is $M_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$ and its reachability graph is shown in figure 2.4.b). Notice then that $SD(G, M_0; t_1, t_3) = 1$. This is because t_1 can only be fired once from marking M_0, which is the only marking that enables that transition, and once t_1 is fired the only way to return to M_0 is firing t_3t_5. Now, $SD(G, M_0; t_3, t_1) = 1$ because t_3 can only be fired once form the marking $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \end{bmatrix}^T$ which is the only marking that enables that transition, and in order to reach that marking again t_1 has to be fired. However, the synchronic distance is not symmetric: notice that $SD(G, M_0; t_1, t_5) = 1$ but $SD(G, M_0; t_5, t_1) = \infty$, i.e. transition t_1 can only be fired once without firing t_5, but t_5 can be fired infinite times without firing t_1.*

2.2.1 PN basic properties

Three basic PN properties will be now presented because this work focuses on the IPN models that exhibit these characteristics.

Definition 2.10 *A PN (G, M_0) is said to be* ***cyclic*** *if $\forall M_i \in \mathbf{R}(G, M_0)$ it holds that $\exists \sigma$ such that $M_i \xrightarrow{\sigma} M_0$.*

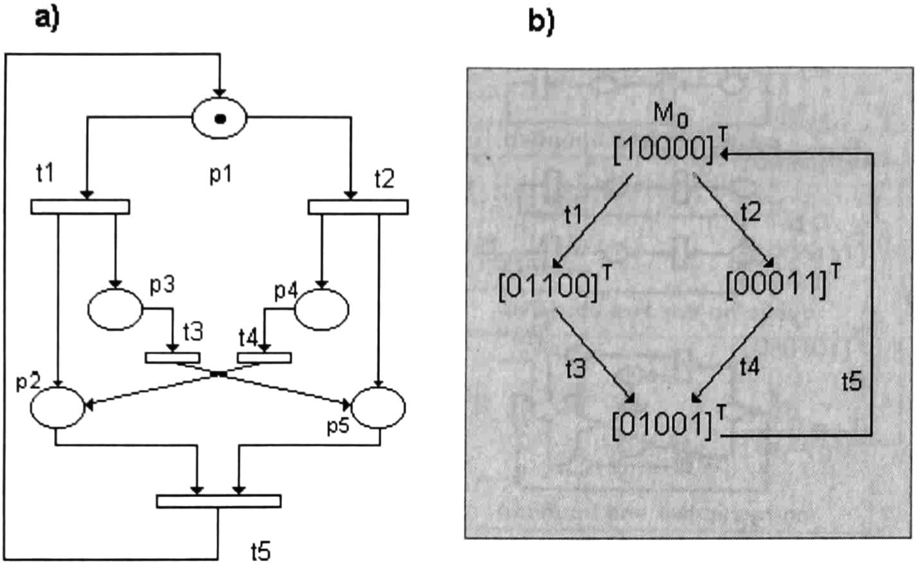


Figure 2.4: Synchronic distance in PN.

Definition 2.11 *A PN (G, M_0) is said to be live if $\forall M_i \in \mathbf{R}(G, M_0)$ and $\forall t \in T$ it holds that $\exists M_j$ such that $M_i \xrightarrow{\sigma} M_j \xrightarrow{t}$.*

As it will be shown in the next chapters, cyclicness and liveness provide conditions that can guarantee that a certain transition sequence can be fired.

Boundedness guaranties that the marking of a net cannot grow indefinitely, for that reason boundedness is considered a security property.

Definition 2.12 *A PN (G, M_0) is said to be **bounded** if $\forall M \in \mathbf{R}(G, M_0)$ it holds that $\forall p_i, M(p_i) \leqslant \infty$. If there is a positive integer L such that $\forall M \in \mathbf{R}(G, M_0)$ it holds that $\forall p_i, M(p_i) \leqslant L$ then (G, M_0) is said to be **L-bounded** and if $L = 1$ then (G, M_0) is said to be **binary** or **safe.***

The fact that none of previous properties (liveness, cyclicness and boundedness) imply any other, can be easily checked by considering the PN in figure 2.5.

Characterizations of the PN that exhibit those properties are available by different methods, herein only the characterizations in terms of the reachability graph are given.

First, a PN is bounded if and only if its reachability graph has a finite number of nodes, a PN is cyclic if and only if its reachability graph is strongly connected (every node can be reached from any other), and a PN is live if in every node begins a path of transition firings containing all transitions.

Example 2.5 *Consider the PN depicted in figure 2.6.a). For that PN, the reachability graph is shown in*

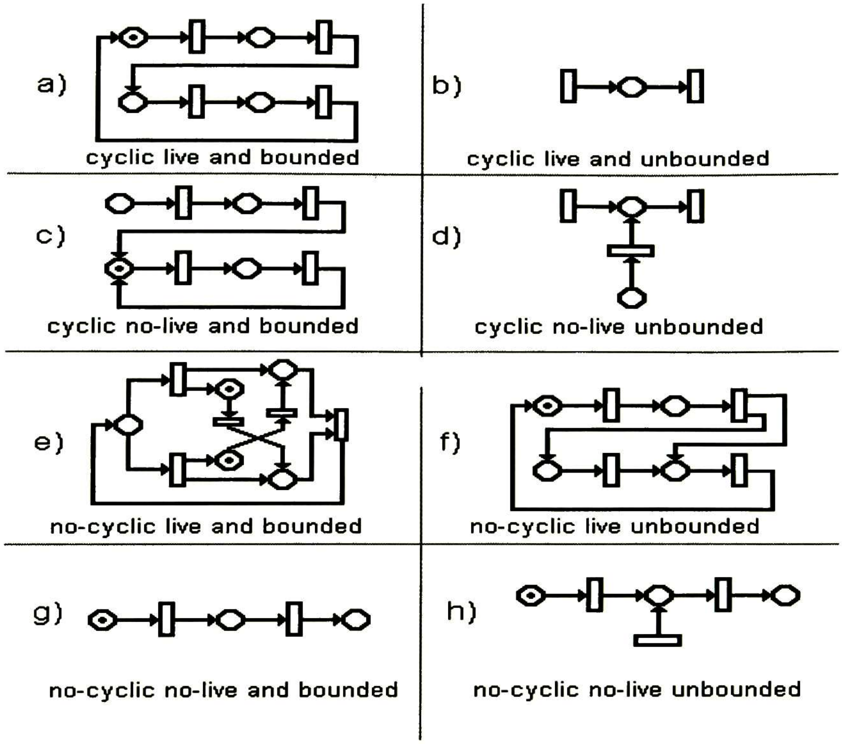


Figure 2.5: Cyclicness, liveness and boundedness in PN.

figure 2.6.b). Using this graph, it is easy to see that the PN is cyclic (because from any reachable marking there is a path ending in the initial marking), live (because every transition is contained in all possible transition firing sequences) and 3-bounded (because any place contains at most three tokens at any reachable marking).

2.3 Interpreted Petri Nets

IPN is an extension to PN that allows to assign input and output languages to the models, adding a physical meaning to them.

Definition 2.13 *An **Interpreted Petri Net** (IPN) is the 6-tuple* $Q = (N, \Sigma, \Phi, \lambda, D, \varphi)$ *where*

- $N = (G, M_0)$ *is a PN,*
- $\Sigma = \{\sigma_1, \sigma_2, ..., \sigma_r\}$ *is the input alphabet of the net, where* σ_i *is an input symbol,*
- $\Phi = \{\phi_1, \phi_2, ..., \phi_s\}$ *is the output alphabet, where* ϕ_i *is an output symbol,*
- $\lambda : T \to \Sigma \cup \{\varepsilon\}$ *is a labeling function of transitions with the following restriction:* $\forall t_j, t_k \in T$, $j \neq k$ *if* $I(p_i, t_j) = I(p_i, t_k) \neq 0$ *and both* $\lambda(t_j)$, $\lambda(t_k) \neq \varepsilon$, *then* $\lambda(t_j) \neq \lambda(t_k)$, *where* ε *represents an internal system event,*

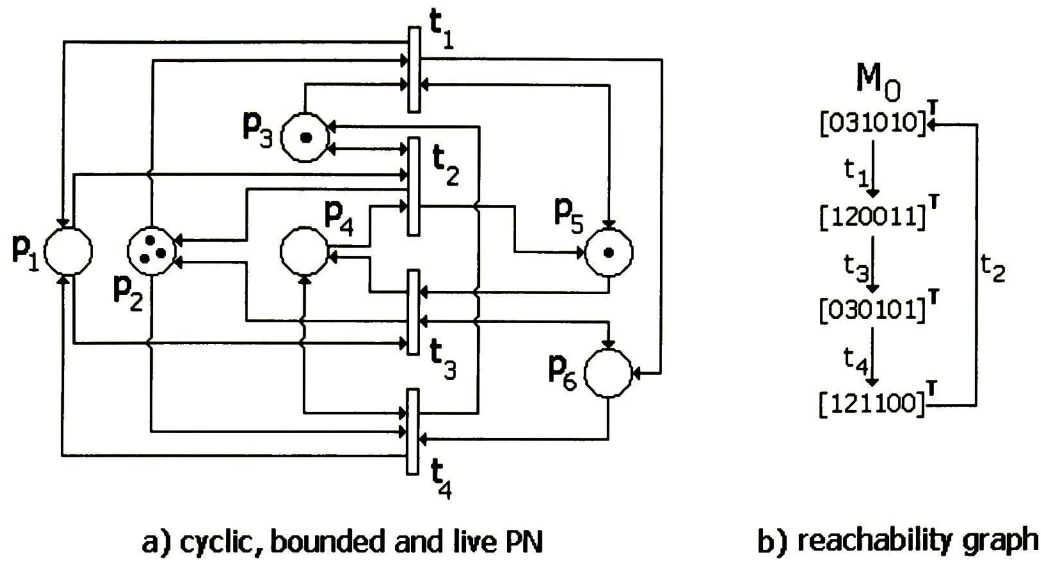


Figure 2.6: Cyclic, live and 3-bounded PN and reachability graph.

- $D : \{\mathbb{Z}^{\geqslant 0}\}^m \rightarrow \{\mathbb{Z}^{\geqslant 0}\}^p$ *is linear output function, represented as a* $m \times p$ *matrix* $D = [D_{ij}]$ *where* m *is the number of transitions,* p *is the number of measurable transitions, and the* i*-th row vector* D_j *of* D *is the transpose of the elemental vector* e_j $(e_j[i \neq j] = 0,\ e_j[j] = 1)$ *if* t_j *is the* i*-th measurable transition according to the order given by the transition labeling.*

- $\varphi : \mathbf{R}(G, M_0) \longrightarrow \{\mathbb{Z}^{\geqslant 0}\}^q$ *is a linear output function, represented as a* $q \times n$ *matrix* $\varphi = [\varphi_{ij}]$ *where* n *is the number of places,* q *is the number of measurable places, and the* i*-th row vector* $\varphi(i, \bullet)$ *of* φ *is the transpose of the elemental vector* e_j *if* p_j *is the* i*-th measurable place according to the order given by the place labeling.*

Remark 2.1 *To enhance the fact that there is an initial marking in an IPN,* (Q, M_0) *will be used instead of* $Q = (N, \Sigma, \Phi, \lambda, D, \varphi)$*.*

For IPN the firing rules are redefined as follows:

A transition $t_j \in T$ of an IPN is enabled at a marking M if $\forall p_i \in P$, $M(p_i) \geqslant I(p_i, t_j)$. If $\lambda(t_j) = a_i \neq \varepsilon$ is present and t_j is enabled, then t_j must fire. If $\lambda(t_j) = \varepsilon$ and t_j is enabled then t_j can be fired. If an enabled transition t_j fires at a marking M_k, then a new marking M_{k+1} is reached which can be computed using the dynamical part of the state equation: $M_{k+1} = M_k + Cv_j$, where C and v_j are defined like in a PN.

The following classification is made over transitions and places, depending on the way that functions λ, φ and D are defined.

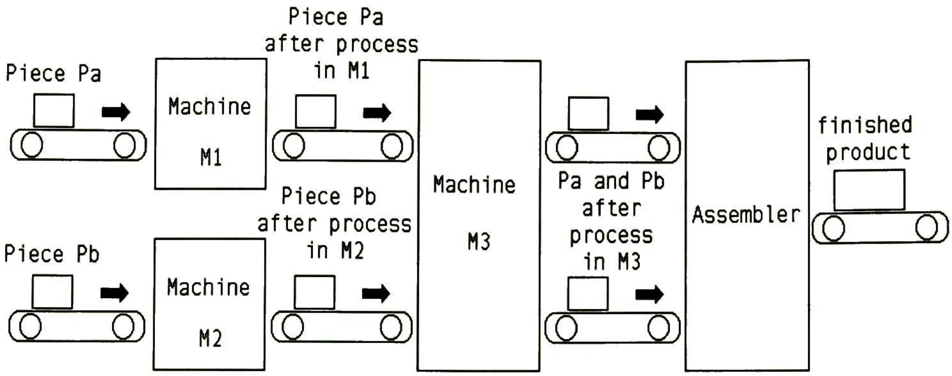


Figure 2.7: Manufacturing cell layout.

Definition 2.14 *If $\lambda(t_i) \neq \varepsilon$ the transition t_i is said to be* ***manipulated****, otherwise it is* ***no-manipulated****. A place $p_i \in P$ is said to be* ***measurable*** *if the i-th column vector of φ is not null (i.e.: $\varphi(\bullet, i) \neq 0$) and* ***no-measurable,*** *otherwise. A transition t_j is said to be* ***measurable*** *if the j-th column vector of D is not null (i.e.: $D(\bullet, j) \neq 0$) and* ***no-measurable,*** *otherwise. In this work, the measurable places of an IPN are depicted as clear circles, the no-measurable ones as dark circles, the measurable transitions as clear bars, while the no-measurable ones as dark bars.*

The following example shows the meaning of measurable and no-measurable place and transition.

2.3.1 Manufacturing cell example

Example 2.6 *Figure 2.7 shows a scheme of a manufacturing cell layout where a product consisting of two parts (Pa, Pb) is processed. Pa requires to use the machines $M1$ and $M2$, and Pb requires $M3$ and $M2$, both in that sequence. After both parts are processed they are assembled and the product is released, then the system is ready to start another cycle. The IPN of figure 2.8 is a model of the system where a token in place p_1 represents an idle state of the system; transition t_1 represents the beginning of a cycle. A token in p_2 (p_5) represents that the machine $M1$ ($M3$) is being used and in p_3 (p_4) that $M1$ ($M3$) is available. t_2 and t_3 represent the ending of the process in $M1$ and $M3$ respectively, a token in p_6 and p_7 represent a piece waiting for $M2$ to be available. t_4 and t_5 represent the beginning of the process of a part in $M2$. A token in p_9 represents that $M2$ is available. p_8 and p_{10} represent a part being processed in $M2$. t_6 and t_7 represent the ending of the process in $M2$. Tokens in p_{11} and p_{12} represent the parts waiting to be assembled. Finally, t_8 represents the assembling and release of the product.*

Different signals are displayed when $M1$ and $M3$ are being used, and $M2$ display two different signals depending on the piece that is being processed. Therefore p_2, p_5, p_8 and p_{10} are measurable places in the model and since the releasing of a product can be detected, t_8 is also measurable.

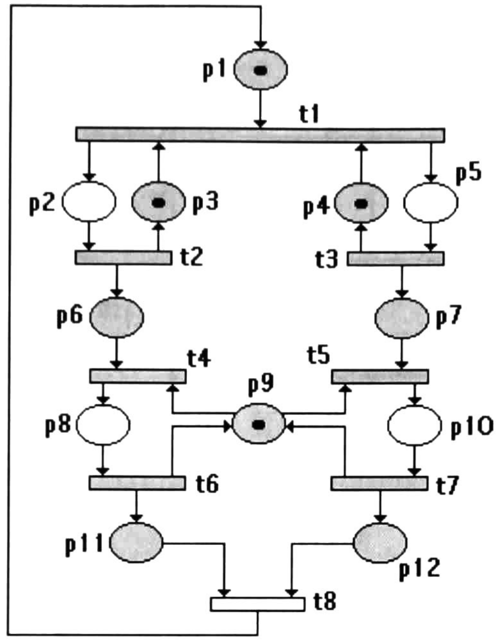


Figure 2.8: Manufacturing cell IPN model.

2.3.2 IPN state equation

A special representation of the state equation of an IPN will be used in the remaining chapters of this work, it is formally established as follows:

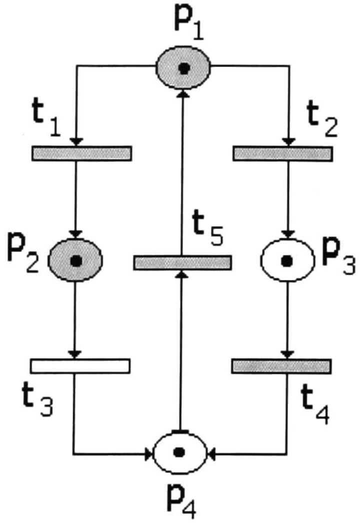
Definition 2.15 *The* ***state equation of an IPN*** *can be written as:*

$$\begin{aligned} M_{k+1} &= M_k + C^{\epsilon} v_k^{\epsilon} + C^D v_k^D \qquad (2.1) \\ y_{k1} &= \varphi \bullet M_k \\ y_{k2} &= D \bullet v_k \end{aligned}$$

where $C = [C^{\epsilon} \vdots C^D]$, C^{ϵ} is formed by the columns of the no-measurable transitions and C^D by the columns of the measurable ones. D is a function on the firing vector that shows if a measurable transition has been fired.

Example 2.7 *In figure 2.9 an IPN is shown along with its state equation. That state equation is composed by a dynamical part that describes the state changing, and an output part that describes the part of the marking that is accessible and the firing of measurable transitions. In this work it is assumed that the output functions φ and D are linear functions represented as matrices.*

In this work the output part y_{k2} corresponding to function D will be often omitted from the IPN state equation and $y\prime$ will be written as the output part corresponding to function φ.



$$M_{k+1} = M_k + \begin{bmatrix} -1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix} v_k$$

$$y_{k1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} M_k$$

$$y_{k2} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} v_k$$

Figure 2.9: IPN, dynamic and output part of state equation.

If in an IPN (Q, M_0) it holds that $\exists C(\bullet, i) = C(\bullet, j)$, $i \neq j$, since transitions t_i and t_j cause the same effect when firing, those transitions t_i and t_j are said to be equivalent.

Remark 2.2 *It is assumed in this work that no IPN (Q, M_0) contains equivalent transitions.*

Example 2.8 *For the IPN depicted in figure 2.10.a)*

$$C = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

is the incidence matrix. Since the columns corresponding to transitions t_1 and t_2 are equal, those transitions are equivalent. That IPN is out of the scope of this work. The IPN depicted in figure 2.10.b) is equivalent to the other net and contains no equivalent transitions.

2.3.3 IPN input and output languages

Note that all the concepts introduced for PN can be extended to IPN. In addition, the following definition presents the input and output languages that can be associated to an IPN.

Definition 2.16 *The **input language** of Q is $\pounds_{in}(Q, M_0) = \{\lambda(t_i)\lambda(t_j)\ldots\lambda(t_k) | t_i t_j \ldots t_k \in \pounds(Q, M_0)\}$, and the **output language** is $\pounds_{out}(Q, M_0) = \{\varphi(M_0)\varphi(M_1)\ldots\varphi(M_w)\ldots | M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \ldots M_w \xrightarrow{t_k} \ldots$ and $t_i t_j \ldots t_k \in \pounds(Q, M_0)\}$.*

Also the output part corresponding to function D can be considered in the output language of an IPN.

Example 2.9 *The input and output languages of the IPN depicted in figure 2.11 respectively are: $\{a\,(\varepsilon + b)\}$ and* $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \left(\varepsilon + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \left(\varepsilon + \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right) \left(\varepsilon + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) \right) \right) \right\}$.

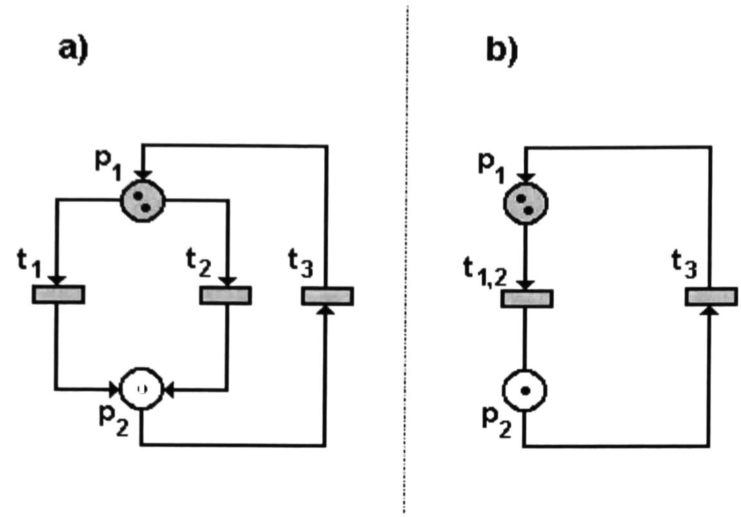


Figure 2.10: IPN with equivalent transitions and equivalent IPN.

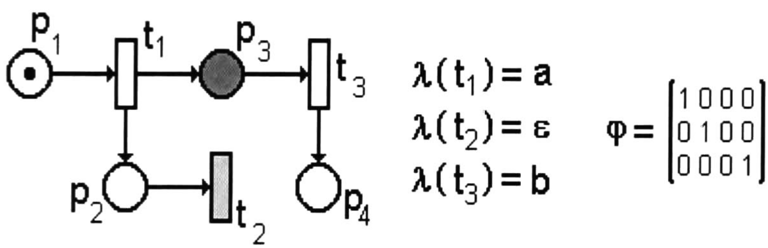


Figure 2.11: IPN, input and output functions.

2.4 Conclusions

In this chapter IPN, an extension to PN is introduced as a DES modeling tool. IPN provide a fine visual representation of DES and compact mathematical models in terms of difference equations. IPN allow to assign a physical meaning to models using input and output alphabets and functions.

Basic notions and properties of IPN useful for the observability study presented in this work were defined.

Chapter 3

Observability in IPN

Summary: Observability deals with the possibility of determining the initial state of a system from the knowledge of its structure, inputs and outputs.

Herein, it is proved that the observability problem is equivalent to solve two problems. The first one deals with determining any sequence of events and the second with computing the marking of the system model. IPN models where any transition firing sequence can be determined are called event-detectable. A simple characterization for the IPN that exhibit this property is given in terms of the structure of the net.

Event-detectability is not enough to provide observability since it does not guarantee that the marking of the IPN can be computed. Physical characteristics of DES can provide some knowledge about the maximum number of tokens that a set of places in the model can hold. Event-detectability, the knowledge of physical characteristics of the system and a condition on the transition sequences that can be fired in the IPN need to be held to provide observability.

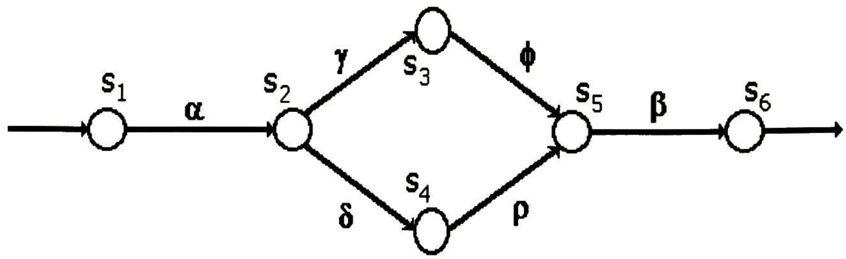


Figure 3.1: Finite automata example.

3.1 Introduction

In system theory it is said that a system is observable if its initial state can be computed in a finite time using the knowledge of the system structure, inputs and outputs[4]: i.e., in an observable system it is possible to know the state of the system even when it is initially unknown.

As it was mentioned above, works like [16], [17] and [22] study the controllability of partially observed DES. Those works present an observability concept that is useful only for control purposes. In the sense of the concept presented in those works, even DES were the system state cannot be determined can be observable. It is only necessary that the control actions required by the system considering partial observation result equal to those required when that consideration is not made, i.e. it is only needed that partial observation does not affect the DES control.

Example 3.1 *As an example consider figure 3.1. The events $\sum_o = \{\alpha, \beta\}$ are "observable" (measurable), events $\sum_u = \{\gamma, \phi, \delta, \rho\}$ are "no observable" (no-measurable) and the desired behavior for that DES is to execute the word $E = \alpha(\gamma\phi + \delta\rho)\beta$.*

Notice that the language of the DES is equal to the desired behavior, then there is a controller that can restrict the behavior of the DES to a desired one. Since the control actions necessary to impose the behavior E to the system when the "observability" of events is not considered do not need to be modified when the "observability" of events is considered, according to [16] the system is observable.

However, since events $\gamma, \phi, \delta, \rho$ are no-measurable, once γ or δ occurs and until β does, it is not possible to determine the state of the system, i.e. once the system evolves from s_1 to s_2 -since no further information is provided by the occurrence of other events-, the system state is not known but when it evolves from s_5 to s_6. Then, according to the observability concept used in systems theory, that system is not observable.

As it has been shown in the previous example, the observability concept used in [16] does not capture the essence of this property: the possibility of computing the initial state of the system.

Works like [8], [6], [1], and [11] deal with determining the initial state of PN models of DES. In [8] and [1] algorithms to determine the current system state are presented, however it is assumed that all transitions are accessible, i.e. the state estimator knows exactly which transition was fired. That hypothesis only holds for systems where the instrumentation necessary to determine the occurrence of any event that may affect the system state is provided. Providing that instrumentation may result in monetary or technologic excessive cost.

In [11], the observability problem is divided into two sub-problems: computing the initial marking of the net and determining the firing sequence that leads the observer from the initial marking to the current state. In that work, algorithms to determine a set of possible initial markings of a PN and the corresponding firing sequence are presented. However, the initial or current state of the system is not uniquelly determined and the information provided by detectable events is not used.

The first topic addressed in this chapter is the statement of necessary conditions to determine every transition firing sequence executed in a system, then the observability concept and the characterization of the IPN that exhibit this property will be introduced.

These results are presented in [23], [24] and [25].

3.2 Event-detectability

In some IPN, the transition firing sequence can be detected from the information that functions φ and D provide when the visible part of the marking changes or a measurable transition fires.

This work presents an IPN classification depending on the detectability of the transition firing sequences. When this sequence can be detected using only the output and the IPN structural information, the IPN will be called **event-detectable** IPN, otherwise it will be called **nonevent-detectable** IPN.

Example 3.2 *Consider the IPN depicted in figure 3.2.a). The state equation for that IPN is:*

$$M_{k+1} = M_k + \begin{bmatrix} -1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix} \bullet v_k$$

$$y_{k1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet M_k$$

$$y_{k1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \bullet v_k$$

Notice that places p_1 and p_2 are no-measurable, and p_3 and p_4 are measurable. t_1 is a measurable transition and t_2, t_3, t_4 and t_5 are no-measurable.

Since t_1 is measurable, its firing can be detected by the value of the function D. The firing of t_2 can be detected because that transition firing is the only one that can increase the marking of the measurable place p_3. t_3 and t_4 are input transitions to p_4 but t_4 is also output transition to p_3, and that provides enough information to uniquely determine their firings. Since t_5 is the only output transition to p_4 which is a measurable place, its firing can also be uniquely determined by the difference in consecutive output values. Then, all transition firing can be computed using the information provided by C and functions φ and D. For that reason, that IPN model is said to be event-detectable.

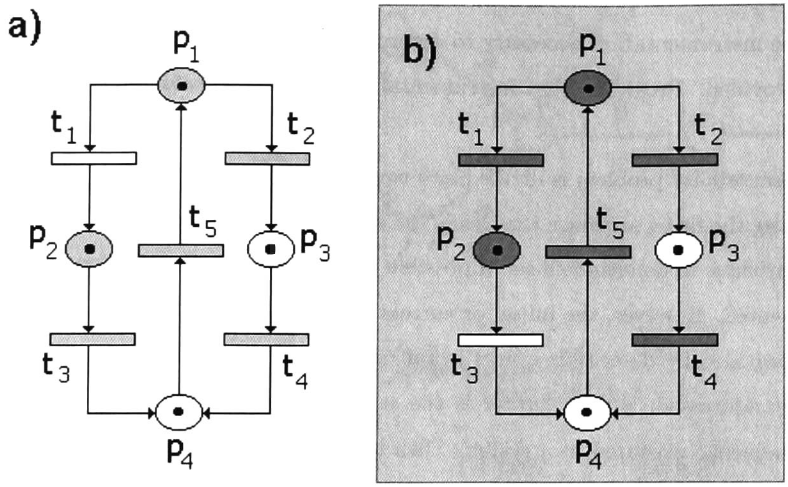


Figure 3.2: a) event-detectable IPN, b) nonevent-detectable IPN.

Consider now figure 3.2.b). Now transition t_1 is no-measurable and t_3 is measurable. The firings of t_2, t_3, t_4 and t_5 still can be detected, but now the firing of t_1 does not produce any signal or visible change in the IPN marking and then that transition firing cannot be detected. Since the information provided by the output functions is not enough to determine the firing of every transition, that is a nonevent-detectable IPN.

Definition 3.1 *An IPN $(Q, M_0) = (N, \Sigma, \Phi, \lambda, D, \sigma)$ described by the state equation (2.1) is said to be* ***event-detectable*** *if every transition firing sequence can be detected for any initial marking M_0 using the information provided by the structure, input and output of the system.*

The following lemma can be considered as an extension to the one presented in [11] for PN. This result states a characterization of event-detectable IPN.

Lemma 3.1 *Let $(Q, M_0) = (N, \Sigma, \Phi, \lambda, D, \sigma)$ be an IPN described by the state equation (2.1). Every transition firing sequence $\sigma \in \pounds(Q, M_0)$ is fully detected for any initial marking M_0 if and only if all $\varphi \bullet C^{\varepsilon}$ columns are not null and different from each other.*

Proof. (If) All $\varphi \bullet C^{\varepsilon}$ columns are not null and different from each other. Assume, without loss of generality that marking M_j is reached firing t_j from marking M_i, i.e. $M_i \xrightarrow{t_j} M_j$. Now, from the state equation, a difference of markings can be computed as

$$M_j - M_i = \begin{cases} C^{\varepsilon}\overrightarrow{t_j} \text{ if } t_j \text{ is no-measurable} \\ C^{D}\overrightarrow{t_j} \text{ if } t_j \text{ is measurable} \end{cases}$$

or using the output $y\prime$

$$y_j' - y_i' = \begin{cases} \varphi \bullet C^{\varepsilon}\overrightarrow{t_j} \text{ if } t_j \text{ is no-measurable} \\ \varphi \bullet C^{D}\overrightarrow{t_j} \text{ if } t_j \text{ is measurable} \end{cases}$$

Also $\varphi \bullet C^{\epsilon}\overrightarrow{t_j}$ ($\varphi \bullet C^{D}\overrightarrow{t_j}$) is a column of $\varphi \bullet C^{\epsilon}$ ($\varphi \bullet C^{D}$), if all $\varphi \bullet C^{\epsilon}$ and all $\varphi \bullet C^{D}$ columns are not null and different from each other then the firing of any transition can be detected (from the $y_j' - y_i'$ value), but since the firing of measurable transitions always can be detected from the output function D, then the condition on $\varphi \bullet C^{D}$ is not needed, reducing previous condition to the following one:

If all $\varphi \bullet C^{\epsilon}$ columns are not null and different from each other then the firing of any transition can be detected.

The same procedure can be applied to any pair of consecutive markings, then any firing transition sequence can be detected if all $\varphi \bullet C^{\epsilon}$ columns are not null and different from each other.

(Only if) Now suppose that there are two columns $\varphi \bullet C^{\epsilon}(\bullet, i)$, $\varphi \bullet C^{\epsilon}(\bullet, j)$, $i \neq j$ such that $\varphi \bullet C^{\epsilon}(\bullet, i) = \varphi \bullet C^{\epsilon}(\bullet, j)$. Then

$$y_j' - y_i' = \varphi \bullet C^{\epsilon}(\bullet, j) = \varphi \bullet C^{\epsilon}(\bullet, i)$$

and the firing of t_j is confused with the firing of t_i, then no sequence containing t_i or t_j can be fully detected. ■

Although it has been determined the cases when any transition firing in an IPN can be detected, that is not enough to guarantee that its state can be computed. Next section is devoted to establish the other conditions that need to be held to provide observability in an IPN.

3.3 Observable IPN

Consider an event-detectable IPN with an unknown initial marking M_0 and assume that the transition firing sequence represented by the set of the of firing vectors $v_1 ... v_n$ is executed in that IPN, then the following set of equations can be obtained for the IPN outputs $y_0' ... y_n'$.

$$\begin{aligned}
y_0' &= \varphi(M_0) \\
y_1' &= \varphi(M_1) = \varphi\left(M_0 + C^{\epsilon}v_1^{\epsilon} + C^{D}v_1^{D}\right) \\
y_2' &= \varphi(M_2) = \varphi(M_1 + C^{\epsilon}v_2^{\epsilon} + C^{D}v_2^{D}) = \varphi(M_0 + C^{\epsilon}v_1^{\epsilon} + C^{D}v_1^{D} + C^{\epsilon}v_2^{\epsilon} + C^{D}v_2^{D}) \\
&\vdots \\
y_n' &= \varphi(M_n) = \varphi(M_{n-1} + C^{\epsilon}v_n^{\epsilon} + C^{D}v_n^{D}) = \varphi\left(M_0 + \sum_{i=1}^{n} C^{\epsilon}v_i^{\epsilon} + \sum_{i=1}^{n} C^{D}v_i^{D}\right)
\end{aligned}$$

Since the IPN is event-detectable, all the firing vectors in those equations can be computed and if the function φ is linear, the equations can be rewritten in the following way:

$$\begin{aligned} y'_0 &= \varphi(M_0) \\ z_1 &= \varphi(M_0) \\ z_2 &= \varphi(M_0) \\ &\vdots \\ z_n &= \varphi(M_0) \end{aligned}$$

Where $z_i = y'_i - \varphi\left(\sum_{j=1}^{i} C^{\epsilon}v_j^{\epsilon} + \sum_{j=1}^{i} C^{D}v_j^{D}\right), i \in [1, ..., n]$ and y'_0 are known. According to that, the initial marking M_0 can be computed only if φ has full rank.

The previous result is too restrictive because φ has full rank only when all places are measurable. Fortunately, sometimes physical constraints of DES provide additional information and this problem can be overcome[8]. This information is provided by a set of conservative marking laws defined in the following way.

Definition 3.2 *A set of* ***Conservative Marking Laws*** *CML for an IPN (Q, M_0) is a set of w equations*

$$\begin{gathered} \sum_{j=1}^{n} \alpha_j^1 \bullet M(p_j) = k_1 \\ \vdots \\ \sum_{j=1}^{n} \alpha_j^w \bullet M(p_j) = k_w \end{gathered}$$

such that $\alpha_j^i \in \mathbb{Z}^{\geqslant 0}$, $\forall p_j$ no-measurable it occurs that $\alpha_j^i \neq 0$ for at least one equation and $\forall \alpha_j^i \neq 0$, k_i/α_j^i is an integer value.

The set of conservative marking laws (CML) of an IPN does not only depend on its structure but also on its initial marking M_0. Fortunately, it is common to know this set in most of the systems [8] even when the initial marking is unknown, because it can be obtained from the knowledge of the p-semiflows of the net and the maximum number of parts that a store can hold, or the capacity of a machine, etc.

Given an CML, it is possible establish upper and lower bounds for all reachable markings of an IPN in the following way.

Definition 3.3 *Let (Q, M_0) be an IPN, where a CML is defined. Then*

$$\begin{array}{c|c} M^{LB}(p_k) = \min M(p_k) & M^{UB}(p_k) = \max M(p_k) \\ s.t. & s.t. \\ \sum_{j=1}^{n} \alpha_j^1 \bullet M(p_j) = k_1 & \sum_{j=1}^{n} \alpha_j^1 \bullet M(p_j) = k_1 \\ \vdots & \vdots \\ \sum_{j=1}^{n} \alpha_j^w \bullet M(p_j) = k_w & \sum_{j=1}^{n} \alpha_j^w \bullet M(p_j) = k_w \\ \forall p_i, M(p_i) \geqslant 0 & \forall p_i, M(p_i) \geqslant 0 \end{array}$$

are the ***minimum*** *and* ***maximum marking bounds of place*** *p_k respectively, $k = 1, ..., n$. Also, the* ***maximum marking gap*** *in place p_k is $\mathbb{D}_k = M^{UB}(p_k) - M^{LB}(p_k)$. These quantities can be arranged as the vectors $M^{UB} = [M^{UB}(p_1)...M^{UB}(p_n)]^T$, $M^{LB} = [M^{LB}(p_1)...M^{LB}(p_n)]^T$ $\mathbb{D} = [\mathbb{D}(p_1)...\mathbb{D}(p_n)]^T$*

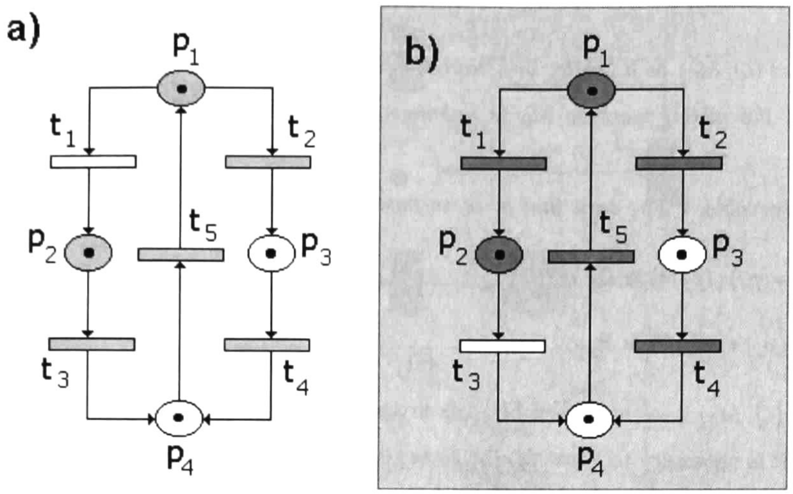


Figure 3.3: a) observable IPN, b) no-observable IPN.

The knowledge of these conservative laws, and upper and lower marking bounds, as stated, depends on the model interpretation and the physical characteristics that the actual system exhibits. Therefore the existence of a CML for an IPN can only be assumed when the IPN is considered as the model of a system.

Now, a definition of observability which is an extension of that used in system theory [4] is presented for IPN modeling DES.

Definition 3.4 *An IPN (Q, M_0) is* ***observable at k steps*** *if and only if $\forall \omega \in \pounds_{in}(Q, M_0)$ $\exists z$, such that $\omega z \in \pounds_{in}(Q, M_0)$, $|\omega z| \leqslant k < \infty$ the information provided by ωz, the output word generated by ωz, a set of conservative marking laws CML and the structure of the system C are enough to compute M_0.*

Previous definition establishes the observability concept that will be used in the remaining of this work. Loosely speaking, an IPN is observable if for all possible transition firing sequences there is another one that when it is executed produces enough information to fully determine the initial marking.

Example 3.3 *Figure 3.3 is useful to understand that event-detectability is a necessary condition for observability. Consider first the IPN depicted in 3.3.a). Suppose for a moment that it is known that the IPN contains 4 tokens, then when a marking M_k such that $M_k(p_3) + M_k(p_4) = 4$ is reached, the marking of all the places can be computed. Moreover, since that IPN is event-detectable, once the current marking of the net is known, the initial or any reached marking for that net can be computed using its state equation. Consider now figure 3.3.b). Let the marking $\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$ be the initial marking for that IPN. Since the firing of t_1 cannot be detected, that marking cannot be distinguished from $\begin{bmatrix} 0 & 2 & 1 & 1 \end{bmatrix}^T$ for example. Moreover, not even if a marking M_k such that $M_k(p_3) + M_k(p_4) = 4$ is reached, the initial marking can be uniquely determined.*

Next result is a characterization of observable IPN.

Theorem 3.1 *Let (Q, M_0) be a cyclic, live, bounded and event-detectable IPN, described by the state equation (2.1), where the initial marking M_0 is unknown, but a CML in the sense of definition 3.2 can be obtained.*

(Q, M_0) is observable, if $\forall p_j$ such that p_j is no-measurable either:

i) $SD(Q, M_0; \bullet(p_j), (p_j)\bullet) \geqslant \mathbb{D}_j$ or

ii) $SD(Q, M_0; (p_j)\bullet, \bullet(p_j)) \geqslant \mathbb{D}_j$.

Proof. Since (Q, M_0) is event-detectable, any firing sequence η_i such that $M_0 \xrightarrow{\eta_i} M_i$ can be computed. To compute M_0, it is necessary to know M_i; however, it cannot be directly obtained because of the existence of no-measurable places.

The proof consist in showing that the marking M_i can be known if either condition $i)$ or condition $ii)$ holds and then, using event-detectability, computing M_0.

Let p_k be a no-measurable place:

Assume that $i)$ holds, then a firing transition sequence σ_k -such that the number of firings of transitions in $\bullet(p_k)$ without firing any transition in $(p_k)\bullet$ is equal to $\mathbb{D}_k$- exists. If σ_k does not occur immediately from M_0 then, since (Q, M_0) is cyclic, it will return to M_0 and eventually σ_k will occur. σ_k can be split as $\sigma_k = \sigma_1\sigma_2$, such that σ_2 does not contain any transition in $(p_j)\bullet$ and the transitions in $\bullet(p_j)$ appears $\mathbb{D}_k$ times. Then $M_j(p_k) = M_0(p_k) + C(p_k, \bullet)\vec{\sigma_1} + C(p_k, \bullet)\vec{\sigma_2} = M_n(p_k) + \mathbb{D}_k = M_n(p_k) + M^{UB}(p_k) - M^{LB}(p_k)$. Now, the claim $M_n(p_k) = M^{LB}(p_k)$ and $M_j(p_k) = M^{UB}(p_k)$ is made. To prove it assume for a moment that $M_n(p_k) = M^{LB}(p_k) + \triangle M$, this implies that $\triangle M > 0$, then $M_j(p_k) = M^{LB}(p_k) + \triangle M + M^{UB}(p_k) - M^{LB}(p_k) = M^{UB}(p_k) + \triangle M > M^{UB}(p_k)$, which is a contradiction. Thus, after firing σ_k the marking of p_k is $M^{UB}(p_k)$. σ_k can be determined from the output because (Q, M_0) is event-detectable.

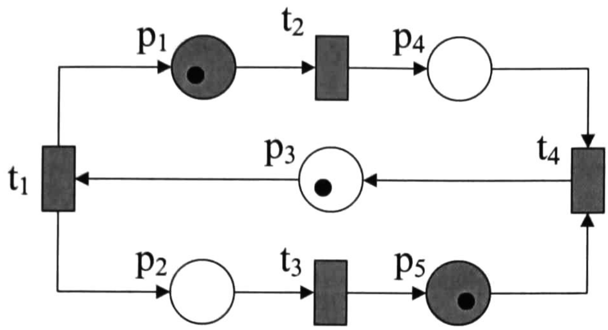
Now, assume that $ii)$ holds, then a firing transition sequence σ'_k -such that the number of firings of transitions in $(p_k)\bullet$ without firing any transition in $\bullet(p_k)$ is equal to $\mathbb{D}_k$- exists. Following a similar procedure like in the previous case, after firing σ'_k the marking of p_k can be determined as $M^{UB}(p_k)$.

Once the actual marking of p_k has been computed, it will remain known for any firing sequence since (Q, M_0) is event-detectable.

Moreover, using this procedure, the marking of the remaining no-measurable places can be determined. Then the whole marking M_i will be known and using event-detectability, M_0 can be computed using the fired sequence and the state equation (2.1). Therefore, (Q, M_0) is observable. ∎

It is important to note that if a no-measurable place is not contained in any CML equation, then there is no way to determine the marking in that place and the IPN is not observable.

Example 3.4 *Figure 3.4 shows an IPN its φC^{ε} matrix and a CML defined for it. In that IPN places p_1 and p_5 and all transitions are no-measurable. Since the CML establishes that each p-semiflow contains two*



CML:

$M(p_1) + M(p_3) + M(p_4) = 1$

$M(p_2) + M(p_3) + M(p_5) = 1$

$$\varphi C^{\varepsilon} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

Figure 3.4: Observable IPN.

tokens, it holds that $\forall k$ $\mathbb{D}_k = 2$, $M^{UB}(p_k) = 2$, $M^{LB}(p_k) = 0$. *The IPN is event-detectable because all columns in the matrix* φC^{ε} *are different from each other and not null. Notice that the IPN is live, cyclic and bounded, then according to previous theorem the IPN is observable if it holds that* $\forall p_j$ *such that* p_j *is no-measurable either:* $SD(Q, M_0; \bullet(p_j), (p_j)\bullet) \geqslant \mathbb{D}_j$ *or* $SD(Q, M_0; (p_j)\bullet, \bullet(p_j)) \geqslant \mathbb{D}_j$.

Notice that the transition firing sequence $\sigma = t_1t_2t_2t_3t_4t_4$ *is enabled at the initial marking.* σ *contains two consecutive firings of both* t_2 *and* t_4*, since those transitions are respectively output transitions of* p_1 *and* p_5*, and those are all the no-measurable places in the IPN, it holds* $\forall p_j$ *no-measurable that* $SD(Q, M_0; \bullet(p_j), (p_j)\bullet) \geqslant \mathbb{D}_j = 2$. *Notice that when* $\sigma_1 = t_1t_2t_2$ *is executed, it is known that* $M_3(p_1) = 0$ *and when* $\sigma_2 = t_3t_4t_4$ *fires it is also known that* $M_6(p_5) = 0$. *At that moment then the marking of all no-measurable places is known, moreover, it is known that* $M_6 = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 \end{bmatrix}^T$. *Since that IPN is event-detectable, the firing sequence* $\sigma = \sigma_1\sigma_2$ *can be computed and then the initial marking can be obtained using the IPN state equation.*

The existence of a firing sequence σ *(enabled at the initial marking) that allows to determine the initial marking in a live, cyclic, bounded and event-detectable IPN is enough to guarantee observability because from any reachable marking the initial one can be reached again; and from there the sequence* σ *can be fired.*

Previous theorem establishes that the condition on the synchronic distance is enough to guarantee observability in a cyclic, live, bounded and event-detectable IPN. Next corollary shows that this condition can be dropped if the IPN is binary. This result was first presented in [23].

Corollary 3.1 *Let (Q, M_0) be a cyclic, live and binary IPN, described by the state equation (2.1), where the initial marking M_0 is unknown.*

(Q, M_0) is observable if it is event-detectable.

Proof. Since (Q, M_0) is cyclic, all places in the IPN are covered by a p-semiflow, then a CML can be obtained. For that CML it holds that $\forall k \; \mathbb{D}_k = 1$ because the net is binary. Liveness implies that each transition can be fired at least once from every reachable marking then, there is a firing sequence that fires an input transition to each place, and then the condition on the synchronic distance holds, and the result follows. ■

The results presented in this chapter provide algorithms to determine the initial state of a DES modeled by IPN. However, computing the initial state of the system in not always necessary. For some control and security purposes it is only needed to determine the current state of the system, that is what the asymptotic observers for IPN presented in the following chapters are devoted to achive.

3.4 Conclusions

The observability problem was divided into the possibility of determining the occurrence of every event in the system and the computation of the IPN marking.

When every event can be detected in an IPN, it is said to be event-detectable. A simple characterization of the IPN that exhibit this property was presented.

To determine the marking of an IPN model, information of the resources contained in the DES is needed. That information provides a set of conservative marking laws.

The event-detectability property, a set of conservative marking laws, and a condition on the synchronic distance between the sets of input and output transitions to each no-measurable place are sufficient conditions for observability in cyclic, live and bounded IPN models.

Chapter 4

Observer in IPN terms

Summary: An observer in IPN terms is presented for IPN modeling DES where no-measurable transitions can exist. The observer firing vectors are obtained using the information provided by the inputs and outputs of the system and the maximum number of tokens that each place in the system can contain. The observer is a copy of the IPN model of the system with a bigger initial marking. An output transition is added to each place to remove the exceeding tokens. A condition on the synchronic distance between the output and input transitions of each place, that needs to be held in order to guarantee that the marking in the observer will become equal to the system marking, is stated.

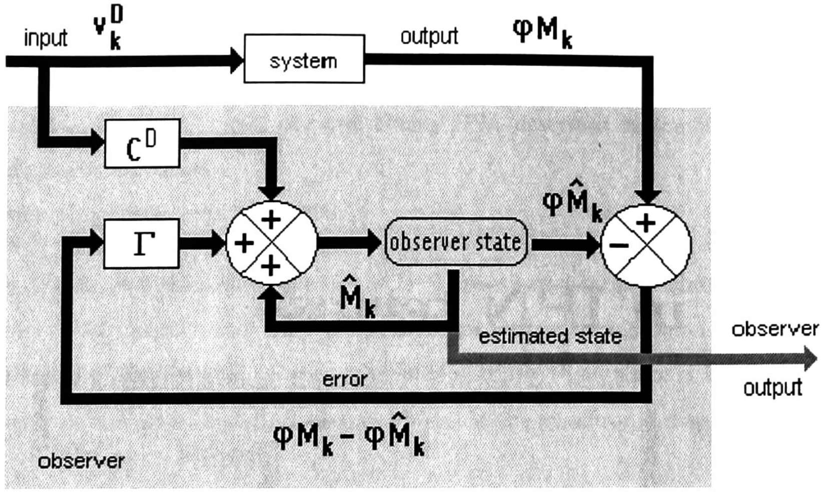


Figure 4.1: Observer scheme.

4.1 Introduction

Computing the current marking of the IPN modeling DES is enough for some control and security purposes. An observer is a device devoted to estimate the current system state and provides useful redundancy to provide fault tolerancy and recovery.

Giua in [8] presents an algorithm to determine the marking of the PN modeling a DES, however, he assumes that all the events in the system are accessible, Aguirre *et al.* in [1] present a similar work making the same assumption. The case when that assumption does not hold has been addressed in [2]. In that work it was shown that the observer firing vectors can be computed by solving a linear programing problem. Herein another approach is taken, the computation of the firing vectors presented in this work require less computational effort and the observer initial marking is defined in a different way that increases the marking convergency speed.

The results presented in this chapter are included in [24].

4.2 Observer for IPN models

In this work it is considered the pair system-observer scheme depicted in figure 4.1. In this case, the system and the observer are IPN and have the same controllable input.

Definition 4.1 *The system IPN model is*

$$N_S = (P_S, T_S, I_S, O_S, \Sigma, \Phi, \lambda, D, \varphi)$$

and the ***observer net*** *is*

$$N_O = (P_O, T_O, I_O, O_O, \Sigma, \Phi, Id, Id, Id)$$

Note that in the observer all transitions are measurable and manipulated and all places are measurable. The state equation of the IPN modeling the system is:

$$M_{k+1} = M_k + C^D v_k^D + C^\varepsilon v_k^\varepsilon$$
$$y_{k1} = \varphi M_k$$
$$y_{k2} = D v_k$$

and the ***state equation of the observer net*** *is:*

$$\hat{M}_{l+1} = \hat{M}_l + C^D v_k^D + \Gamma(\varphi M_k - \varphi \hat{M}_l)$$
$$\hat{y}_k^l = \hat{M}_l$$

The number of places in the observer is the same that the number of places of the system IPN model, however the observer structure is still unknown because Γ has not being defined. Fortunately, using the observation error, Γ can be computed. The **observation error** is defined as:

$$e_k^{l_k} = M_k - \hat{M}_{l_k}$$

That error can be expressed as the difference equation:

$$e_{k+1}^{l+1} = e_k^l + \begin{bmatrix} C^\varepsilon & -\Gamma \end{bmatrix} \begin{bmatrix} v_k^\varepsilon \\ \varphi(e_k^l) \end{bmatrix} \tag{4.1}$$

where $\begin{bmatrix} v_k^\varepsilon \\ \varphi(e_k^l) \end{bmatrix}$ is the "firing vector" of the observer. To make the marking of the observer equal to the marking of the system, then special values must be assigned to $\Gamma(\varphi(e_k^l))$. Next proposition states sufficient conditions to perform this task.

Proposition 4.1 *If* $\Gamma(\varphi(e_k^l)) = \begin{bmatrix} C^\varepsilon \gamma_k & F\beta_k \end{bmatrix}$ *such that* $C^\varepsilon v_k^\varepsilon = C^\varepsilon \gamma_k$ *and* $e_k^l = F\beta_k$ *then the error* e_{k+1}^{l+1} *is equal to* 0.

Proof. As $\Gamma(\varphi(e_k^l)) = \begin{bmatrix} C^\varepsilon \gamma_k & F\beta_k \end{bmatrix}$ such that $C^\varepsilon v_k^\varepsilon = C^\varepsilon \gamma_k$ and $e_k^l = F\beta_k$ then introducing these equations in equation (4.1)

$$e_{k+1}^{l+1} = e_k^l + \begin{bmatrix} C^\varepsilon & -C^\varepsilon & -F \end{bmatrix} \begin{bmatrix} v_k^\varepsilon \\ \gamma_k \\ \beta_k \end{bmatrix} = e_k^l + C^\varepsilon v_k^\varepsilon - C^\varepsilon \gamma_k - F\beta_k = 0.$$

■

As $\Gamma(\varphi(e_k^l)) = \begin{bmatrix} C^\varepsilon \gamma_k & F\beta_k \end{bmatrix}$ is a solution, then the observer state equation becomes:

$$\hat{M}_{l+1} = \hat{M}_l + \begin{bmatrix} C^D & C^\varepsilon & F \end{bmatrix} \begin{bmatrix} v_k^D \\ \gamma_k \\ \beta_k \end{bmatrix}$$
$$\hat{y}_k^l = \hat{M}_l$$

Now, as $l+1$ depends only on k, it is not necessary to maintain two subscripts, so the observer structure is redefined as follows.

Definition 4.2 *The* ***observer net*** $N_O = (P_S, T_S \cup T', I_O, O_O, \Sigma, \Phi, Id, Id, Id)$ *has the* ***state equation:***

$$\hat{M}_{k+1} = \hat{M}_k + \begin{bmatrix} C^D & C^\epsilon & F \end{bmatrix} \begin{bmatrix} v_k^D \\ \gamma_k \\ \beta_k \end{bmatrix}$$

$$\hat{y}'_k = \hat{M}_k$$

Note that the observer is a copy of the system IPN model with some extra transitions defined in F To guarantee that $e_{k+1}^{l+1} = e_{k+1} = 0$, $F\beta_k$ and γ_k must be adequately computed. As the error equation is $e_{k+1} = e_k + C^\epsilon v_k^\epsilon - C^\epsilon \gamma_k - F\beta_k = 0$ then one solution is to propose that $C^\epsilon v_k^\epsilon = C^\epsilon \gamma_k$ and that $e_k = F\beta_k$. Next proposition states how to compute γ_k; $F\beta_k$ will be analyzed later on.

Proposition 4.2 *Let* $(Q, M_0) = (N, \Sigma, \Phi, \lambda, D, \sigma)$ *be an event-detectable IPN then* γ_k *can be computed to be equal to* v_k^ϵ.

Proof. As (Q, M_0) is an event-detectable IPN then any firing transition sequence can be detected. So, the sequence σ such that $\overrightarrow{\sigma} = v_k^\epsilon$ can be detected and this value is assigned to γ_k. ■

The computation of $F\beta_k$ is more difficult because it is related with the initial error over all system-observer pair and just the error over measurable places is known, then $F\beta_k$ cannot be directly computed to fullfil $e_k = F\beta_k$. This problem can be solved when a CML is known and the following special initial marking is given to the observer.

Definition 4.3 *Let* N_S *be an IPN model of a system and* N_O *be its observer IPN, where a* CML *is defined. The* ***initial admissible marking*** $\hat{M}_0$ *of* N_O *is*

$$\hat{M}_0(p_i) = \begin{cases} M_0(p_i) & \text{if } p_i \text{ is measurable} \\ M^{UB}(p_i) & \text{if } p_i \text{ is no-measurable} \end{cases}$$

where $M^{UB}(p_i)$ *is the upper marking bound.*

Using this initial admissible marking, by the Monotonicity lemma [5], if t_j is an enabled transition in the system then it is enabled in the observer. Also, the observer has more tokens that the system, so one way to eliminate these tokens is adding one output transition to each place in the observer and firing this output transition at the appropriate moment. F is devoted to perform this task and it is defined as follows.

Definition 4.4 $F : \{1, 2, ..., n\} \times \{1, 2, ..., n\} \longrightarrow \{-1, 0\}$, *such that* $F[i, i] = -1$ *and* $F[i, j] = 0, i \neq j$

F is a matrix (a Petri net incidence matrix) representing the fact that one output transition is added to each place. Let us define β_k as follows.

Definition 4.5 *Let* $N_S = (P_S, T_S, I_S, O_S, \Sigma, \Phi, \lambda, D, \varphi)$ *be the system IPN model where a* CML *is defined,* $N_O = (P_S, T_S \cup T', I_O, O_O, \Sigma, \Phi, Id, Id, Id)$ *be the observer net with an state equation:*

$$\hat{M}_{k+1} = \hat{M}_k + \begin{bmatrix} C^D & C^\epsilon & F \end{bmatrix} \begin{bmatrix} v_k^D \\ \gamma_k \\ \beta_k \end{bmatrix}$$

$$\hat{y}'_k = \hat{M}_k$$

where v_k^D is the same that is given to the system, γ_k was computed as in proposition 4.2. Then β_k is defined as $\beta_k = \begin{bmatrix} \varpi_1 \\ \vdots \\ \varpi_n \end{bmatrix}$, *where*

$$\varpi_i = \begin{cases} \max\{\hat{M}_k(p_i) - M^{UB}(p_i), 0\} \text{ if } p_i \text{ is no-measurable} \\ \max\{\hat{M}_k(p_i) - M_k(p_i), 0\} \text{ if } p_i \text{ is measurable} \end{cases}$$

Previous definition states that only when the number of tokens into an observer place is greater than its bound or the marking in the corresponding measurable place in the system, then the added output transition must be fired eliminating the exceeding tokens. Using β_k the error $e_{k+1} = e_k - F\beta_k$ will eventually be cancelled.

As it is not easy to see that there is a $k < \infty$ such that $e_k - F\beta_k = 0$, the remainder of this chapter is devoted to prove it.

Lemma 4.1 *Let N_S be a system and N_O be an observer with an initial marking according to definition 4.3. If the same input word is given to both IPN, then the marking $\hat{M}_k$ is always greater than or equal to the marking M_k of N_S (i.e. $\forall k\ \hat{M}_k \geqslant M_k$).*

Proof. With the initial marking $\hat{M}_0 = M_0 + \hat{M}\prime$, where $\hat{M}\prime \geqslant 0$, because of Monotonicity Lemma[5], any enabled transition in N_S is also enabled in N_O. If the same transition t_i is fired in N_S and in N_O then the markings $M_1 = M_0 + C\vec{t_i}$ and $\hat{M}_1 = \hat{M}_0 + C\vec{t_i} = \hat{M}_0 + \hat{M}\prime + C\vec{t_i}$, with $C = \begin{bmatrix} C^D & C^\epsilon \end{bmatrix}$, are reached in N_S and N_O respectively and $\hat{M}_1 \geqslant M_1$. The same procedure can be performed for all reachable markings. ■

Note that one result of previous lemma is that $\gamma_k = v_k^\epsilon$ is an enabled transition sequence in the observer. Now a more complex firing word will be given to the net. First, the same input word will be given to the system and to the observer, and afterwards β_k will be applied to the observer to eliminate exceeding tokens and it will be proved that the marking of the observer asymptotically tends to the marking of the system.

Lemma 4.2 *Let N_S be a system and N_O be an observer with an initial marking according to definition 4.3. If the same input word is given to both IPN and afterwards β_k is given to the observer, then the marking $\hat{M}_k$ is always greater than or equal to the marking M_k of N_S (i.e. $\forall k\ \hat{M}_k \geqslant M_k$).*

Proof. Again the observer initial marking is $\hat{M}_0 = M_0 + \hat{M}\prime$, where $\hat{M}\prime \geqslant 0$. The part when the same input word is given to both IPN was proved before, now the part of β_k will be proved. When the same input word is applied to both nets the marking in the system is $M_1 = M_0 + C\vec{t_i}$ and in the observer is $\hat{M}_1 = M_0 + \hat{M}\prime + C\vec{t_i} = M_1 + \hat{M}\prime$. Then, the error is $e_1 = -M\prime$, then β_1 will be fired and some tokens will be removed; note that at most $M\prime(p_i)$ tokens can be removed in $\hat{M}_1 = M_1 + \hat{M}\prime$ because M_1 is the actual marking and it is never bigger than the bound. Assume that $M\prime_1$ tokens are removed so $\hat{M}_2 = M_0 + \hat{M}\prime + C\vec{t_i} - M\prime_1$, i.e. $\hat{M}_2 = M_1 + \hat{M}\prime - M\prime_1$. For convenience, the subindex of previous marking will be renamed to 1, i.e. $\hat{M}_1 = M_1 + \hat{M}\prime - M\prime_1$. Since $\hat{M}\prime - M\prime_1 \geqslant 0$, $\hat{M}_1 \geqslant M_1$. The same procedure can be performed for all k and then $\hat{M}_k \geqslant M_k$. ■

Note that one result of previous lemma states that $\gamma_k\beta_k$ is an enabled transition sequence in the observer and also that the observer marking tends to the system marking. Now, the next theorem uses previous lemmas to show that when the system is observable, then the observer structure and firing sequences $\gamma_k\beta_k$ proposed can be used to estimate the system marking.

Theorem 4.1 *Let N_S be a cyclic, live and bounded IPN modelling a system, where a CML is defined. Let N_O be an observer with an initial marking according to definition 4.3. If $\forall p_i \in P_S$, $SD(N, M_0; \bullet(p_i), (p_i)\bullet) = M^{UB}(p_i)$ and the same input word is given to both IPN and afterwards β_k is given to the observer, then*

$$\lim_{\left\|v_k^u+v_k^c\right\|\to\infty}\left\|\hat{M}_{k+1}-M_{k+1}\right\|=0$$

Proof. In Lemma 4.2 it was proved that if the same input word is given to both IPN and afterwards β_k is given to the observer, then $\forall k\ M_k \leqslant \hat{M}_k$ and that the enabled transitions in N_S are also enabled in N_O. Again the observer initial marking is $\hat{M}_0 = M_0 + \hat{M}\prime$, where $\hat{M}\prime \geqslant 0$. Notice that the initial marking in the observer is equal to the initial marking in the system for every measurable place, then $\hat{M}\prime$ corresponds to no-measurable places. Let p_i be a measurable place in the system such that $p_i \in \bullet(t_j) \cup (t_j)\bullet$ and that t_j fires in the system. Then since the same input word is given to both nets, t_j also fires in the observer. Therefore $\forall k\ M_k(p_i) = \hat{M}_k(p_i)$. Let p_j be a no-measurable place in the system such that $p_j \in (t_j)\bullet$. Suppose that t_j is the first transition to fire in the system. Then it is also fired in the observer and the markings $M_1(p_j) = M_0(p_j)+1$, $\hat{M}_1(p_j) = \hat{M}_0(p_j)+1$ are reached. Then, by the effect of β_1 the marking $\hat{M}_1(p_j)$ is reduced ϖ_j units, until $M^{UB}(p_j)$ is reached. Then $\left\|\hat{M}_0(p_j) - M_0(p_j)\right\| = \left\|\hat{M}_1(p_j) - 1 - \varpi_j - M_1(p_j) + 1\right\|$ and $\left\|\hat{M}_0(p_j) - M_0(p_j)\right\| \geqslant \left\|\hat{M}_1(p_j) - M_1(p_j)\right\|$. Suppose now that t_j repeatedly fires until $M_k(p_j) = M^{UB}(p_j)$ then $\left\|\hat{M}_k(p_j) - M_k(p_j)\right\| = 0$. Since $\forall p_i \in P_S$, $SD(N, M_0; \bullet(p_i), (p_i)\bullet) = M^{UB}(p_i)$, such sequence exists. This reasoning can be applied to each place, and then $\left\|\hat{M}_{|\sigma|} - M_{|\sigma|}\right\| = 0$ for a transition firing sequence σ.
■

Since it is considered in this work that no-manipulated transitions can exist in the IPN model of the system, the transition firing sequence σ cannot be imposed to the system. For that reason, it is only said that the marking in both nets will be equal when the lenght of the input word tends to infinite, assuming that at that moment the condition on the synchronic distance will be held for all places.

4.2.1 Observer example

Example 4.1 *Let the IPN depicted in figure 4.2.a) be a model of a DES, where the $CML = \{M(p_1) + M(p_2) + M(p_4) = 1, M(p_1) + M(p_3) + M(p_5) = 1\}$ is defined. This IPN is cyclic, live and bounded, since the IPN is event detectable and the synchronic distance properties are fulfilled, then the net is observable. This is true because the IPN is live and binary so $\forall p_j$ no-measurable place $SD(Q, M_0; \cdot(p_j), (p_j)\cdot) \geqslant 1$, and all columns in the matrix*

$$\varphi C^{\varepsilon} = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 & -1 \end{bmatrix}$$

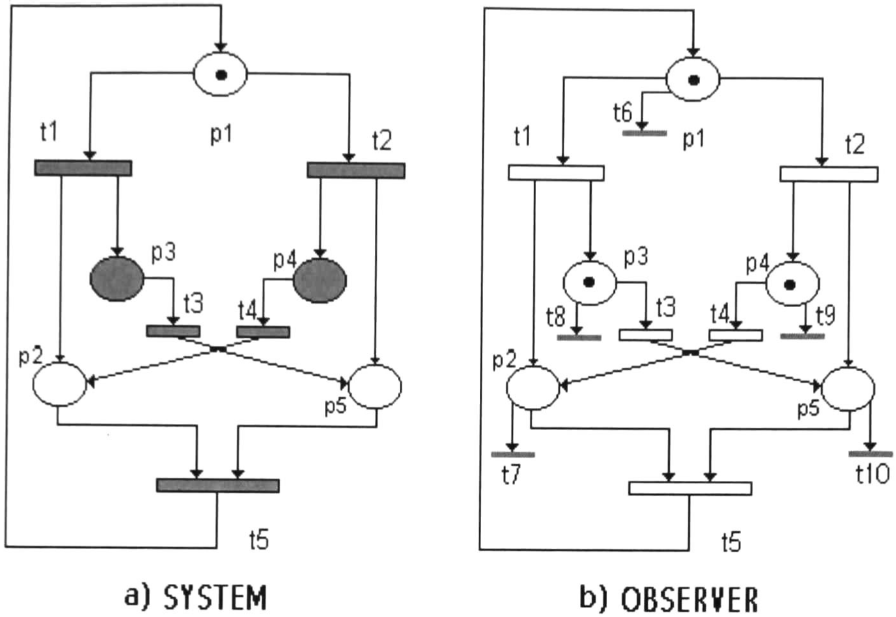


Figure 4.2: Observable system with asymptotic observer.

are not null and different from each other. Figure 4.2.b) represents the asymptotic observer. As an example, assume that in the system the sequence t_1, t_3, t_5, t_2, t_4 *is fired, then in the observer, the sequence* $\gamma_1\beta_1\ \gamma_2\beta_2\ \gamma_3\beta_3\ \gamma_4\beta_4\ \gamma_5\beta_5$ *is computed as* $t_1t_8, t_3, t_5, t_2t_9, t_4$*. Firing it, the marking of the observer will be equal to the marking of the system.*

Note that the marking of the system also can be computed using theorem 3.1.

4.3 Conclusions

In this chapter a method for designing asymptotic observers for IPN modeling DES was presented. The observer is a copy of the system with output transitions added to each place and it allows no-measurable transitions to exist in the system IPN model.

It is shown that the same measurable inputs can be given to the system and the observer if the marking in the observer is supposed to be bigger than in the system; and that the estimation error is reduced by the firing of the output transitions added to the observer.

Measurable inputs are given to both nets at the same time and the rest of the observer firing vector are computed by comparing the incidence matrix of the IPN modeling the system with the difference in the outputs of the observer and system nets.

Chapter 5

Observer improvement

Summary: The observer presented in the previous chapter is improved to accept an initial marking that does not need to be the maximum number of tokens that each place in the system can contain. Another set of transitions are included in the observer, these transitions are devoted to add tokens to the places when it is known that the marking in an observer place is less than in the corresponding system place. A less restrictive condition on the synchronic distance needs to be held for this observer scheme.

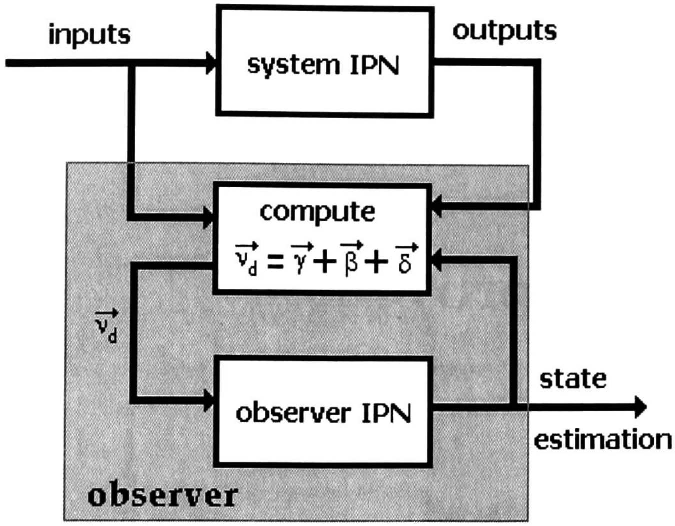


Figure 5.1: Observer block diagram.

5.1 Introduction

In this chapter another observer scheme is presented. This observer requires a less restrictive condition on the synchronic distance to be held in order to guarantee that the observer marking will be equal to the system marking. An input transition is added to each place in the observer and the observer initial marking can be choosen as any value between the minimum and maximum marking that each place can reach.

The marking estimation method presented herein includes the incremental approach taken by Giua in [8] and the decremental one used in the previous chapter.

The observer scheme design technique herein presented is based on the block diagram depicted in figure 5.1 and is presented in [25].

5.2 Observer design

Let us begin by giving the following definition.

Definition 5.1 *The system IPN model is*

$$N_S = (P_S, T_S, I_S, O_S, \Sigma, \Phi, \lambda, D, \varphi)$$

*and the **observer net** is*

$$N_O = (P_S, T_O, I_O, O_O, \Sigma, \Phi, Id, Id, Id)$$

*Note that in the observer all transitions are manipulated and measurable and all places are measurable. The state equation of the IPN system is as the state equation (2.1) and the **state equation of the observer***

net *is:*

$$\hat{M}_{k+1} = \hat{M}_k + \begin{bmatrix} C & F & \Upsilon \end{bmatrix} \begin{bmatrix} \gamma_k \\ \beta_k \\ \delta_k \end{bmatrix}$$

$$\hat{y}'_k = \hat{M}_k$$

where $F[i,i] = -1$ *and* $F[i,j] = 0, i \neq j$; *and* $\Upsilon[i,i] = 1$ *and* $\Upsilon[i,j] = 0, i \neq j$.

Now the observer initial marking $\hat{M}_0$ and the firing vectors γ_k, β_k, δ_k will be defined.

Definition 5.2 *Let N_S be an IPN model of a system and N_O be its observer IPN, where a CML is defined. The* ***initial admissible marking*** *$\hat{M}_0$ of N_O is the following $\hat{M}(p_i) = M(p_i)$ if p_i is measurable and $\hat{M}(p_i)$ is any value fulfilling that $M^{UB}(p_i) \geqslant \hat{M}_0(p_i) \geqslant M^{LB}(p_i)$ if p_i is no-measurable.*

Notice that this observer initial marking definition contains those presented in [8], [1] and in [23].

Now the following proposition is used to compute the firing vectors γ_k, β_k, and δ_k.

Definition 5.3 *Let N_S be an IPN model of the system and N_O be its observer net. If transition t_k fires in N_S then*

- $\gamma_k = \begin{cases} \overrightarrow{t_k} & \text{if } t_k \text{ is enabled in the observer} \\ 0 & \text{other case} \end{cases}$

- $\beta_k = \begin{bmatrix} \varpi_1 \\ \vdots \\ \varpi_n \end{bmatrix}$, *where*

$$\varpi_i = \begin{cases} 1 & \text{if} \quad \begin{array}{l} \gamma_k = \overrightarrow{t_k}, \text{ and } \hat{M}_k(p_i) + C(p_i, \bullet)\gamma_k > M^{UB}(p_i) \text{ or} \\ \gamma_k \neq \overrightarrow{t_k}, \ p_i \in \bullet(t_k), \text{ and } \hat{M}_k(p_i) > M^{LB}(p_i) \end{array} \\ 0 & \text{other case} \end{cases}$$

- $\delta_k = \begin{bmatrix} \upsilon_1 \\ \vdots \\ \upsilon_n \end{bmatrix}$, *where*

$$\upsilon_i = \begin{cases} 1 & \text{if} \quad \begin{array}{l} \gamma_k = \overrightarrow{t_k}, \text{ and } \hat{M}_k(p_i) + C(p_i, \cdot)\gamma_k < M^{LB}(p_i) \text{ or} \\ \gamma_k \neq \overrightarrow{t_k}, p_i \in (t_k)\cdot, \text{ and } \hat{M}_k(p_i) < M^{UB}(p_i) \end{array} \\ 0 & \text{other case} \end{cases}$$

The previous definition states that when the number of tokens in the observer are not enough to fire the same transition t_j that fires in the system then the added input transitions in the observer will be fired to add tokens into the output places of t_j; or when the marking of a place exceeds the known marking bound for that place, those tokens are removed by the firing of the added output transitions in the observer.

Using these firing vectors the state of the observer will tend to the state of the system as will be proved in the following theorem.

Theorem 5.1 *Let N_S be a cyclic, live, bounded and event detectable IPN modelling a system, where a CML is defined. Let N_O be an observer with an initial marking according to definition 5.2. If $\forall p_i \in P_S$ either, $SD(N, M_0; \bullet(p_i), (p_i)\bullet) = \mathbb{D}_i$ or $SD(N, M_0; (p_i)\bullet, \bullet(p_i)) = \mathbb{D}_i$ and the firing vectors of the observer γ_k, β_k and δ_k are computed as in definition 5.3, then*

$$\lim_{\|\alpha\|\to\infty} \left\| \hat{M}_{k+1} - M_{k+1} \right\| = 0$$

where α is the transition sequence fired in N_S.

Proof. The observer initial marking is $\hat{M}_0 = M_0 + \hat{M}\prime$, where $\left|\hat{M}\prime\right| < \mathbb{D}$.

Assume that $SD(N, M_0; \bullet(p_k), (p_k)\bullet) = \mathbb{D}_k$ for some p_k, and the firing vectors of the observer γ, β and δ are computed as in definition 5.3, then, a transition firing sequence σ_k -such that the number of firings of transitions in $\bullet(p_k)$ without firing any transition in $(p_k)\bullet$ is equal to $\mathbb{D}_k$- exists. Then, as in theorem 3.1, when σ_k is fired, a new marking is reached: $M_j(p_k) = M_n(p_k) + \mathbb{D}_k$, where $M_j(p_k) = M^{UB}(p_k)$ and $M_n(p_k) = M^{LB}(p_k)$. Three cases must be studied:

a) $\hat{M}\prime(p_k) = 0$. If a transition $t_j \in \bullet(p_k)$ is fired in σ_k, then by definition 5.3 either $\gamma = \overrightarrow{t_j}$ or $\upsilon_k = 1$ and $\varpi_k = 0$, then the estimation error of p_k remains zero over the execution of σ_k.

b) $\hat{M}\prime(p_k) > 0$. If a transition $t_j \in \bullet(p_k)$ is fired in σ_k, then by definition 5.3 the following cases are possible:

i) $\gamma = \overrightarrow{t_j}$ or $\upsilon_k = 1$ and $\varpi_k = 0$ when $\hat{M}(p_k) < M^{UB}(p_k)$, the estimation error of p_k does not change.

ii) $\gamma = \overrightarrow{t_j}$ or $\upsilon_k = 1$, and $\varpi_k = 1$ and when $\hat{M}(p_k) \geqslant M^{UB}(p_k)$, then the estimation error of p_k decreases, and will be equal to zero over the execution of σ_k.

Note that in both cases $\gamma = \overrightarrow{t_j}$ is mutually exclusive with $\upsilon_k = 1$ and that case ii) occurs exactly $\hat{M}\prime(p_k)$ times because marking $M^{UB}(p_k)$ is reached and the observer marking is not allowed to exceed it.

c) $\hat{M}\prime(p_k) < 0$. During the firing of σ_k, the marking $M_n(p_k) = M^{LB}(p_k)$ was reached. By definition 5.3, when the firing of any transition reduces the marking $\hat{M}(p_k)$ below $M^{LB}(p_k)$, then $\upsilon_k = 1$ "*freezing*" this marking in $M^{LB}(p_k)$, so the estimation error of p_k is reduced to zero when the marking $M_n(p_k)$ is reached.

Then in all the cases the estimation error of p_k is reduced to zero.

Now assume that $SD(N, M_0; (p_k)\bullet, \bullet(p_k)) = \mathbb{D}_k$ for some p_k, and the firing vectors of the observer γ, β and δ are computed as in definition 5.3, then a transition firing sequence σ_k -such that the number of firings of transitions in $(p_k)\bullet$ without firing any transition in $\bullet(p_k)$ is equal to $\mathbb{D}_k$- exists. Using a similar reasoning it can be proved that the estimation error will be equal to zero for place p_k when σ_k has been fired.

Since at least one of the conditions on the synchronic distance holds for each place and N_S is a cyclic IPN, then the estimation error will be zero in all places when a transition sequence $\alpha = \sum\limits_{i=1..n} \sigma_i$ fires in the system. Therefore the theorem holds. ■

Theorem 5.1 formally establishes that the markings in the observer and in the system become equal after a finite sequence of events occur in the system. However, since the net can contain uncontrollable events (no-manipulated transitions), the length of this sequence cannot be established.

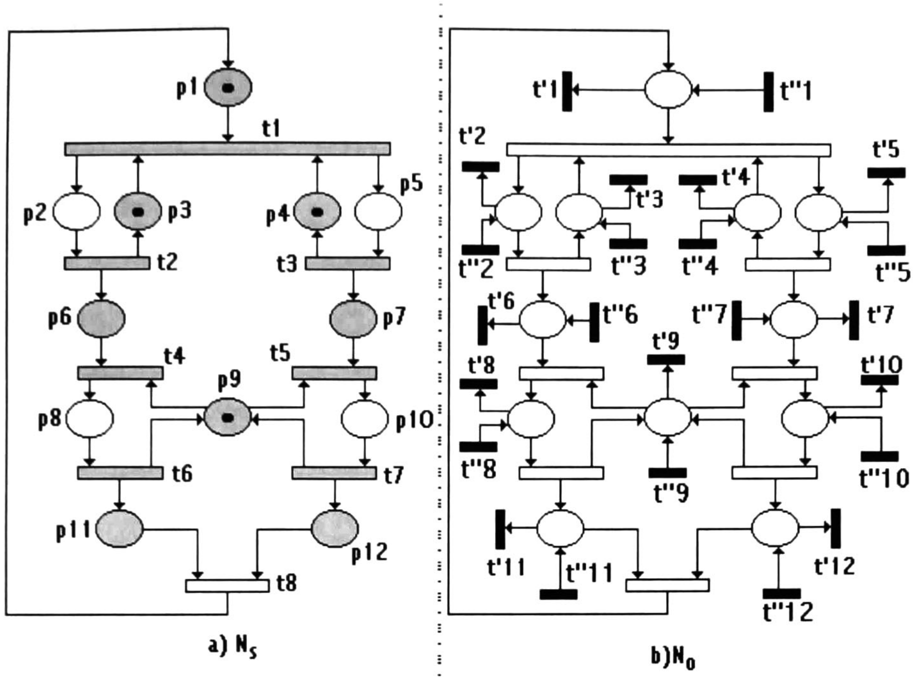


Figure 5.2: System and Observer IPN.

5.2.1 Manufacturing cell observer example

The manufacturing cell example 2.6, whose description is transcript here for the readers comfort, is used for showing the operation of the observer presented in this chapter.

Example 5.1 *Consider a manufacturing cell where a product consisting of two parts (Pa, Pb) is processed. Pa requires to use the machines $M1$ and $M2$, and Pb requires $M3$ and $M2$, both in that sequence. After both parts are processed they are assembled and the product is released, then the system is ready to start another cycle. The IPN of figure 5.2a) is a model of the system where a token in place p_1 represents an idle state of the system; transition t_1 represents the beginning of a cycle. A token in p_2 (p_2) represents that the machine $M1$ ($M3$) is being used and in p_3 (p_4) that $M1$ ($M3$) is available. t_2 and t_3 represent that the process in $M1$ and $M3$ has finished respectively, a token in p_6 and p_7 represent a piece waiting for $M2$ to be available. t_4 and t_5 represent the beginning of the process of a part in $M2$. A token in p_9 represents that $M2$ is available. p_8 and p_{10} represent a part being processed in $M2$. t_6 and t_7 represent that the process in $M2$ has finished. Tokens in p_{11} and p_{12} represent the parts waiting to be assembled. Finally, t_8 represents the assembling and release of the product.*

Different signals are displayed when $M1$ and $M3$ are being used, and $M2$ display two different signals depending on the piece that is being processed. Therefore p_2, p_5, p_8 and p_{10} are measurable places in the model and since the releasing of a product can be detected also t_8 is measurable.

The model for this example is a binary IPN where a CML is given by the following p-semiflows:

$$\begin{aligned} M(p_2) + M(p_3) &= 1 \\ M(p_4) + M(p_5) &= 1 \\ M(p_4) + M(p_5) &= 1 \\ M(p_1) + M(p_2) + M(p_6) + M(p_8) + M(p_{11}) &= 1 \\ M(p_1) + M(p_5) + M(p_7) + M(p_{10}) + M(p_{12}) &= 1 \end{aligned} \tag{5.1}$$

For this model all columns in the matrix

$$\varphi C^{\epsilon} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

are not null and different; then any transition firing can be determined. Using the information provided by the CML set 5.1 for the initial marking depicted in the figure 5.2b) the transition firing sequence $\sigma = t_1$ is enough to compute the system markings M_1 and M_0 since it is known that p_2 and p_5 are marked at M_1 and using 5.1 the number of tokens in all other places can be computed.

If the observer with the initial marking $\hat{M}_0 = 0$, shown in figure 5.2b) is used and if, for instance, in the system the sequence t_1, t_2, t_3, t_4 is executed and in consequence $t_2'', t_5'', t_2, t_3, t_8'', t_6'$ is executed in the observer, then the markings in both IPN become equal.

5.3 Conclusions

Any marking between the minimum and maximum number of tokens that each place can reach can be chosen to be the initial marking of the observer introduced in this chapter. Because of this, the observer presented in the previous chapter and some of those included in the works cited herein can be consider as special cases of this observer.

The observer scheme presented in this chapther requires a less restrictive condition on the synchronic distance to be held and for that reason achives to determine the marking of more IPN than the observer presented in the previous chapter.

Chapter 6

Conclusions and future work

6.1 Conclusions

In this work, the observability problem for DES was addressed. A definition that clearly extends the meaning of this property as it is understood in systems theory to DES was presented. That definition establishes that a DES is observable if the information provided by the inputs, outputs and structure of the system is enough to compute its initial state when a finite sequence of inputs is given to it. Using this definition a characterization of the IPN models that exhibit this characteristic was given. Unfortunately that characterization is not completely given in terms of the structure of the IPN and only provides sufficient conditions to guarantee that the initial system state can be computed.

In order to determine the marking of an IPN some knowledge on physical characteristics of DES is needed, however the assumption on the knowledge of all events that affect the state of the system has been dropped.

An observer in IPN terms that achieves to compute the marking of observable IPN was given. Since observable IPN are event-detectable, once the current marking is known, the initial one can be easily computed. This observer is improved to accept different initial markings, which can be useful to perform further analysis to increase the state estimation speed.

6.2 Future work

The following extension of the results herein presented are left as future work:

- Determining all necessary conditions for an IPN to be observable.
- The extension of this work to colored, generalized, stochastic and timed PN.
- The application of this observer scheme to state feedback control and fault tolerant systems.
- Finding the minimum number of measurable places needed in an IPN model to be observable.
- Determining the best way to define the observer initial marking in order to increase the convergency speed of the state estimation.

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Appendix A

Symbols and abbreviations

A.1 Symbols

G	a PN structure.
P	the set of places of a PN, $P = \{p_1, p_2, ..., p_n\}$.
p_i	the i-th place of a PN.
T	the set of transitions of a PN, $T = \{t_1, t_2, ..., t_m\}$.
t_j	the j-th transition of a PN.
I	the function representing the arcs going from places to transitions $I : P \times T \longrightarrow \{0, 1\}$.
O	the function representing the arcs going from transitions to places $O : P \times T \longrightarrow \{0, 1\}$.
C	the incidence matrix of a PN.
b_{ij}	the ij-th element of a matrix B.
$\mathbb{Z}^{\geqslant 0}$	the nonnegative integer numbers.
$\{x\}^k$	a k-dimentional vector of x.
M	the marking function that assigns a nonnegative integer to each place of the net ,$M : P \longrightarrow \{\mathbb{Z}^{\geqslant 0}\}^n$
N	a PN, $N = (G, M_0)$, where M_0 is an initial token distribution.
v	a firing vector of a PN.
$\mathbf{R}(G, M_0)$	the reachability set of a PN.
$\bullet(t_j)$	the set of all places p_i such that $I(p_i, t_j) \neq 0$.
$(t_j)\bullet$	the set of all places p_i such that $O(p_i, t_j) \neq 0$.
$\bullet(p_i)$	the set of all transitions t_j suchthat $O(p_i, t_j) \neq 0$.
$(p_i)\bullet$	the set of all transitions t_j such that $I(p_i, t_j) \neq 0$.
Q	an IPN.
Σ	the input alphabet of an IPN, $\Sigma = \{\sigma_1, \sigma_2, ..., \sigma_r\}$.
σ_i	an input symbol.
Φ	the output alphabet of an IPN, $\Phi = \{\phi_1, \phi_2, ..., \phi_s\}$.
ϕ_i	an output symbol.
ε	a null value in an input or output alphabet.
λ	a transition labeling function, $\lambda : T \rightarrow \Sigma \cup \{\varepsilon\}$.
τ	the set of names of the transitions.
D	feed-forward function, $D : T \rightarrow \top$, where $\top = \tau \cup \{\varepsilon\}$.
φ	output function, $\varphi : \mathbf{R}(G, M_0) \rightarrow \{\Phi \cup \{\varepsilon\}\}^q$.

$M_k \xrightarrow{t_j}$	transition t_j is enabled at marking M_k.
$M_k \xrightarrow{t_j} M_{k+1}$	transition t_j is enabled at marking M_k and the firing of this transition in this marking leads to the marking M_{k+1}.
$M_j \xrightarrow{\sigma} M_k$	the firing of the transition sequence σ is enabled at marking M_j and leads to the marking M_k.
e_j	elemental vector, $e_j[i \neq j] = 0$, $e_j[j] = 1$.
C^ε	submatrix of C formed by the columns of the no-measurable transitions in an IPN.
C^D	submatirx of C formed by the columns of the measurable transitions in an IPN.
$\pounds(Q, M_0)$	the firing language of the IPN (Q, M_0).
$\pounds_{in}(Q, M_0)$	the input language of (Q, M_0).
$\pounds_{out}(Q, M_0)$	the output language of (Q, M_0).
$SD(Q, M_0; t_i, t_j)$	the synchronic distance of a transition t_i with respect to a transition t_j.
CML	a set of conservative marking laws.
$M^{LB}(p_k)$	marking lower bound for place p_k.
$M^{UB}(p_k)$	marking upper bound for place p_k.
$\mathbb{D}_k$	maximum marking gap for place p_k.
$\hat{M}$	observer marking function.
e	observation error $M - \hat{M}$.
$\digamma$	a PN incidence matrix containing output transition.
Υ	a PN incidence matrix containing input transition.
$\alpha, \beta, \gamma, \delta$	observer firing vectors.
$\|G\|$	the size of a set G.
$Y(\bullet, x)$	the x-th column vector of the matrix Y.
$Y(x, \bullet)$	the x-th row vector of the matrix Y.

A.2 Abbreviations

DES	Discrete Event System
FA	Finite Automata
PN	Petri Net
IPN	Interpreted Petri Net

CENTRO DE INVESTIGACION Y DE ESTUDIOS AVANZADOS DEL IPN
UNIDAD GUADALAJARA

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