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Despacho óptimo de reactivos mediante una formulación multi-objetivo

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Reactive optimal management by a multi-objective formulation

A thesis presented by:
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DEDICATION

To the memory of my beloved mother, Flora Maria

To my father, Miguel Angel

To my brother, Camilo

To my sisters, Wendy and Claudia

To my aunt, Guadalupe

Acknowledgements

I would like to express my profound gratitude

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Resumen

El problema de optimización de la potencia reactiva es muy importante para la seguridad del sistema de potencia y el funcionamiento económico. El objetivo fundamental es encontrar los ajustes de las variables de control, tales como el voltaje de los generadores, la derivación de los transformadores, los capacitores e inductores en derivación, que ayudan a mantener los perfiles de voltaje aceptable y minimizar las pérdidas de energía.

La estabilidad de voltaje es una preocupación importante en la planificación y operación de los sistemas de potencia. Es bien sabido que el colapso y la inestabilidad de voltaje dan lugar al principal fallo del sistema. El problema de la estabilidad de voltaje puede explicarse simplemente como la incapacidad del sistema de potencia para proporcionar el consumo de potencia reactiva.

Varios modelos matemáticos para el despacho de la potencia reactiva han sido reportados en la literatura. La mayoría de ellos adoptan una función de un solo objetivo. La minimización de las pérdidas de potencia activa es a menudo considerada como una función objetivo para el envío de potencia reactiva en sistemas de potencia. Dado que esta función objetivo tiende a elevar los voltajes del generador, el estado resultante puede dar lugar a una falta de reservas necesarias para proporcionar potencia reactiva durante las contingencias. Reducir al mínimo las desviaciones de los voltajes de los valores deseados es ampliamente utilizado.

Recientemente, los métodos de optimización multi-objetivo para el control de la potencia reactiva se han hecho populares. Convencionalmente, la atención se ha centrado en las pérdidas de potencia y la desviación de voltaje. Relativamente, pocos esfuerzos han estado directamente involucrados con la mejora de la estabilidad de voltaje.

Hasta ahora, varios algoritmos de optimización matemáticos, tales como los algoritmos de gradiente, programación lineal, programación no lineal y los métodos de puntos interiores, han sido ampliamente utilizados para resolver el problema. Sin embargo, el despacho de la potencia reactiva es un problema de optimización de funciones no continuas y no lineales. Estas técnicas convencionales necesitan muchos supuestos matemáticos, tales como la continuidad, convexidad, etc., y muchas veces quedan atrapados en soluciones óptimas locales. En los últimos años, los algoritmos evolutivos (AE), tales como algoritmos genéticos, programación evolutiva y estrategia evolutiva, se han aplicado al problema de optimización de la potencia reactiva. Teóricamente, estas técnicas convergen a la solución óptima global con alta probabilidad. Son útiles especialmente cuando fallan otros métodos de optimización para encontrar la solución óptima.

En esta tesis se propone una formulación multi-objetivo del problema de la potencia reactiva y el control de voltaje, resuelto mediante una técnica evolutiva multi-objetivo. Los objetivos son la desviación de voltaje y un índice de estabilidad de voltaje del sistema. Las restricciones de las cargas y operación son también tomadas en cuenta. El enfoque propuesto se evalúa en tres sistemas de potencia de diferente complejidad.

Abstract

The reactive power optimization problem is very important to power system security and economical operation. The basic objective is to find proper adjustments on the control variables, such as generator voltages, transformer taps, shunt capacitors and inductors, that help to maintain acceptable voltage profiles and minimize power losses.

Voltage stability is a major concern in power systems' planning and operation. It is well known that voltage instability and collapse have led to major system failure. The problem of voltage stability may be simply explained as the power system inability to provide the reactive power consumption.

Several mathematical models for the reactive power dispatch have been reported in the literature. Most of them adopt a single-objective function. The active power losses minimization is often considered as an objective function for reactive power dispatch in power systems. Since this objective function tends to raise generator voltages, the resulting state may give rise to a lack of required reserves to provide reactive power during contingencies. Minimizing the deviations of voltages from desired values is widely used.

Recently, multi-objective optimization approaches for reactive power control have become popular. Conventionally, the attention has been focused upon power losses and voltage deviation. Relatively, little effort has been directly involved with voltage stability improvement.

Up to now, several mathematical optimization algorithms, such as gradient-based algorithms, linear programming, non-linear programming and interior point methods, have been widely used to solve the problem. However, the reactive power dispatch is an optimization problem of non-continuous and non-linear functions. These conventional techniques need many mathematical assumptions, such as continuity, convexity, etc., and often they become stuck into local optimal solutions. In recent years, evolutionary algorithms (EAs), such as genetic algorithm, evolutionary programming and evolutionary strategy, have been applied to reactive power optimization problem. Theoretically, these techniques converge to the global optimum solution with high probability. They are useful especially when other optimization method fail in finding the optimal solution.

In this thesis, a formulation multi-objective of the reactive power and voltage control problem solved by a multi-objective evolutionary technique is proposed. The objectives are voltage deviation and a voltage stability index of the system. The load constraints and operational constrains are also taken into account. The proposed approach is evaluated in three power systems of different complexity.

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Chapter 1

Introduction

Recently, the problem of the reactive power and voltage control or volt/var control (VVC) to improve the power system economy and security, has attracted special attention. The basic objective is to find proper adjustments on the control variables, such as generator voltages, transformer taps, shunt capacitors and inductors, which help to maintain acceptable voltage profiles and to minimize power losses. A proper volt/VAr control can increase system efficiency, decrease system power losses, and improve voltage profile.

Several reactive power dispatch's formulations have been reported. There are two general approaches to solve this complex problem.

The first approach employs conventional optimization techniques, such as gradient-based algorithms, linear programming, non-linear programming and interior point methods [1.1-1.4]. Unfortunately, the VVC is a non-linear and non-continuous optimization problem and may have several local minima. Since these conventional techniques require many mathematical assumptions, such as differential properties of the objective functions and a unique minimum within the variables' domain, they are susceptible to be trapped into local minima.

The second approach is based on the heuristic methods to search for the optimal solution in the problem space [1.5-1.7]. Theoretically, these techniques converge to the global optimum solution with high probability. These heuristic methods have been applied to solve the reactive power dispatch problem with impressive success.

Nowadays, multi-objective optimization approaches for reactive power control have become popular. However, some approaches do not address the problem as a true multi-objective problem, because it converts to a single objective problem [1.8-1.10].

The recent direction is to handle the objectives simultaneously as competing objectives instead of simplifying the multi-objective problem to a single objective problem. Conventionally, the attention has been focused upon power losses, fuel cost, atmospheric emissions, and voltage deviation [1.11-1.14]. Relatively, little effort has been directly involved with voltage stability improvement. In this thesis, this is one of our major purposes.

Any real-power production, transport, and/or consumption are at the same time accompanied by the production, transport, and/or consumption of reactive power. In particular, each load consumes certain reactive power QL as it consumes real power PL. Because of this, it is necessary to balance reactive power.

The reactive-power compensation for the power consumed by loads and reactive power lost in transport could be provided by the loads themselves, by the reactive-power-compensating devices on the transmission system (such as shunt and series capacitor banks), and/or by the power plants.

The problem of reactive-power dispatch is generally bundled with the problem of maintaining load voltages within the pre-specified limits. The generator voltage set-point values VGiref are optimized with respect to certain performance criteria subject to the reactive-power-balance constraints, the load voltage acceptable limits, and the available limits on the reactive power generated and the limits on generator voltages. The generation-based reactive-power dispatch falls under the category of the optimal power flow (OPF).

The optimal power flow (OPF) problem, which was introduced in 1960s is an important and powerful tool for power system operation and planning [1.15]. Reactive power optimization is a sub-problem of OPF calculation, which determines all the controllable variables, such as tap ratio of transformers, output of shunt capacitors/reactors, reactive power output of generators and static reactive power compensators etc., and minimizes transmission losses or other appropriate objective functions, while satisfying a given set of physical and operational constraints.

Since transformer tap ratios and outputs of shunt capacitor/reactors have a discrete nature, while reactive power output generators, bus voltage magnitudes and angles are, on the other hand, continuous variables, the reactive power optimization problem is formulated as mixed-integer, nonlinear problem [1.16].

Modern algorithms are considered as effective tools for nonlinear optimization problems with applications to power systems scheduling, e.g. economic load dispatch (ELD). The algorithms do not require that the objective functions and the constraints have to be differentiable and continuous.

Algorithms based on the principles of natural evolution have been applied successfully to a set of problems of numerical optimization. With a good degree of parallelism and stochastic characteristics, they are adequate for solving complicated problems of optimization, such as those found in reactive optimization, distribution systems planning, expansion of transmission systems, etc. [1.17-1.21]. The literature presents an extensive list of works concerning the application of evolutionary techniques to problems of power systems [1.22]. In general, these applications concentrate primarily on power system planning, followed by distribution systems.

Lai and Ma [1.17] have presented a modified evolutionary program to solve the reactive power dispatch, obtaining good results. Other authors [1.19, 1.20] have applied the same algorithm for other power system problems, reporting results using the IEEE30 system. A simplified evolution strategy has been used in [1.20] and compared with genetic algorithms and the Lai and Ma algorithm. More recently, a proposal quite similar to [1.17] has been presented in [1.21]. In spite of these efforts, evolutionary techniques have not yet been explored completely for power system applications [1.23].

Recently, different heuristic algorithms have been analyzed to solve optimization problems. Generally speaking, all population-based optimization algorithms suffer from long computational times because of their evolutionary/stochastic nature. This crucial drawback sometimes limits their application to offline problems with little or no real-time constraints.

In a population of potential solutions to an optimization problem within an n-dimensional search space, a fixed number of vectors are randomly initialized, and then new populations are evolved over time to explore the search space and locate the minima of the objective function. If the fitness of the trial vector is better than that of the target, the target vector is replaced by the trial vector in the next generation.

Optimization techniques are applied to determine the steady-state optimal operating conditions, where voltage magnitudes and angles at all buses are evaluated for specific levels of load and generation. Evidently the results of any optimization technique will impact on the power system stability. Therefore, in this paper the implication on steady state voltage stability is taken into account.

1.1. Thesis Objectives

The aim of this thesis is the proposition of a multi-objective formulation for the reactive power and voltage control or volt/var control (VVC) optimization problem. The objectives are the buses' voltage deviations and the system's voltage stability, measured through an index. The load and operational constraints are also taken into account.

It also proposes the application of multi-objective evolutionary techniques as effective tools to solve the multi-objective VVC problem.

1.2. Thesis Outline

The thesis is broke down into five chapters to report on the whole research activities and the results obtained.

- Chapter 1 presents a brief introduction, as well as the main objectives of the thesis.
- Chapter 2 introduces the general concept of the multi-objective problem, and several concepts and definitions related to multi-objective optimization. Also presents an overview on the evolutionary algorithm for multi-objective optimization.
- Chapter 3 presents and describes the statement of the problem to be handled with, as well as the voltage stability index definition and the voltage deviation function. The utilized multi-objective evolutionary approaches and the *spacing metric* used to evaluate quantitatively the performance of the proposed algorithms are also summarized.
- Chapter 4 exposes the application of the proposed multi-objective evolutionary techniques to the 9-bus power system, 26-bus power system, and the 39-bus system.
- Finally, some concluding remarks and recommendations for future works are presented.

- Finally, some concluding remarks and recommendations for future works are presented.

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Chapter 2

Multi-Objective Optimization

As the name suggest, a multi-objective optimization problem (MOP) deals with more than one objective function. In most of the real world problems, multiple objectives or multiple criteria are possible. If these objectives are conflicting, then the problem becomes one of finding the best possible designs that satisfy the competing objectives under different tradeoff scenarios [2.1]. With these multiple objectives and constraints taken into account, an optimum design problem can then be formulated. This type of problems is known as multi-objective, multi-criteria, or vector optimization problems [2.2].

Multi-objective optimization (MO) is a very demanding research topic because most real-world problems have not only a multi-objective nature, but also many open issues to be answered qualitatively and quantitatively [2.1]. In fact, there is not even a universally accepted definition of *optimum* as in single-objective optimization [2.3], because the solution to a MOP is generally more than a single point. It consists of a family of points in a feasible solution space, which describes the tradeoff among contradicted objectives.

2.1 The general multi-objective optimization problem

Multi objective optimization (MO) is a methodology to look for an optimal solution to multivariable problems with multiple, often conflicting, objectives [2.4]. The problem can be formulated as follows [2.5]:

Find the vector $\bar{x}^* = [x_1^*, x_2^*, ..., x_n^*]^T$ which will satisfy the m inequality constraints:

$$g_i(\bar{x}) \leq 0 \qquad i = 1, 2, ..., m \tag{2.1}$$

the p equality constraints:

$$h_i(\bar{x}) = 0 \qquad i = 1, 2, ..., p \tag{2.2}$$

and optimizes the vector function:

$$\vec{f}(\bar{x}) = [f_1(\bar{x}), f_2(\bar{x}), ..., f_k(\bar{x})]^T \tag{2.3}$$

where $\bar{x} = [x_1, x_2, ..., x_n]^T$ is the vector of decision variables.

In other words, we wish to determine the set of decision variables which satisfy (2.1) and (2.2). That is, the particular set $x_1^*, x_2^*, ..., x_n^*$ which also yields the optimum values of all the objective functions.

Constraints expressed through (2.1)-(2.2) define the *feasible region F* and any point $\bar{x}$ in *F* defines a feasible solution. The vector function $f(\bar{x})$ is a function which maps the set *F* into the set *X*, which represents all possible objective functions' values.

The problem usually has no unique, perfect (or utopian) solution, but a set of non-dominated, alternative solutions, known as the Pareto-optimal set [2.6].

2.1.1 Background concepts

Several concepts and definitions related to multi-objective optimization are described in the following [2.7].

Decision variables:

The decision variables are a set of *n* parameters whose values give a solution (can be valid or not) to a problem. These parameters are denoted as $x_j, j = 1, 2, ..., n$ In this work, these variables will be represented by:

$$\bar{x} = [x_1, x_2, ..., x_n]^T \tag{2.4}$$

Constraints:

Most real world optimization problems have (natural and problem dependant) constraints to be satisfied (they draw up the boundaries of the feasible set). Constraints depend on the decision variables and can be expressed in form of mathematical inequalities (2.5)-(2.6).

$$g_i(\bar{x}) \leq 0 \qquad i = 1, 2, ..., m \tag{2.5}$$

$$h_i(\bar{x}) = 0 \qquad i = 1, 2, ..., p \tag{2.6}$$

We say that an inequality constraint is active at $\bar{x}$ if $g_i(\bar{x}) = 0$. All equality constraints h_i (regardless of the value of $\bar{x}$ used) are considered active at all points of *F*.

Objective functions:

The objective functions are the evaluation criteria used to estimate how good a solution is. As in the case of constraints, objective functions depend on the decision variables. In multi-objective optimization problems there are k (≥ 2) objective functions $f_1(\bar{x}), f_2(\bar{x}), ..., f_k(\bar{x})$. In this document, we will represent $\vec{f}$ in the following way:

$$\vec{f}(\bar{x}) = [f_1(\bar{x}), f_2(\bar{x}), . . f_k(\bar{x})]^T \tag{2.7}$$

Pareto Dominance:

A vector $\bar{x}^* = (x_1, x_2, ..., x_k)$ is said to dominate $\bar{y}^* = (y_1, y_2, .., y_k)$ (denoted by $\bar{x} \preceq \bar{y}$) if and only if x is partially less than y, i.e., $\forall i \in \{1,...,k\}$, $x_i \le y_i \wedge \exists i \in \{1,...,k\} : x_i < y_i$.

As an example, for the case of two decision vectors $\bar{x}^*, \bar{y}^* \in X$,

$\bar{x}^* \prec \bar{y}^*$	*iff* $f_i(\bar{x}^*) < f_i(\bar{y}^*)$
($\bar{x}^*$ strictly dominates $\bar{y}^*$)	for every $i = 1,...,k$
$\bar{x}^* \preceq \bar{y}^*$	*iff* $f_i(\bar{x}^*) \le f_i(\bar{y}^*)$
($\bar{x}^*$ weakly dominates $\bar{y}^*$)	for every $i = 1,...,k$
$\bar{x}^* \sim \bar{y}^*$	*iff* $f_i(\bar{x}^*) \npreceq f_i(\bar{y}^*) \wedge f_i(\bar{y}^*) \npreceq f_i(\bar{x}^*)$
($\bar{x}^*$ and $\bar{y}^*$ are non-dominated between themselves)	for every $i = 1,...,k$

These definitions are analogous for maximization problems ($\succ, \succeq, \sim$). To reinforce these concepts, let's consider the following example:

$\bar{x}^* = [2,5,4.5]^T$, $\bar{y}^* = [2,5,4.8]^T$ and $\bar{z}^* = [3,6,4.7]^T$

$\bar{x}^* \prec \bar{z}^*$ because $2 < 3$, $5 < 6$ and $4.5 < 4.7$

$\bar{x}^* \preceq \bar{y}^*$ because $2 = 2$, $5 = 5$ and $4.5 < 4.8$ and,

$\bar{y}^* \sim \bar{z}^*$ because $2 < 3$, $5 < 6$ and $4.8 > 4.7$

Figure 2.1 shows the difference between the decision variable space and the objective function space.

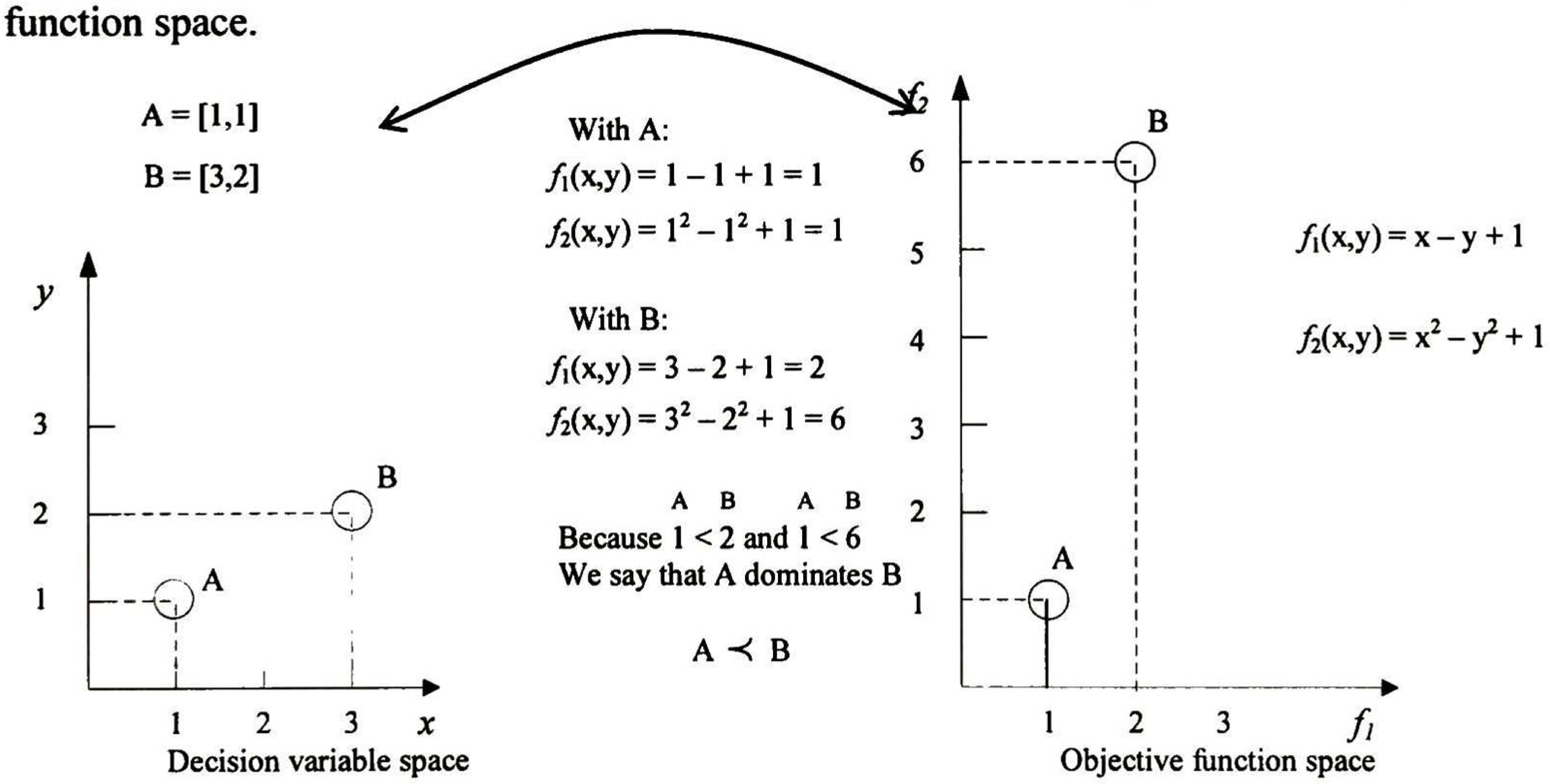

Figure 2.1: Graphical illustration of the decision variable space (left) and objective function space (right).

Pareto optimum:

The concept of Pareto optimum was formulated by Vilfredo Pareto [2.8] in the XIX century, and constitutes by itself the origin of research in multi-objective optimization. The formal definition is as follows [2.5]:

A point $\bar{x}^* \in F$ is *Pareto Optimal* if for every $\bar{x} \in F$ and $I = \{1,2,...,k\}$ either

$$f_i(\bar{x}^*) = f_i(\bar{x}), \quad \forall i \in I \tag{2.8}$$

Or, there is at least one $i \in I$ such that

$$f_i(\bar{x}^*) < f_i(\bar{x}) \tag{2.9}$$

In words, this definition says that $\bar{x}^*$ is Pareto optimal if there exists any feasible vector $\bar{x}$ which would decrease some criterion without causing a simultaneous increase in at least one other criterion. Unfortunately, the Pareto optimum almost always gives not a single solution, but rather a set of solutions called *non-inferior* or *non-dominated* solutions.

Pareto front:

When plotted in the objective space, the non-dominated points are collectively known as the *Pareto front*. The minima in the Pareto sense are going to be in the boundary of the design region, or in the locus of the objective functions' tangent points. In figure 2.2 a bold line is used to mark this boundary for a bi-objective problem which is known as the *Pareto front* [2.5].

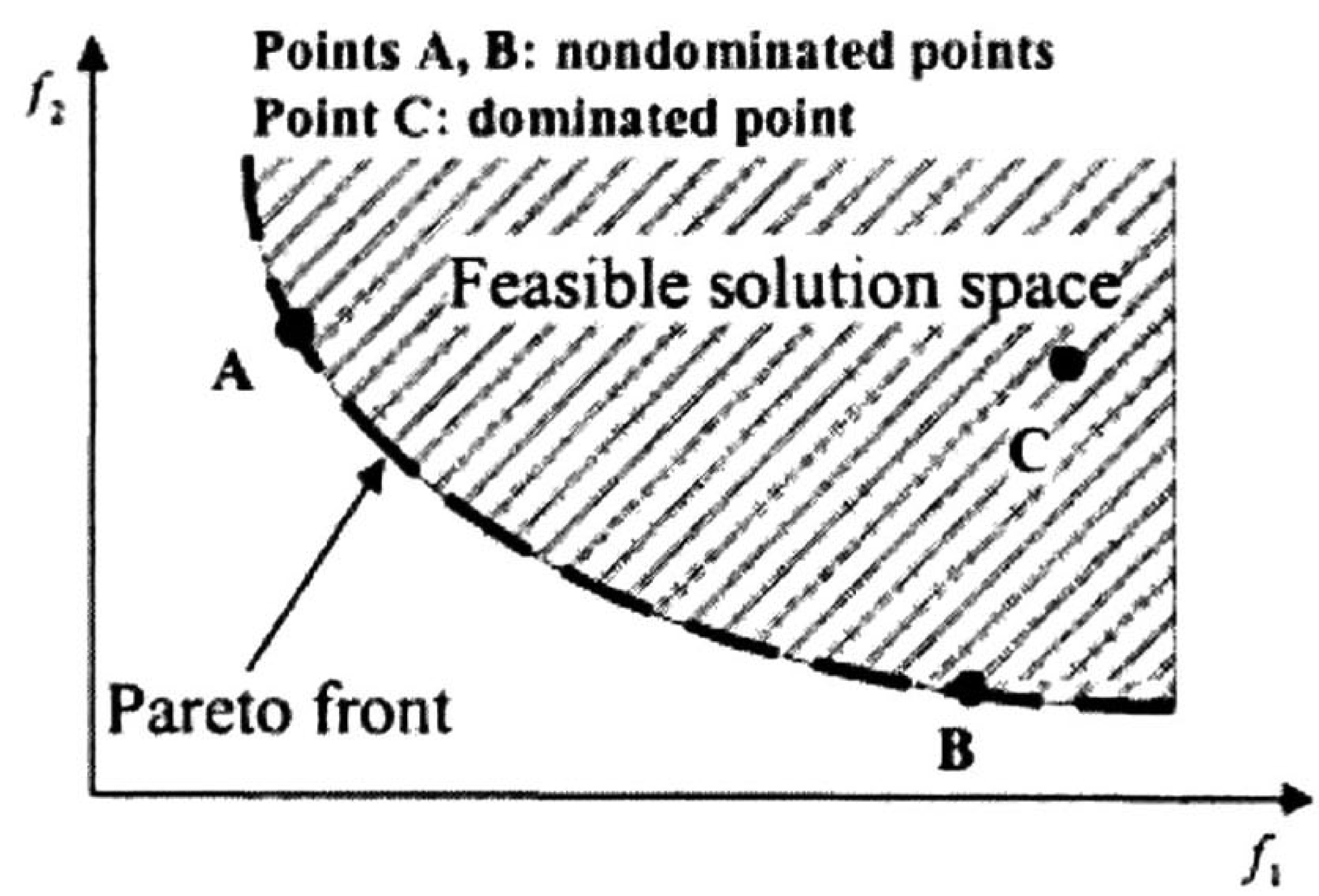

Figure 2.2: Graphical illustration of the Pareto front of a two objective minimization problem.

2.2 Examples under different test functions

A set of multi-objective test functions taken from the specialized literature are presented below, each one presents a different complexity.

2.2.1 Deb's test function

This example is a bi-objective test function proposed by Deb [2.9]:

$$\text{Minimize } f_1(x) = x_1 \quad (2.10)$$

$$\text{Minimize } f_2(x) = g(x) \cdot h(x) \quad (2.11)$$

where:

$$g(x) = 1 + 10 \cdot x_2 \quad (2.12)$$

$$h(x) = 1 - \left(\frac{f_1}{g(x)}\right)^2 - \frac{f_1}{g(x)} \cdot \sin\left(12 \cdot \pi \cdot f_1\right) \quad (2.13)$$

and $0 \le x_i \le 1, \quad i = 1,2$

This problem has the Pareto front and the Pareto optimal set disconnected. Figure 2.3 shows the Pareto optimal set and the Pareto front of the Deb's test function.

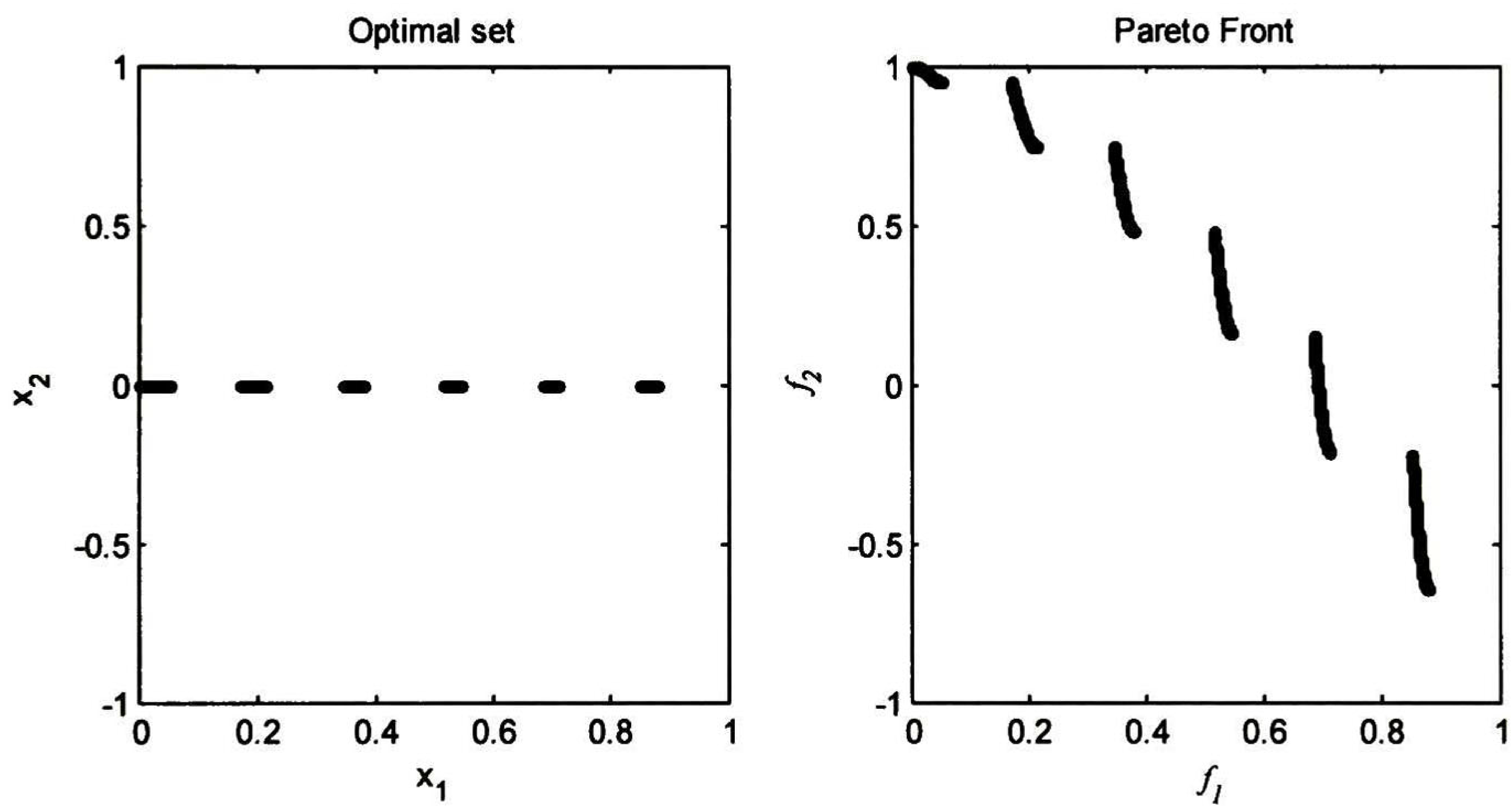

Figure 2.3: Pareto front and optimal set for Deb's test function.

2.2.2 Kursawe's test function

This test function was proposed by Kursawe [2.10]:

$$\text{Minimize } f_1(x) = \sum_{i=1}^{n-1}\left(-10e^{(-0.2)*\sqrt{x_i^2+x_{i+1}^2}}\right) \quad (2.14)$$

$$\text{Minimize } f_2(x) = \sum_{i=1}^{n}\left(\left|x_i\right|^{0.8} + 5\sin\left(x_i\right)^3\right) \quad (2.15)$$

where $-5 \le x_1, x_2, x_3 \le 5$.

Figure 2.4 shows the Pareto optimal set and the Pareto front for the Kursawe's test function.

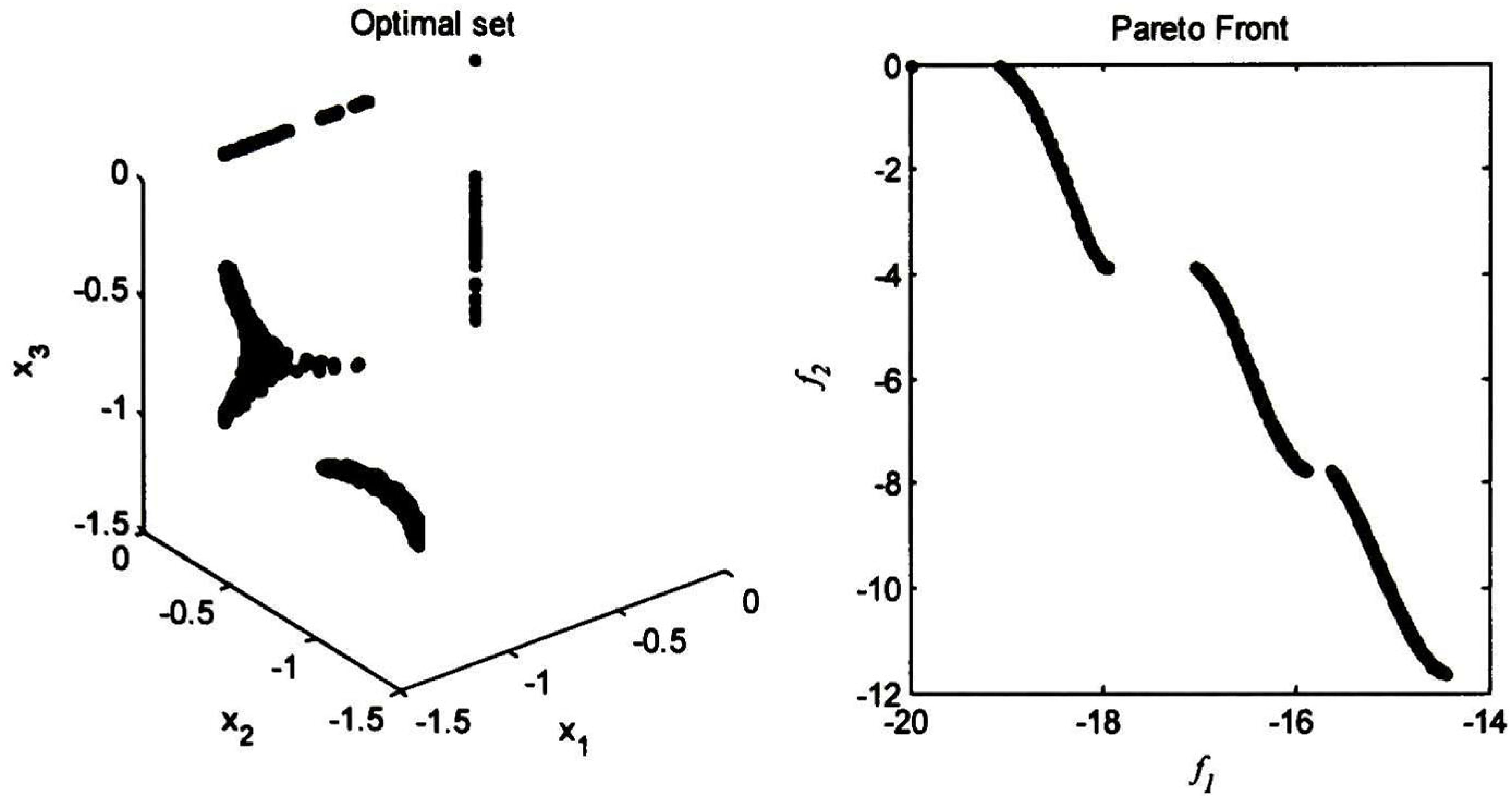

Figure 2.4: Pareto front and optimal set for Kursawe's test function.

2.2.3 Schaffer's test function

This test function is a two objective problem proposed by Schaffer [2.11]:

$$\text{Minimize} \quad f_1(x) = \begin{cases} x & \text{if } x \le 1 \\ -2+x & \text{if } 1 < x \le 3 \\ 4-x & \text{if } 3 < x \le 4 \\ -4+x & \text{if } x > 4 \end{cases} \tag{2.16}$$

$$\text{Minimize} \quad f_2(x) = (x-5)^2 \tag{2.17}$$

where $-5 \le x \le 10$.

The graphic on the left in figure 2.5 shows the Pareto optimal set, and the graphic on the right the Pareto front for the Schaffer's test function.

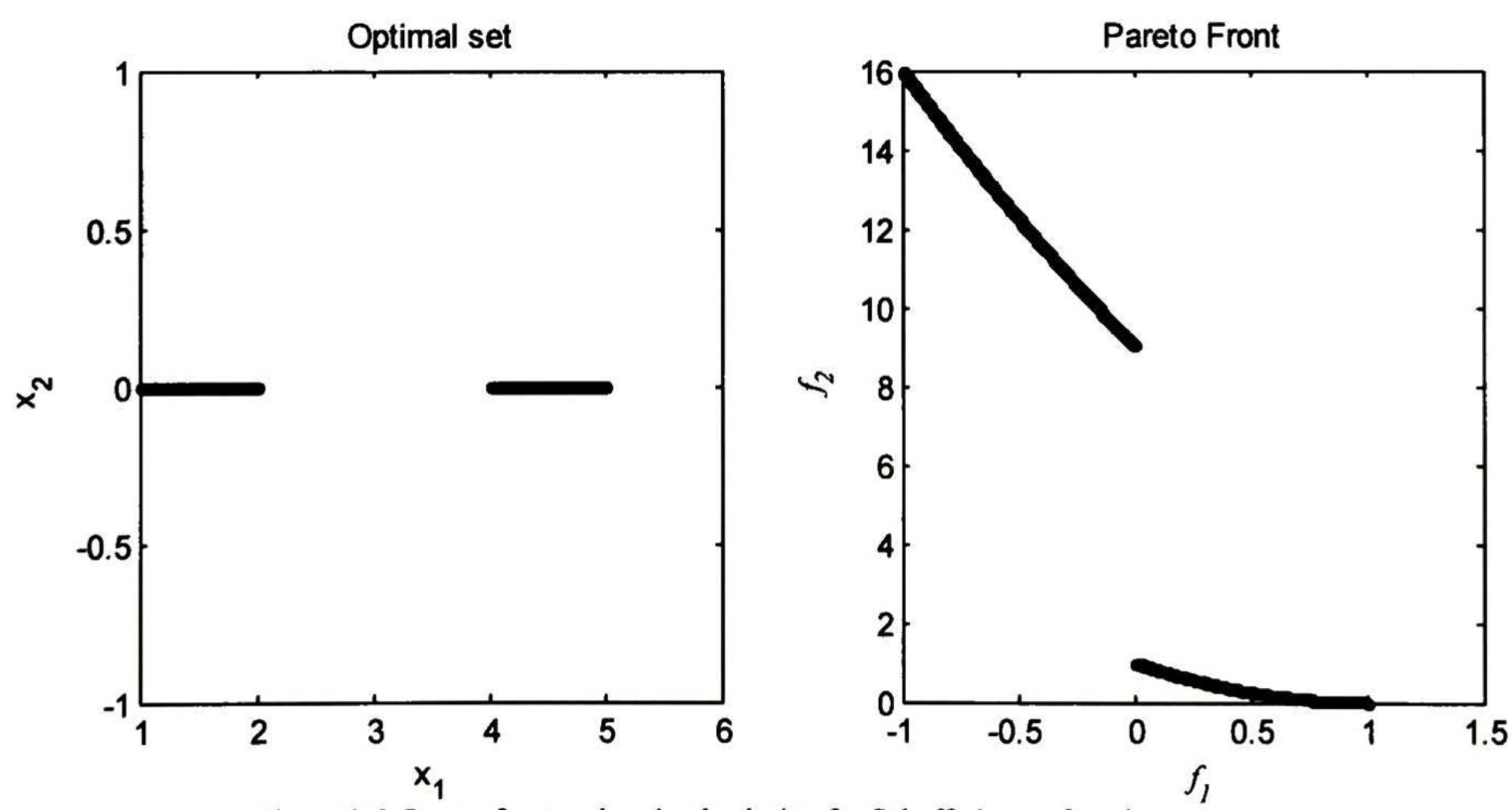

Figure 2.5: Pareto front and optimal solution for Schaffer's test function.

2.2.4 Kitta's test function

This is a two objective problem proposed by [2.12]:

$$\text{Maximize} \quad f_1(x,y) = -x^2 + y \tag{2.18}$$

$$\text{Maximize} \quad f_2(x,y) = \frac{1}{2}x + y + 1 \tag{2.19}$$

Subject to:

$$\frac{1}{6}x + y - \frac{13}{2} \le 0 \tag{2.20}$$

$$\frac{1}{2}x + y - \frac{15}{2} \le 0 \tag{2.21}$$

$$\frac{5}{x} + y - 30 \le 0 \tag{2.22}$$

and $0 \le x, y \le 7.0$.

Figure 2.6 displays the optimal solution and the Pareto front of the Kitta's test function.

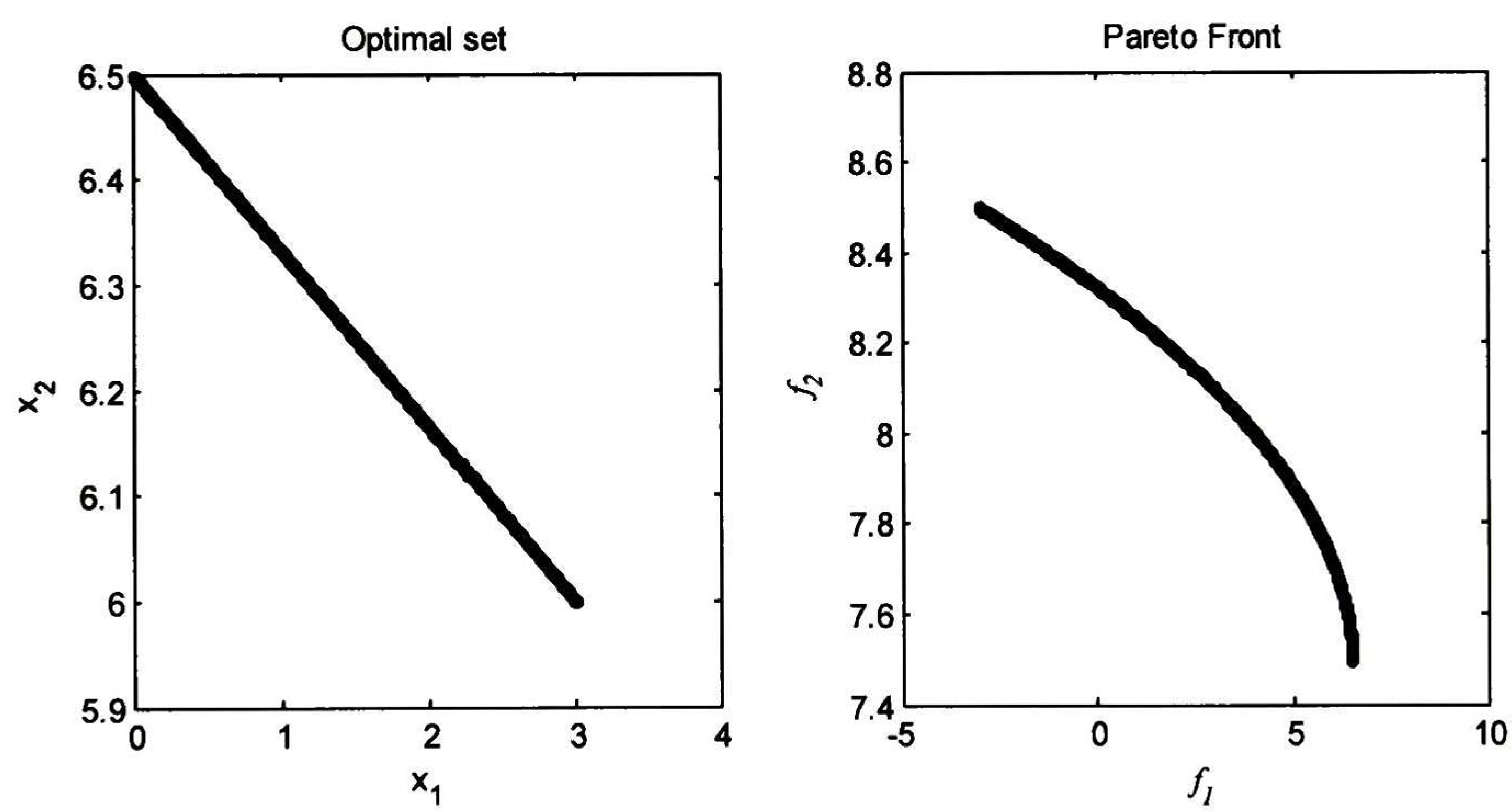

Figure 2.6: Pareto front and optimal solution for Kitta's test function.

2.2.5 Belegundu's test function

This test function was proposed by Belegundu [2.13]:

$$\text{Minimize} \quad f_1(x) = -2x + y \tag{2.23}$$

$$\text{Minimize} \quad f_2(x) = 2x + y \tag{2.24}$$

Subject to

$$0 \ge -x + y - 1 \tag{2.25}$$

$$0 \ge x + y - 7 \tag{2.26}$$

The decision variables' bounds are the following:

$$0 \leq x \leq 5 \tag{2.27}$$

$$0 \leq y \leq 3 \tag{2.28}$$

The graphic of the left in figure 2.7 depicts the Pareto optimal set, and the graphic of the right shows the Pareto front for Belegundu's test function.

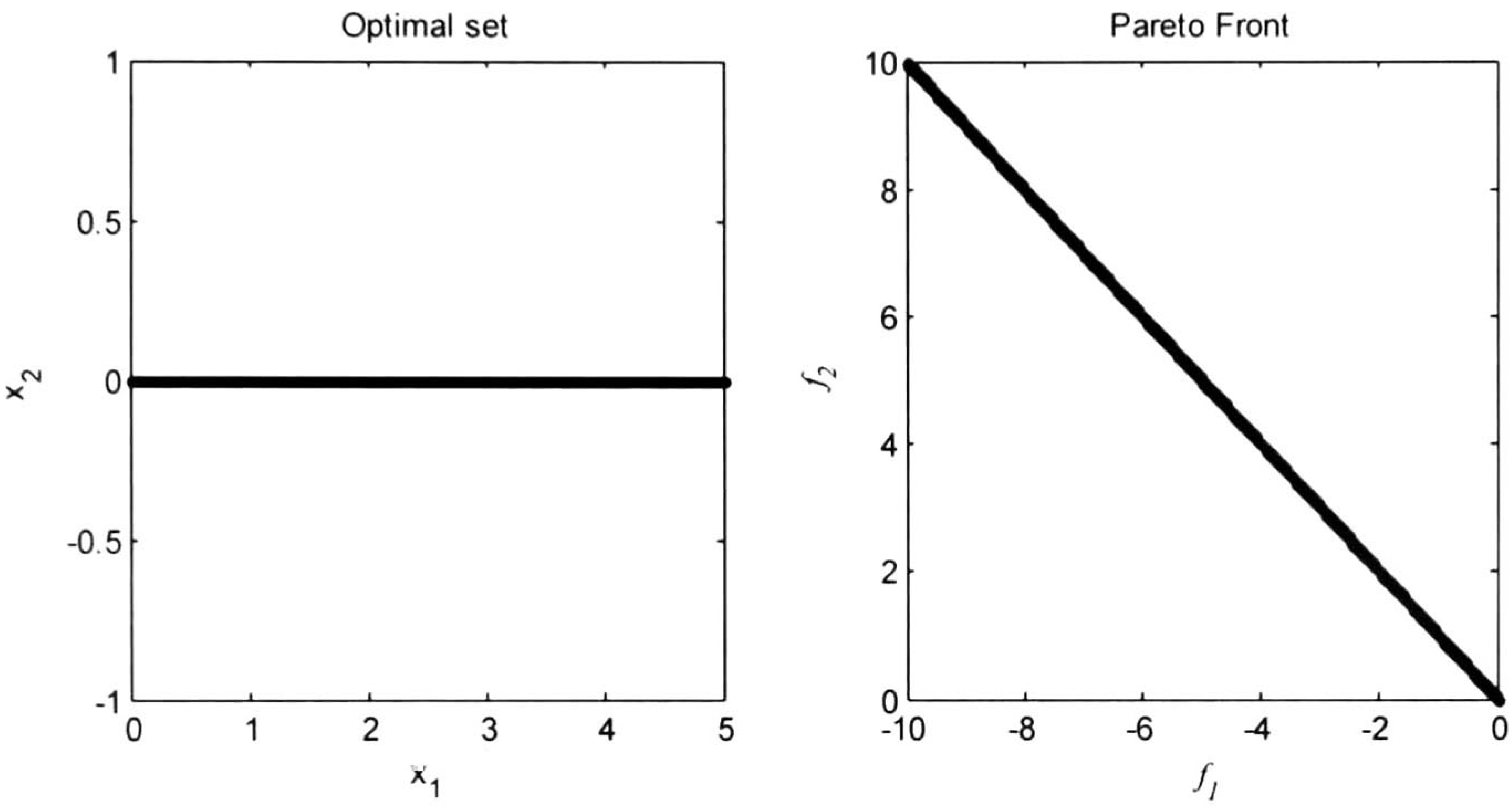

Figure 2.7: Pareto front and optimal solution for Belegundu's test function.

The first three problems involve unconstrained optimization: Deb, Kursawe and Schaffer with its Pareto Front and Pareto optimal set disconnected. The other two are constrained optimization problems: Kitta and Belegundu with its Pareto Front and Pareto optimal set connected. All these test problems have different levels of complexity like convexity, non-convex and disconnected Pareto optimal solutions.

It is noteworthy that the Pareto optimal set lies within the decision variable space, whereas the Pareto Front lies in the objective space.

2.3 Classical optimization methods

In any growing field of research and application, it becomes difficult to call any study '*classical*'. We are going to refer all search and optimization algorithms that use a single solution updating within each iteration and mainly use a deterministic transition rule as *classical* methods [2.14]. Such optimization algorithms can be found in standard textbooks [2.15-2.17].

Most classical point by point algorithms utilize a deterministic procedure for approaching the optimum solution. Such algorithms start from a random guess solution. Thereafter, based on a pre-specified transition rule, the algorithm suggests a search direction, which is often arrived at by considering local information. A unidirectional search is performed along the search direction to find the best solution. This best solution becomes the new solution and the above procedure is continued for a number of times [2.14].

Classical optimization methods can be classified into two distinct groups: direct methods and gradient-based methods [2.16]. In direct search methods, only the objective function $f(\bar{x})$ and the constraint values (2.1)-(2.2) are used to guide the search strategy, whereas gradient-based methods employ the first and/or second order derivatives of the objective function and/or constraints to guide the search process. Since derivative information is not used, the direct search methods are usually slow, requiring many function evaluations in order to converge. On the other hand, gradient-based methods quickly converge near an optimal solution, but are not efficient in non-differentiable or discontinuous problems. There are some common difficulties with most classical direct and gradient-based techniques, as follows [2.14]:

- The convergence to an optimal solution depends on the chosen initial solution.
- Most algorithms tend to get stuck in a suboptimal solution.
- An efficient algorithm to solve one optimization problem may not be efficient to solve a different optimization problem.
- Algorithms are not efficient in handling problems having a discrete search space.
- Algorithms cannot be efficiently used on a parallel machine.

In general, the difficulties associated to the classical optimization methods can be summarized as follows [2.18]:

1. An algorithm has to be applied many times to find multiple Pareto-optimal solutions.
2. Most algorithms require some knowledge about the problem being solved.
3. Some algorithms are sensitive to the shape of the Pareto-optimal front.

Since 1980s, several multi-objective evolutionary algorithms (MOEAs) have been proposed and applied in multi-objective optimization problems [2.2]. These algorithms share the same purpose-searching for a uniformly distributed, near-optimal, and near-complete Pareto front for a given MOP.

Generally, the approximation of the Pareto-optimal set involves two conflicting objectives: the distance to the true Pareto front is to be minimized, while the diversity of the generated solutions is to be maximized [2.19]. To address the first objective, a Pareto-based fitness assignment method is usually designed in many existing MOEAs [2.20] in order to guide the search toward the true Pareto-optimal front. For the second objective, some successful MOEAs provide density estimation methods to preserve the population density.

2.4 Evolutionary algorithm for multi-objective optimization

The first attempts to adapt evolutionary algorithm to solve multi-objective optimization problems relied on straightforward transformations of a multi-objective optimization problem into a single-objective one, known as the *aggregating approaches*.

Fonseca and Fleming categorized several MOEAs and compared different fitness assignment approaches. They classify these approaches in *non-Pareto-based approaches*, and *Pareto-based approaches* [2.20].

2.4.1 Plain aggregating approaches

The approach of combining objectives into a single function that is used for fitness calculation is normally named *aggregating functions*. Although these approaches have the advantage of producing one single solution, they require well-known domain knowledge that is often not available. In addition, multiple runs are required to find a family of non-dominated solutions and to identify the Pareto optimal solution. The most popular aggregating approaches are described in the following [2.5].

2.4.1.1 Weighted sum approach

This method consists of adding up all the objective functions together using different weighting coefficients for each one. This means that our multi-objective optimization problem is transformed into a scalar optimization problem through:

$$\min \sum_{i=1}^{k} w_i f_i(\overline{x}) \tag{2.29}$$

where $w_i \geq 0$ are the weighting coefficients representing the relative importance of the objectives. It is usually assumed that

$$\sum_{i=1}^{k} w_i = 1 \tag{2.30}$$

If we want that w_i reflects closely the importance of the objectives, all functions should be expressed in units of similar numerical values. Additionally, we can also transform (3.6) to the form:

$$\min \sum_{i=1}^{k} w_i f_i(\overline{x}) c_i \tag{2.31}$$

where c_i are constant multipliers that will scale properly the objectives.

This method is computationally efficient and can be applied to generate strongly non-dominated solutions that can be used as an initial solution for other techniques. The problem with this approach is how to determine the appropriate weights when we do not have enough information about the problem.

2.4.1.2 Goal programming

Charnes-Cooper [2.21] and Ijiri [2.22] propose the development of the goal programming method for a linear model. In this method, the decision maker has to assign targets or goals that one's wishes to achieve for each objective. These values are incorporated into the problem as additional constraints. The objective function will try to minimize the absolute deviations from the targets to the objectives. The simplest form of this method may be formulated as follows:

$$\min \sum_{i=1}^{k} \left| f_i(\overline{x}) - T_i \right|, \qquad \text{subject to } \overline{\mathrm{x}} \in F \tag{2.32}$$

where T_i denotes the ith objective function $f_i(\bar{x})$ target or goal set, and F represents the feasible region. Thus, the criterion is to minimize the sum of the absolute values of the differences between target values and the actually achieved values.

This technique is computationally efficient if we know the desired goals, and if they are into a feasible region. However, the decision maker has the task of devising the appropriate weights or priorities for the objectives that will eliminate the non-commensurable characteristics of the problem, which in most cases is difficult, unless there is prior knowledge about the shape of the search space. Also, if the feasible region is difficult to approach, this method could become very inefficient. Nevertheless, this technique may be useful for linear or piecewise-linear approximation objective function. On the other hand, in non-linear cases, alternative approaches may be more efficient.

2.4.1.3 The ε-constraint method

This method is based on minimization of one (the most preferred or primary) objective function, considering the other objectives as constraints, bounded by some allowable value ε_i. Hence, a single objective minimization is carried out for the most relevant objective function f_1 subject to additional constraints. Values ε_i are then altered to generate the entire Pareto optima set. The method may be formulated as follows:

I. Find the minimum of the r th objective function, i.e. find $\bar{x}^*$ such that

$$f_r(\bar{x}^*) = \min_{x \in F} f_r(\bar{x}) \tag{2.33}$$

subject to additional constraints of the form

$$f_i(\bar{x}) \le \varepsilon_i \qquad for\ i = 1, 2, \ldots, k \quad and \quad i \ne r \tag{2.34}$$

where ε_i are assumed values of the objective functions which should not to be exceeded.

II. Repeat I, for different values of ε_i. The information derived from a well chosen set of ε_i can be useful to make decisions. The search is stopped when the decision maker finds a satisfactory solution.

The drawback of this approach is that it is time-consuming, and the coding of the objective functions may be difficult for certain problems, particularly if there are too many objectives. Furthermore, finding weakly non-dominated solutions may not be appropriate in some applications.

2.4.2 Population-based non-Pareto approach

To overcome the difficulties involved in the aggregating approaches, alternative techniques based on population strategies, selection criteria, or special objectives' handling has been developed [2.23]. These approaches are known as non-Pareto-based approaches. The advantage of these approaches is that multiple non-dominated solutions can be simultaneously evolved in a single run. The most popular approaches that fall into this category are described in the following [2.5].

2.4.2.1 VEGA

David Schaffer [2.11] extended the Grefenstette's GENESIS program [2.24] to include multiple objective functions. Schaffer's approach is an extension of the Simple Genetic Algorithm (SGA) that he called the *Vector Evaluated Genetic Algorithm* (VEGA), and that differs from the first only in the way the selection is performed. This operator is modified so that at each generation a number of sub-populations are generated by performing proportional selection, according to each objective function in turn. Thus, for a problem with k objectives, k sub-populations of size N/k each would be generated (assuming a total population size of N). These sub-populations would be shuffled together to obtain a new population of size N, on which GA would apply the crossover and mutation operators as usual.

Richardson et al [2.25] noted that the shuffling and merging of all sub-populations corresponds to averaging the fitness components associated with each of the objectives. Therefore, the resulting expected fitness corresponds to a objectives' linear combination, where the weights depend on the distribution of the population at each generation. The main consequence of this is that when we have a concave trade-off surface, certain points in concave regions will not be found through this optimization procedure.

2.4.2.2 Lexicographic ordering

In this method, the objectives are ranked by the designer in order of importance. The optimum solution $\bar{x}^*$ is then obtained by minimizing the objective functions, starting with the most important one and proceeding according to the assigned order of importance.

Let the subscripts of the objectives indicate the priority of the objectives. Thus $f_1(\bar{x})$ and $f_k(\bar{x})$ denote the most and least important objective functions, respectively. Then the first problem is formulated as:

$$Minimize \quad f_1(\bar{x}) \tag{2.35}$$

subject to

$$g_j(\bar{x}) \le 0, \quad j = 1,2,...,m \tag{2.36}$$

and its solution $\bar{x}_1^*$ and $f_1^* = (\bar{x}_1^*)$ is calculated. Then the second problem is formulated as:

$$Minimize \quad f_2(\bar{x}) \tag{2.37}$$

Subject to

$$g_j(\bar{x}) \le 0, \quad j = 1,2,...,m \tag{2.38}$$

$$f_1(\bar{x}) = f_1^* \tag{2.39}$$

The solution to this problem is $\bar{x}_2^*$ and $f_2^* = f_2(\bar{x}_2^*)$. This procedure is repeated until all k objectives have been taken into account. The ith problem is given by

$$Minimize \quad f_i(\bar{x}) \tag{2.40}$$

subject to

$$g_j(\bar{x}) \le 0, \quad j = 1,2,...,m \tag{2.41}$$

$$f_l(\overline{x}) = f_l^* \quad l = 1,2,\ldots,i-1 \tag{2.42}$$

The solution obtained at the end, i.e., x_k is taken as the desired solution x^* of the problem.

The use of tournament selection within this approach makes an important difference with respect to other approaches such as VEGA. This technique may be able to see as convex a concave trade-off surface, although really depends on the distribution of the population and on the problem itself. Its main drawback is that this approach will tend to favor more certain objectives, because the randomness involved in this process, and this will have the undesirable consequence to converge to a particular part of the Pareto front, rather than to delineate it completely [2.26].

2.4.2.3 Use of Game Theory

This technique involves a simple optimization problem with two objectives and two design variables which graphical representation is shown in figure 2.8. Let $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$ represent two scalar objectives and x_1 and x_2 two scalar design variables. It is assumed that one player is associated with each objective. The first player wants to select a design variable x_1 which will minimize his objective function f_1, and similarly the second player seeks a variable x_2 which will minimize his objective function f_2. If f_1 and f_2 are continuous, then the contours for constant values of f_1 and f_2 appear as shown in Figure 3.8. The dotted lines, passing through O_1 and O_2, represent the loci of rational (minimizing) choices for the first and second player for a fixed value of x_2 and x_1, respectively. The intersection of these two lines, if it exists, is a candidate for the two objectives minimization problem, assuming that the players do not cooperate with each other (*non-cooperative game*). In figure 2.8, the point $N(x_1^{'},x_2^*)$ represents such intersection point. This point, known as a *Nash equilibrium* solution, represents a stable equilibrium condition in the sense that any player can deviate unilaterally from this point for further improvement of his/her own criterion [2.27].

This point is characterized by

$$f_1(x_1\,,x_2^*) \le f_1(x_1,x_2^*) \tag{2.43}$$

and

$$f_2(x_1\,,x_2^*) \le f_2(x_1\,,x_2) \tag{2.44}$$

where x_1 may be located to the left or right of x_1 in equation (2.43), while x_2 may lie above or below x_2 in (2.44).

This approach seems to be computationally very efficient, but is not possible to generate more than one non-dominated solution, which hopefully will be the best overall solution to the problem [2.28]. However, it is indeed possible to extend this approach to k players, and to have several Nash equilibrium points, with which the Pareto front can actually be found, although a cooperative game may be preferred in that case over a non-cooperative approach [2.29].

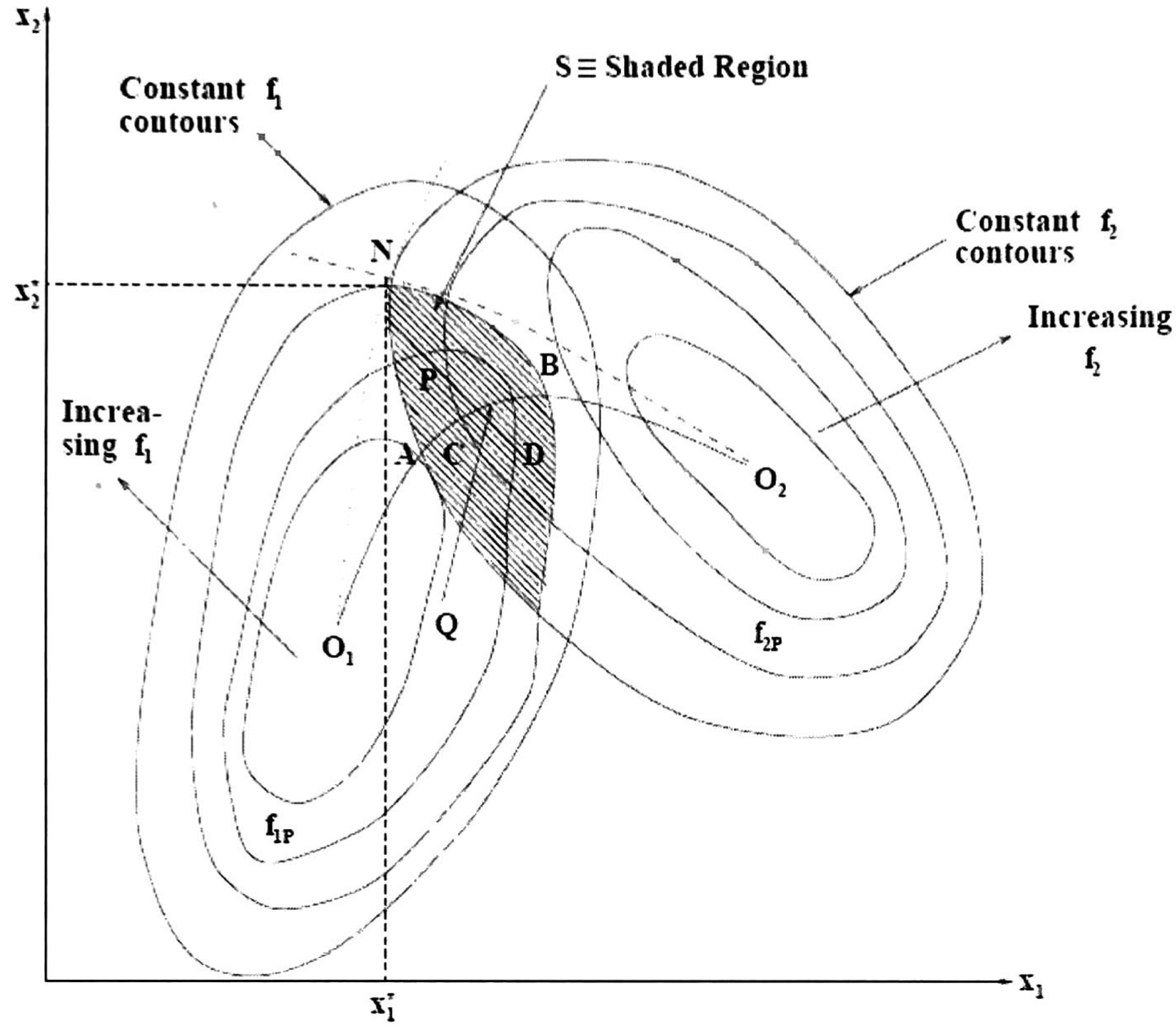

Figure 2.8: Cooperative and non-cooperative game solutions.

2.4.3 Pareto-based approaches

The basic idea of the Pareto-based fitness assignment is to find a set of solutions in the population that are non-dominated by the rest of the population. The highest rank to these solutions are assigned and eliminated from further contention. Generally, all approaches of this class explicitly use Pareto dominance in order to determine the reproduction probability of each individual. Some Pareto-based approaches are described in the following [2.18].

2.4.3.1 Non-dominated Sorted Genetic Algorithm (NSGA)

Srinivas and Ded [2.30] propose the NSGA, which is based on several layers of individuals' classifications. Before the selection is performed, the population is ranked on the basis of non-domination. All non-dominated individuals are classified into one category, with a dummy fitness value, which is proportional to the population size, to provide an equal reproductive potential for these individuals. To maintain the population's diversity, these classified individuals are associated with their dummy fitness values. Then this group of classified individuals is ignored and another layer of non-dominated individuals is considered. The process continues until all individuals in the population are classified.

Since individuals in the first front have the maximum fitness value, they always get more copies than the rest of the population. This allows searching for non-dominated regions, and results in quick convergence.

2.4.3.2 Niched Pareto Genetic Algorithm (NPGA)

Horn et al [2.31] propose a tournament selection schema based on Pareto dominance. Two competing individuals and a comparison set of other individuals are picked at random from the population. The number of individuals in the comparison set is given by the parameter t_{dom}.

Generally, the tournament selection is carried out as follows. If one candidate is dominated by the comparison set while the other is not, then the latter will be selected for reproduction. If neither of both candidates is dominated by the comparison set, then the winner will be decided by sharing. Phenotypic sharing on the attribute space is used in this technique.

Since this approach does not apply Pareto selection to the entire population, but only to a segment of it at each run, the technique is very fast and produces good non-dominated runs that can be kept for a large number of generations [2.26]. However, besides requiring a sharing factor, this approach also requires a good choice of the value of t_{dom} to perform well, complicating its appropriate use in practice.

2.4.3.3 Strength Pareto Evolutionary Algorithm (SPEA)

Zitzler and Thiele [2.32] presented SPEA as a potential algorithm for multi-objective optimization. This technique stores externally the individuals that represent a non-dominated front among all solutions considered so far. All individuals in the external set participate in selection. SPEA uses the concept of Pareto dominance in order to assign scalar fitness values to individuals in the current population.

The procedure starts with assigning a real value s in [0, 1) called strength for each individual in the Pareto-optimal set. The strength of an individual is proportional to the number of individuals covered by it. The strength of a Pareto solution is at the same time its fitness. Subsequently, the fitness of each individual in the population is the sum of the strengths of all external Pareto solutions by which it is covered. In order to guarantee that Pareto solutions are most likely to be produced, one is added to the resulting value. This fitness assignment ensures that the search is directed toward the non-dominated solutions and, at the same time, the diversity among dominated and non-dominated solutions is maintained.

Since non-dominated sorting of the whole population is not used for assigning fitness, the fitness values do not favor all non-dominated solutions of the same rank equally. This bias in fitness assignment in the solutions of the same front is dependent on the exact population and densities of solutions in the search space. Moreover, in the SPEA fitness assignment, an external solution which dominates more solutions gets a worse fitness.

2.5 Remarks

Most of the real world problems involve more than one objective, making the multiple conflicting objectives interesting to solve. Classical optimization methods are inconvenient to solve multi-objective optimization problems, as they could at best find one solution in one simulation run. However, evolutionary algorithms can find multiple optimal solutions in one single simulation run due to their population-based search approach. Thus, EAs are ideally suited for multi-objective optimization problems.

The use of evolutionary algorithms for multi-objective optimization, an area called evolutionary multi-objective optimization, has significantly grown in the last few years, giving rise to a wide variety of algorithms. In this work, we will have the opportunity to apply some of them.

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Chapter 3

Reactive power and voltage control

Voltage stability is a major concern in power systems' planning and operation. It is well known that voltage instability and collapse have led to major system failure. The voltage stability problem may be simply explained as the power system inability to provide the reactive power consumption.

The reactive power optimization problem is very important in power system security and due to economical operation. The basic objective is to find proper control variables' adjustments, such as generator voltages, taps of transformers, shunt capacitors and inductors, which help to maintain acceptable voltage profiles and minimize power losses.

Recently, multi-objective optimization approaches for reactive power control have become popular [3.1–3.4]. Conventionally, attention has been focused upon power losses and voltage deviations. Less effort has been given to voltage stability improvement. In this chapter, one way to take into account such aspect is proposed.

3.1 Problem formulation

The reactive power and voltage control or volt/var control (VVC) optimization problem may be formulated as the minimization of two competing objective functions: (*i*) the voltage deviation; (*ii*) the voltage stability index, while satisfying several equality and inequality constraints. Generally the problem is formulated as follows.

3.1.1 Voltage stability index minimization

Among different indexes for voltage stability and voltage collapse prediction, a voltage stability index, *L-index* [3.5], is chosen as an indicator related to voltage stability. This index is chosen because of a compromise between simplicity and suitability.

Apart from its fast calculation, this index is able to evaluate in each bus its steady state voltage stability margin. Likewise, the chosen index can also take into account generator buses reaching reactive power limits.

The L-index value ranges between zero (no load) and one (voltage collapse). This value implicitly incorporates the effect of all loads on its evaluation at individual load buses. The bus with the highest L-index value will be the most vulnerable and hence this method helps to identify the weak areas requiring critical reactive power support. The general theory and algorithm of the L-index are summarized in the following.

The network equations in terms of the node admittance matrix can be written as:

$$I_{bus} = Y_{bus} \cdot V_{bus} \tag{3.1}$$

Segregating the nodes into two categories: (*i*) the set of loads' buses (α_L); and (*ii*) the set of generators' buses (α_G). Thus, equation (4.1) becomes:

$$\begin{bmatrix} I^L \\ I^G \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} \cdot \begin{bmatrix} V^L \\ V^G \end{bmatrix} \tag{3.2}$$

It is assumed that the transmission system is linear and allows a representation in terms of a hybrid matrix H:

$$\begin{bmatrix} V^L \\ I^G \end{bmatrix} = H \cdot \begin{bmatrix} I^L \\ V^G \end{bmatrix} = \begin{bmatrix} Z^{LL} & F^{LG} \\ K^{GL} & Y^{GG} \end{bmatrix} \cdot \begin{bmatrix} I^L \\ V^G \end{bmatrix} \tag{3.3}$$

where V^L and I^L are voltage's and current's vectors for load buses; V^G and I^G are voltage's and current's vectors for generator buses; Z^{LL} F^{LG} K^{GL} Y^{GG} are sub-matrices of the hybrid matrix H.

The H matrix can be evaluated from the admittance matrix (Y_{bus}) by a partial inversion, where the load buses' voltage vector is exchanged for the current vector. This representation can then be utilized to define a voltage stability indicator in the load bus, namely L_j which is defined by [3.5],

$$L_j = \left| 1 - \frac{\sum_{i \in \alpha_G} F_{ji} V_i}{V_j} \right| \tag{3.4}$$

For stable conditions, $0 \le L_j \le 1$ must not be violated for any j. Hence, a global indicator L describing the whole system's stability is defined by [3.5],

$$L_{index} = \max_{j \in \alpha_L} \left(L_j \right) \tag{3.5}$$

Pragmatically, L must be lower than a given threshold value. The predetermined threshold value is specified depending on the system configuration and on the utility policy regarding service quality and allowable margin. Therefore, in this thesis, the first objective to take into account within the VVC optimization problem is to minimize the system voltage indicator:

$$\min \quad f_1 = L_{index} \tag{3.6}$$

3.1.2 Voltage deviation minimization

The bus voltage is one of the most important security and service quality indexes. Treating the bus voltage limits as constraints often makes all voltages move toward their maximum limits after optimization. Thus, the optimization results may lack the required reserves to provide reactive power during contingencies. One of the effective ways to avoid this issue is to choose the voltage deviation from the desired value as an objective function [3.3]. Therefore, the second objective function of the VVC optimization problem becomes,

$$\min \quad f_2 = \frac{\sum_{i=1}^{N_L} \left| V_i - V_i^* \right|}{N_L} \tag{3.7}$$

where f_2 is the per unit average voltage deviation; N_L is the total number of load buses; V_i and V_i^* are the actual voltage magnitude and the desired voltage magnitude at bus i, respectively.

3.1.3 Equality constraints

The equality constraints are the active and reactive power balance described by a set of power flow equations, which can be expressed as follows:

$$P_{Gi} - P_{Di} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos\left(\delta_i - \delta_j + \theta_{ij}\right) = 0 \tag{3.8}$$

$$Q_{Gi} - Q_{Di} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin\left(\delta_i - \delta_j + \theta_{ij}\right) = 0 \tag{3.9}$$

$$\forall\, i, j \in \{1, 2, \ldots, n\}$$

where P_{Gi} and Q_{Gi} are the net real and reactive power at the i-th bus, respectively; the load demand at the same bus is represented by P_{Di} and Q_{Di}; the elements of the bus admittance matrix are represented by $|Y_{ij}|$ and θ_{ij}.

These constraints can be expressed in a compact form by,

$$H(x, u) = 0 \tag{3.10}$$

3.1.4 Inequality constraints

In this thesis, the set of constraints representing the system operational and security limits is described in the following.

3.1.4.1 *Generating constraints*

Generator voltages (V_G), generator real (P_G), and reactive power (Q_G) outputs are restricted by their lower and upper limits as follows:

$$V_{Gi}^{\min} \leq V_{Gi} \leq V_{Gi}^{\max} \quad i = 1,\ldots,NG \tag{3.11}$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad i = 1,\ldots,NG \tag{3.12}$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max} \quad i = 1,\ldots,NG \tag{3.13}$$

where NG is the number of generators.

3.1.4.2 *Transformer constraints*

Transformer tap T settings are bounded as follows:

$$T_i^{\min} \leq T_i \leq T_i^{\max}, \quad i = 1,\ldots,NT \tag{3.14}$$

where NT is the number of transformers.

3.1.4.3 *Switchable VAR sources constraints*

Switchable VAR compensations (Q_C) are restricted by their limits as follows:

$$Q_{Ci}^{\min} \leq Q_{Ci} \leq Q_{Ci}^{\max}, \quad i = 1,\ldots,NC \tag{3.15}$$

where NC is the number of switchable VAR sources.

3.1.4.4 *Load bus voltage*

These include the constraints of load bus voltages (V_L) as follows:

$$V_{Li}^{\min} \leq V_{Li} \leq V_{Li}^{\max} \quad i = 1,\ldots,NL \tag{3.16}$$

where NL is the number of load buses.

3.1.4.5 *Transmission constraints*

Transmission lines loading are restricted by:

$$S_i \leq S_i^{\max} \quad i = 1,\ldots,NB \tag{3.17}$$

where NB is the number of transmission lines.

These constraints can be expressed in the following compact form:

$$G(x,u) \leq 0 \tag{3.18}$$

3.1.5 Problem statement

In general, adding up the objectives and constraints, the VVC problem can be mathematically formulated as a non-linear constrained multi-objective optimization problem as follows:

$$\text{minimize} \quad f_1 = L_{index} \tag{3.19}$$

$$\text{minimize} \quad f_2 = \frac{\sum_{i=1}^{N_L} \left| V_i - V_i^{*} \right|}{N_L} \tag{3.20}$$

Subject to:

$$H(x,u) = 0 \tag{3.21}$$

$$G(x,u) \leq 0 \tag{3.22}$$

where x is the vector of state variables consisting of slack bus real power P_{G1}, load bus voltages V_L, generator reactive power outputs Q_G, and transmission line loadings S_l. Therefore, vector x becomes:

$$x^T = \left[P_{G1}, V_{L1} \ldots V_{LNL}, Q_{G1} \ldots Q_{GNG}, S_{l1} \ldots S_{lNB} \right] \tag{3.23}$$

u is the vector of control variables consisting of generator voltages V_G, real power outputs P_G (except at the slack bus), transformer tap settings T. Hence, u can be expressed as:

$$u^T = \left[V_{G1} \ldots V_{GNG}, P_{G2} \ldots P_{GNG}, T_1 \ldots T_{NT} \right] \tag{3.24}$$

3.2 Multi-objective Particle Swarm Optimizer (MOPSO)

3.2.1 Overview

Particle Swarm Optimization (PSO) seems particularly suitable for multi-objective optimization mainly because of its high speed of convergence in single-objective optimization [3.6].

Based on such behavior, one would expect that a multi-objective PSO (MOPSO) to be computationally efficient. However, there is no standard (unique) version of the MOPSO algorithm that had been adopted in the specialized literature.

There have been several recent proposals to extend PSO to handle multiple objective problems. We will review some of the most important.

- ***The algorithm of Moore and Chapman*** [3.7]: This algorithm was the first extension of PSO for handling multi-objective problems (MOPSO). It is based on Pareto dominance. The authors emphasize the importance to perform both an individual and a group search (a cognitive component and a social component). Nevertheless, the authors did not adopt any scheme to maintain diversity.

- ***Dynamic Neighborhood PSO proposed of Hu and Eberhart*** [3.8]: In this algorithm, only one objective is optimized at a time using a scheme similar to the lexicographic ordering. This one tends to be useful only when few objective functions are used (two or three), and it may be sensitive to the ordering of the objectives. The idea of the dynamic neighborhood is, however, quite interesting and is novel in this context.

- ***The Multi-objective Particle Swarm Optimizer (MOPSO) of Coello & Lechuga*** [3.9]: This proposal is based on the idea of having a global repository in which every particle will deposit its flight experiences after each flight cycle. Additionally, the updates to the repository are performed considering a geographically-based system, defined in terms of the objective function values of each individual. This repository is employed by the particles to identify a leader that will guide the search. This approach also uses a mutation operator that acts both on the particles of the swarm and on the range of each design variable of the problem to be solved.

- ***The PS-EA of Srinivasan and Hou*** [3.10]: The Particle Swarm Inspired Evolutionary Algorithm (PS-EA) is a hybrid between PSO and an evolutionary algorithm. The authors claim that the conventional PSO equations are too restrictive when applied to multi-constrained search spaces. Thus, they propose to replace the PSO equations with a self-updating mechanism. Such mechanism uses an inheritance probability tree to update each individual within the population. An additional mechanism to dynamically adjust the inheritance probabilities, based on the status of the algorithm at a certain moment in time, is proposed. The approach uses a memory to store the elite particles and does not use a recombination operator.

- ***The approach of Baumgartner et al.*** [3.11]: This approach utilizes weighted sums (i.e., linear aggregating functions) to solve multiobjective optimization problem. In this approach, the swarm is equally partitioned into n subswarms, each of which uses a different set of weights and evolves into the direction of its own swarm leader. The approach adopts a gradient technique to identify the Pareto optimal solutions.

- ***The MOPSO-CD of Raquel and Naval*** [3.12]: This algorithm handles multiobjective optimization problems by incorporating the mechanism of crowding distance computation into the algorithm of PSO. Specifically on global best selection and in the deletion method of an external archive of nondominated solutions. The crowding distance mechanism together with a mutation operator maintains the diversity of non-dominated solutions in the external archive.

- ***The TV-MOPSO of Praveen et al.*** [3.13]: The algorithm called Time Variant Multi-objective Particle Swarm Optimization (TV-MOPSO) is made adaptive in nature by allowing its vital parameters (inertial weight and acceleration coefficients) to change with iterations. This adaptiveness helps the algorithm to explore the search space more efficiently. A new diversity parameter has been used to ensure sufficient diversity amongst the solutions of the non-dominated fronts, while retaining at the same time the convergence to the Pareto-optimal front.

- ***Micro-MOPSO of Fuentes & Coello*** [3.14]: This MOPSO is characterized for using a very small population size, which allows it to require a very low number of objective function evaluations to produce reasonably good approximations of the Pareto front in problems of moderate dimensionality. The proposed approach first selects the leader and then selects the neighborhood for integrating the swarm. The leader selection scheme adopted is based on Pareto dominance and uses a neighbor's density estimator. Additionally, the proposed approach performs a re-initialization process to preserve diversity and uses two external archives: one for storing the solutions that the algorithm finds during the search process and another for storing the final solutions. Furthermore, a mutation operator is incorporated to improve the algorithm's exploratory capabilities.

3.2.2 The proposed MOPSO technique

The MOPSO strategy utilized in this thesis in order to solve the multi-objective reactive power and voltage control problem can be described in the following steps [3.12].

1. **For** $i = 1$ to M (M is the population size)
    a. Initialize *P[i]* randomly. (P is the population)
    b. Initialize *V[i]* = 0. (V is the speed if each particle)
    c. Evaluate *P[i]*.
    d. Initialize the personal best of each particle:
    PBEST[i] = *P[i]*.
    e. *GBEST[i]* = Best particle found in *P[i]*.

    End for.

2. Initialize the iteration counter ($t = 0$)

3. Store the non-dominated vectors found in *P* into *A*. (*A* is the external archive)

4. **Repeat**
    a. Compute the crowding distance values of each non-dominated solution in *A*.
    b. Sort the non-dominated solutions in *A* in descending crowding distance values.

For i = 1 to M

i. Randomly select the global best guide for *P[i]* from a specified top portion of *A* and store its position to *GBEST*.

ii. Compute the new velocity:
V[i] = *W* x *V[i]* + *R1* x (*PBEST[i]* – *P[i]*) + *R2* x (*A[GBEST]* – *P[i]*)
(*W* is the inertial weight equal to 0.4)
(*R1* and *R2* are random numbers in the range [0 1])
(*PBEST[i]* the best position of the particle i have reached)
(*A[GBEST]* is the global best guide for each non-dominated solution)

iii. Calculate the new position of *P[i]*:
P[i] = *P[i]* + *V[i]*.

iv. If *P[i]* goes beyond the boundaries. Then, it is reintegrated by having the decision variable take the value of its corresponding lower or upper boundary, and its velocity is multiplied by (-1) so that it searches in the opposite direction.

v. If (t < *MAXT* x *PMUT*)
Then perform mutation on *P[i]*.
(*MAXT* is the maximum number of iterations)
(*PMUT* is the probability of mutation)

vi. Evaluate *P[i]*.

End for.

c. If they are not dominated by any of the stored solutions, insert all new non-dominated solution in *P* into *A*. All dominated solutions in *A* are removed from the archive by the new solution. If the archive is full, the solution to be replaced is determined by the following steps:

i. Compute the crowding distance values of each non-dominated solution in the archive *A*.

ii. Sort the non-dominated solutions in *A* in descending crowding distance values.

iii. Randomly select a particle from a specified bottom portion which comprise the most crowded particles in the archive then replace it with the new solution.

d. Update the personal best solution of each particle in *P*. If the current *PBESTS* dominates the position in memory, the particles position is updated using *PBESTS[i]* = *P[i]*.

e. Increment iteration counter t.

Until maximum number of iterations is reached.

The flowchart of MOPSO based reactive power and volt control problem is depicted in Figure 3.1.

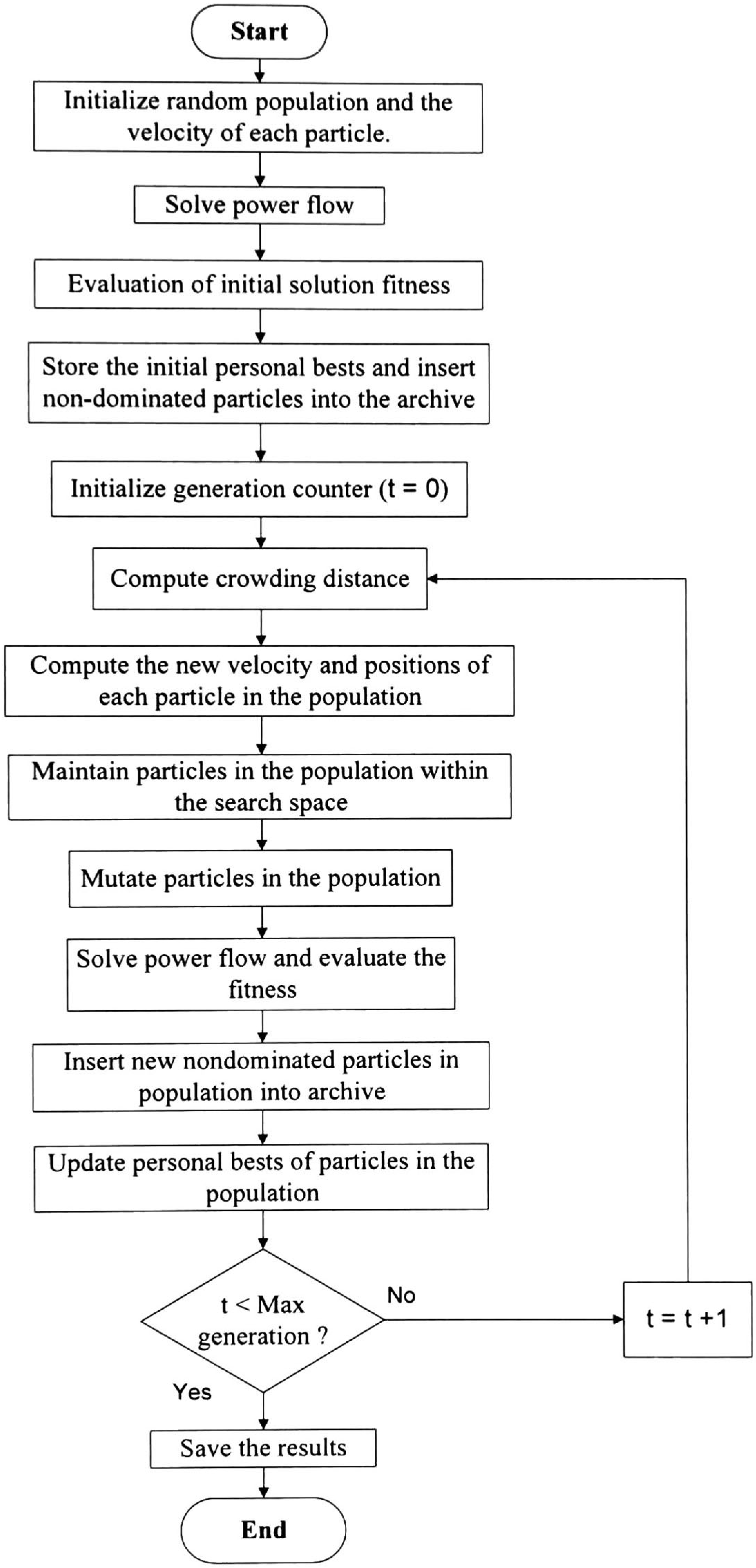

Figure 3.1: Flowchart of MOPSO based VVC problem.

3.3 Multi-objective Micro Genetic Algorithm (MicroGA)

3.3.1 Overview

The term micro-genetic algorithm (micro-GA) refers to a small-population genetic algorithm with re-initialization. The approach was derived from some theoretical results obtained by Goldberg [3.15], according to which a population size of three was sufficient to converge, regardless of the chromosome's length. The process suggested by Goldberg was to start with a small randomly generated population, then apply to it the genetic operators until reaching nominal convergence (e.g., when all the individuals have their genotypes either identical or very similar), and then to generate a new population by transferring the best individuals of the converged population to the new one. The remaining individuals would be randomly generated.

Krishnakumar [3.16] was the first in report a micro-GA implementation. A population size of five, a crossover rate of 1, and a mutation rate of 0, were used. His approach also adopted an elitist strategy that copied the best string found in the current population to the next generation. Selection was performed by holding four competitions between strings that were adjacent in the population array, and declaring to the individual with the highest fitness as the winner. After him, several other researchers have developed applications of micro-GAs (e.g., [3.17, 3.18]). However, one of the first attempts to use a micro-GA for multi-objective optimization was presented by Jaszkiewicz [3.19]. In which a small population initialized from a large external memory is used for a short period of time. However, this approach has been used only for multi-objective combinatorial optimization. Another MicroGA approach for multi-objective optimization was developed by Coello and Toscano [3.20]. The population in this approach is divided in two parts: a replaceable and non-replaceable portion. The non-replaceable portion will never change during the entire run and provides the required diversity for the algorithm. In contrast, the replaceable portion will experience changes after each cycle of the MicroGA.

3.3.2 The proposed MicroGA technique

A summary of the MicroAG algorithm utilized in this thesis in order to solve the multi-objective reactive power and voltage control problem is described in the following [3.21].

Generate randomly the initial population P of size N and store P in the population memory M_R and M_{NR}.

$i = 0$

While $i < GMax$ **do**

Get P^e from M

Repeat

Apply binary tournament selection

Apply two-point crossover

Apply uniform mutation (usually 1/ number of bits of the chromosome)

Apply elitism (retain only one non-dominated vector per generation)

Produce a new population

```
        Until nominal convergence is reached
        Copy solutions to external memory (first form of elitism)
    if external memory is full then
        Apply the Adaptive_grid and Copy two non-dominated vectors to M.
        ( second form of elitism ).
    if i mod replacement cycle then
        Move points from external memory to M_R
        ( third form of elitism )
    end if
    i=i+1
end while
```

where, M_R and M_{NR} are the replaceable and non-replaceable population memory, respectively. GMax is the maximum number of generations, P^e is the evolutionary population, and M is the random mixture of non-replaceable and replaceable portion.

First, a random population is generated. This random population feeds the population memory, which is divided into two parts: a replaceable and a non-replaceable portion. The non-replaceable portion of the population memory never changes during the entire run and is aimed to provide the required algorithm's diversity. In contrast, the replaceable portion experiences changes after each microGA's cycle. The population of the microGA at the beginning of each cycle is taken (with a certain probability) from both portions of the population memory, so that there is a mixture of randomly generated individuals (non-replaceable portion) and evolved individuals (replaceable portion). During each cycle, the microGA undergoes conventional genetic operators.

After the microGA finishes one cycle, two non-dominated vectors are chosen from the final population, and both of them are compared with the contents of the external memory (this memory is initially empty). If either of them (or both) remains as non-dominated after comparison respect to vectors in the external memory, then it is included there (i.e., in the external memory). This is the historical archive of non-dominated vectors. All dominated vectors contained in the external memory are eliminated.

The microGA utilizes three types of elitism: (1) it retains non-dominated solutions found within the microGA internal cycle, (2) it uses a replaceable memory which contents is partially "refreshed" at certain intervals, and (3) it replaces the population of the microGA by the nominal solutions produced (i.e., the best solutions found after a full microGA internal cycle).

3.4 Metrics of Performance

Comparing different optimization techniques experimentally always involves the notion of performance. In the case of multi-objective optimization, the definition of quality is substantially more complex than in a single-objective optimization problem, because of the optimization goal itself consists of multiple objectives.

In order to allow a quantitative assessment of the performance on a multi-objective optimization algorithm, three issues are normally taken into consideration [3.22].

1. Minimize the distance of the Pareto front produced by the algorithm with respect to the true Pareto front (assuming we know its location).

2. A good distribution of the solutions.

3. Maximize the spread of solutions found, so that we can have a distribution of vectors as smooth and uniform as possible.

However for the VVC problem previously defined, the true Pareto front is unknown, therefore one metric to evaluate quantitatively the performance of the proposed algorithms is adopted as follows.

3.4.1 Spacing metric (SM)

Schott [3.23] proposed a metric which is calculated through a relative distance measure between consecutive solutions in the obtained non-dominated set. This metric is defined by,

$$S = \sqrt{\frac{1}{|N-1|}\sum_{i=1}^{|N|}\left(\bar{d}-d_i\right)^2} \tag{3.25}$$

where,

$$d_i = \min_{i,j\neq i}\left\{\left|f_{1,i}-f_{1,j}\right|+\left|f_{2,i}-f_{2,j}\right|\right\} \tag{3.26}$$

and $\bar{d}$ is the mean value of d_i.

The distance measure is the minimum value of the sum of the absolute difference in the objective function values between the *i-th* solution and any other solution in the resultant non-dominated set. Notice that the distance measure is different from the minimum Euclidean distance between two solutions.

The above metric measures the standard deviations of different values d_i. When the solutions are almost uniformly spaced, the corresponding distance measure will be small. Thus, an algorithm finding a set of non-dominated solution having a smaller spacing (SM) is better.

3.5 Remarks

In this chapter, a multi-objective formulation to solve the reactive power and voltage control problem by a multi-objective evolutionary technique is proposed. The objectives are the voltage deviations in buses and the system's voltage stability index. The load and operational constraints are also taken into account.

A review of the most important proposals to extend the PSO algorithm to handle multi objective problems is presented. Also we present a review of the MicroGA algorithms for multi-objetive problems.

The metric to evaluate quantitatively the performance of the proposed algorithms is described.

3.6 References

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Chapter 4

Applications and results

The aim of this chapter is to prove the effectiveness and feasibility of the proposed algorithms to solve the VVC problem. Three studies were performed. In the first study, we consider the nine-bus test system [4.1]. In the second case study, we consider the IEEE 26 bus-test system [4.2]. Finally, in the third case study we consider the ten generator test system [4.3]. In all cases, bus 1 is considered as the slack bus. For each study, two sets of 10 test runs for solving the VVC problem were performed. The first set is based on the MOPSO algorithm and the second one is based on the MGA algorithm. On all optimizations runs, the maximum size of the Pareto Optimal set was chosen as 100 solutions. For the power flow convergence, the tolerance was set to 10^{-4} p.u.

Techniques used in this thesis were developed and implemented using the C++ language.

4.1 Case 1: nine-bus test system

The nine-bus test system consists of 9 transmission lines and 3 generating units. The single-line diagram of this system is depicted in figure. 4.1. Data are given in [1].

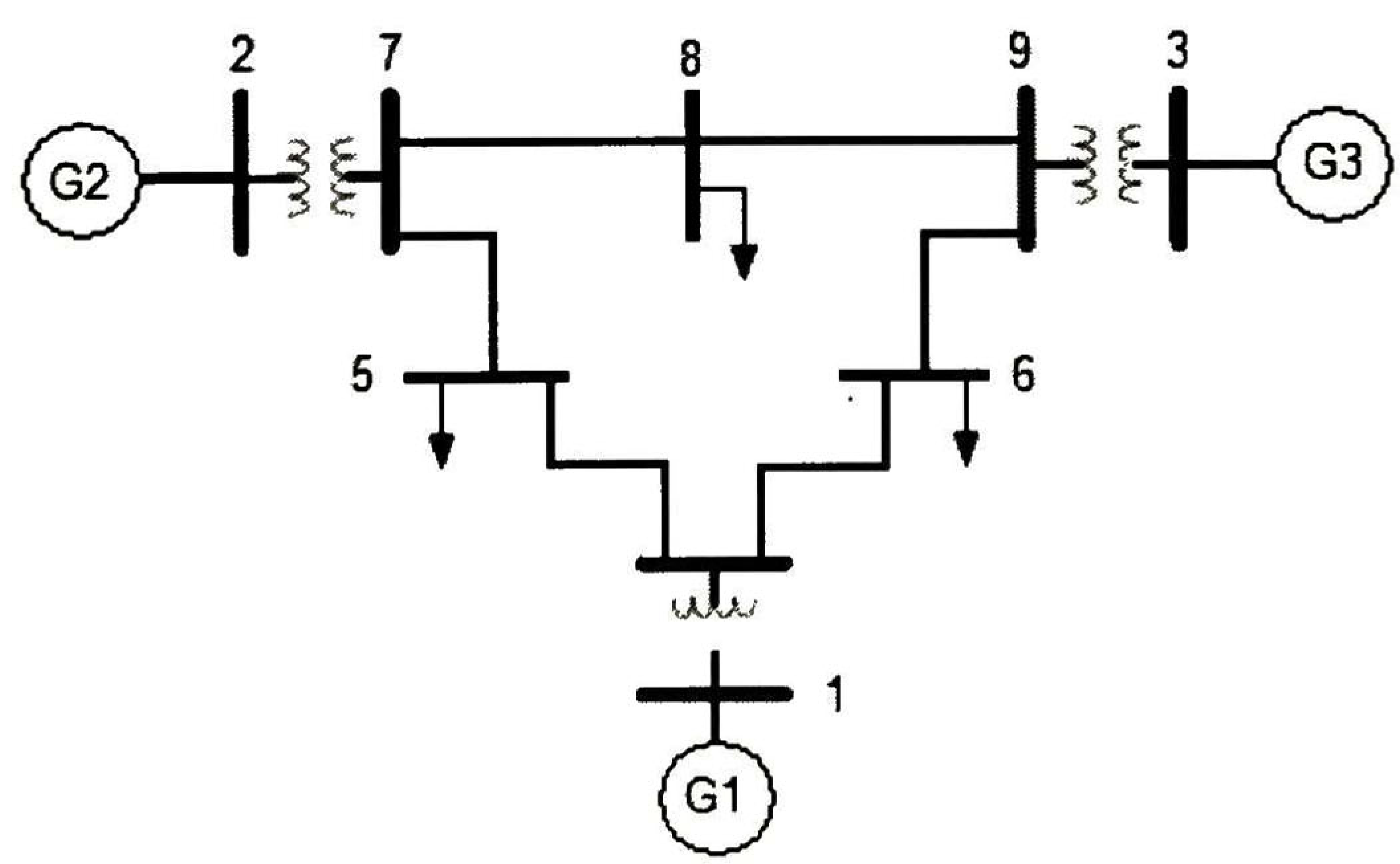

Figure 4.1: The 9-bus test system

For the purpose of comparison between the proposed techniques, two different tests over this system have been considered as follows.

4.1.1 Case 1.1: System without constraints

In this case, network constraints are no considered. The voltage stability function and the Voltage Deviation function were treated simultaneously as competing objectives. The control parameter settings used by the MOPSO and MGA algorithms in this case are given in Table 4.1.

Table 4.1
Control parameter settings of MOPSO and MGA, case 1.1

	MOPSO	MGA
Parameter	Setting	
Population size	100	4
Mutation rate	0.6	0.03
Crossover rate		0.7
Adaptive grid subdivision		25
Second elitism		150
Maximum number of generationts	100	1500

The diversity of the Pareto optimal front of the two proposed algorithms over the voltage stability (L_{index}) and voltage deviation (VD) functions is shown in Figure 4.2. It is worth mentioning that the Pareto optimal front obtained by MOPSO and MGA algorithms has 100 non-dominated solutions. The two non-dominated solutions that represent the best L_{index} and VD found by both algorithms are compared in Table 4.2.

Table 4.2
Comparison of the best solutions of MOPSO and MGA algorithms for case 1.1

Variable	Limits		MOPSO		MGA	
	lower	upper	Lindex	VD	Lindex	VD
Vg1(p.u)	1	1.05	1.05	1.0243	1.05	1.0064
Vg2(p.u)	1	1.05	1.05	1.0059	1.05	1.0214
Vg3(p.u)	1	1.05	1.05	1.0024	1.05	1.0103
Pg2 (MW)	100	240	100	110.9374	104.31	141.99
Pg3 (MW)	45	125	64.1152	73.4475	57.51	66.04
Ploss(MW)			2.604	2.94	2.625	3.504
L_{index}			**0.1474**	0.1599	**0.1474**	0.1614
VD			0.0421	**0**	0.042	**0**
SM			**1.95E-04**		2.90E-04	

The results show that the MOPSO is able to proportionate the minimum value on the Spacing Metric (SM). Thus, this algorithm has better diversity characteristics, regarding the MGA method.

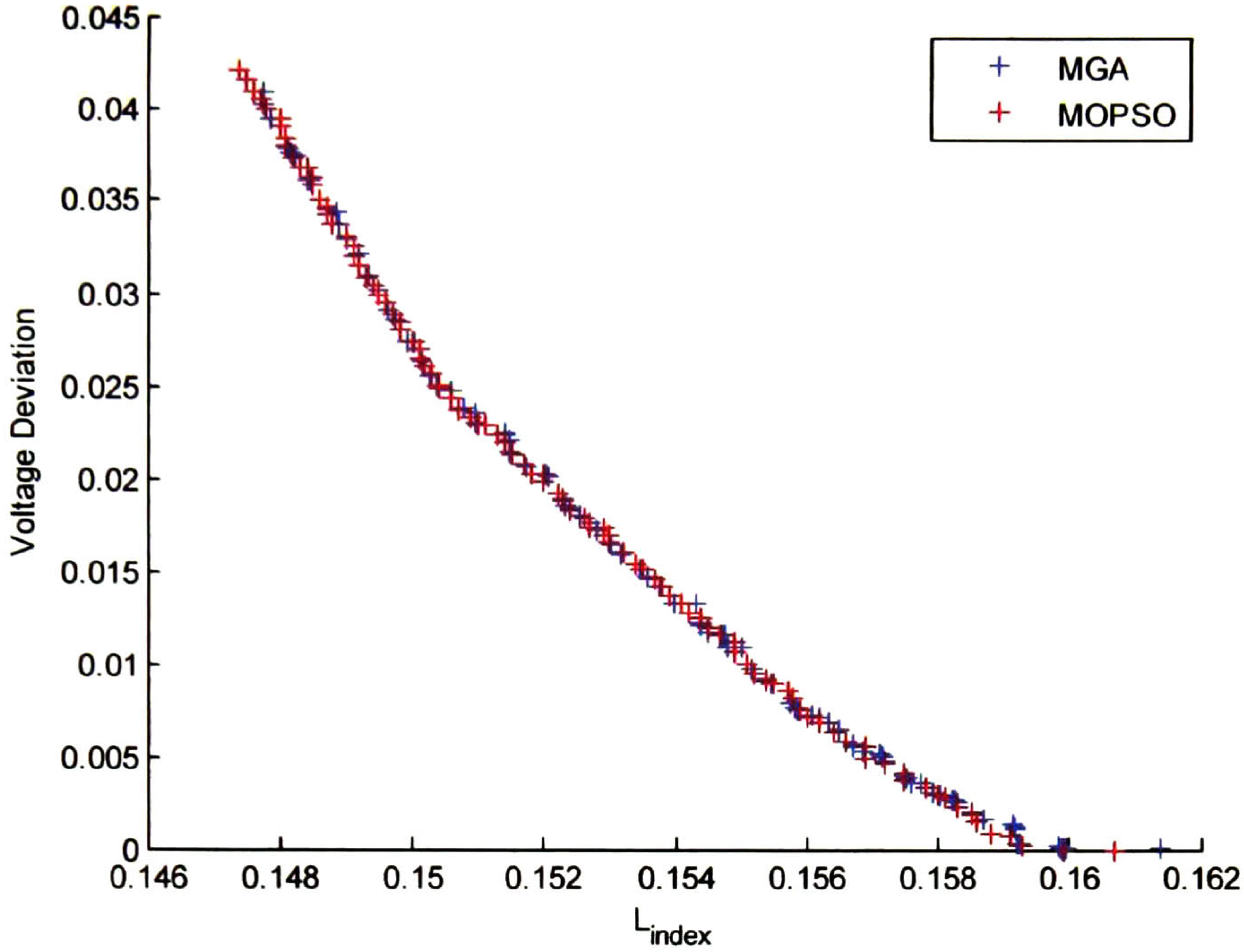

Figure 4.2: Comparison of the Pareto optimal fronts, case 1.1

It is evident that the Pareto-optimal front of these algorithms have good diversity characteristics of the non-dominated solutions. It can be observed that there is good agreement between MOPSO and MGA, which prove that the problem is efficiently solved by both techniques.

4.1.2 Case 1.2: System with constraints

In this case, the transmission line constraint is included. The load limit for line (8-9) is 80 MW. The control parameter settings used by MOPSO and MGA algorithms for this case are described in Table 4.3.

Table 4.3
Control parameter settings of MOPSO and MGA, case 1.2

	MOPSO	**MGA**
Parameter	**Setting**	
Population size	100	4
Mutation rate	0.65	0.03
Crossover rate		0.7
Adaptive grid subdivision		25
Second elitism		300
Maximum number of generationts	110	3000

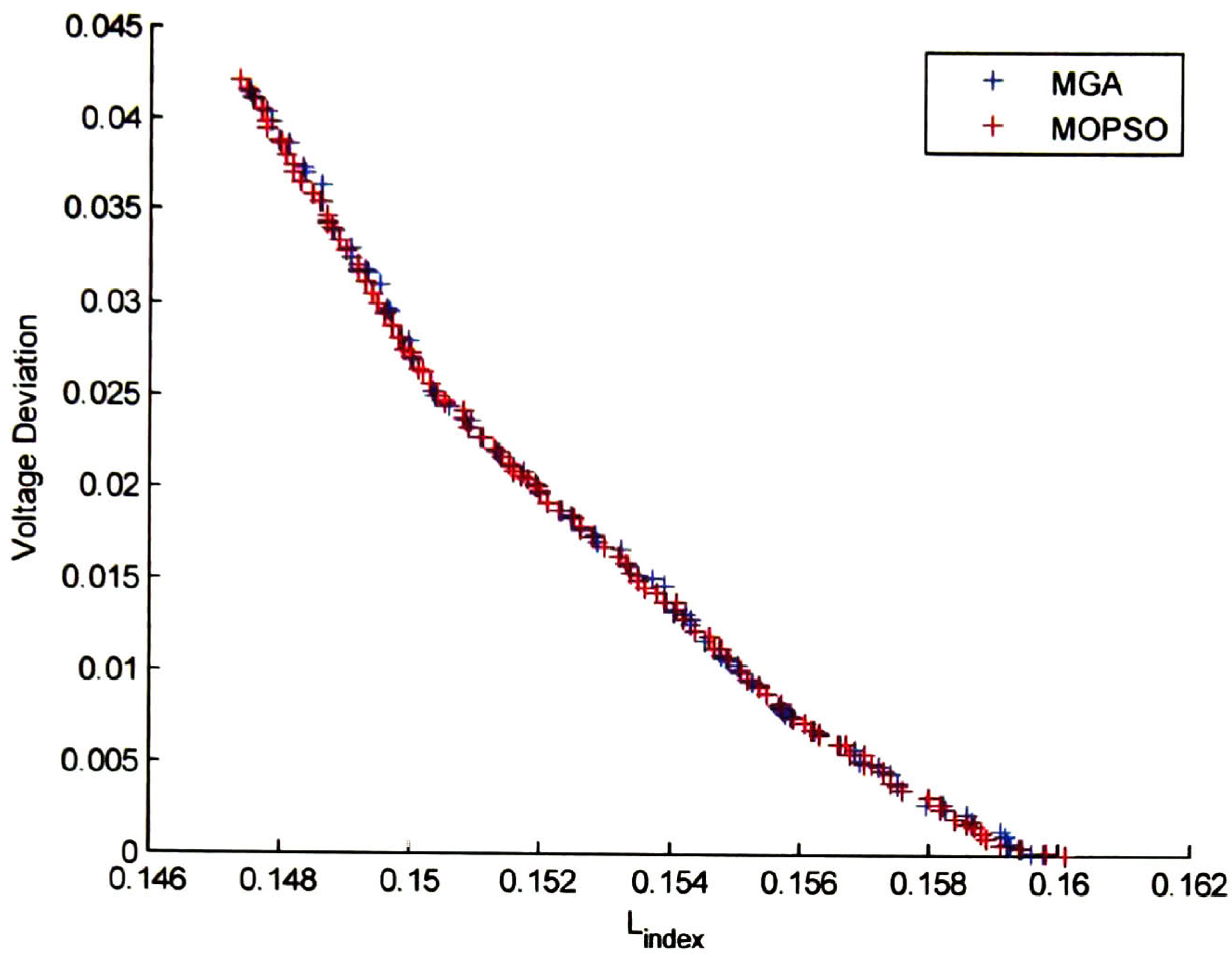

Figure 4.3: Comparison of the Pareto optimal fronts, case 1.2

The Pareto optimal fronts obtained by the MOPSO and MGA methods for this case are illustrated in figure 4.3. The closeness of the non-dominated solutions of these algorithms demonstrates good performance to solve the problem. The best solutions obtained out of ten runs applying the two different algorithms are given in Table 4.4.

Table 4.4
Comparison of the best solutions of MOPSO and MGA algorithms for case 1.2

Variable	Limits		MOPSO		MGA	
	lower	upper	Lindex	VD	Lindex	VD
Vg1(p.u)	1	1.05	1.05	1.0254	1.05	1.0278
Vg2(p.u)	1	1.05	1.05	1.0023	1.0492	1.0064
Vg3(p.u)	1	1.05	1.05	1.0055	1.0492	1.0008
Pg2 (MW)	100	240	101.356	118.3644	103.08	102.33
Pg3 (MW)	45	125	61.7054	79.6647	53.52	116.24
$S_{8\text{-}9}$(MW)	0	80	38.39	56.17	36.31	60.12
Ploss(MW)			2.609	3.184	2.641	3.642
Lindex			**0.1474**	0.1601	**0.1475**	0.1598
VD			0.0421	**0**	0.0414	**0**
SM			**1.90E-04**		2.40E-04	

Likewise, the SM's minimum value is obtained through the MOPSO. It can be observed that this technique has better diversity characteristics in both cases.

4.2 Case 2: IEEE 26-bus test system

This system has 26 buses, 46 branches, six generators, seven transformers, and nine shunt capacitors. The detailed data are given in [2]. The single-line diagram of this system is shown in figure. 4.4.

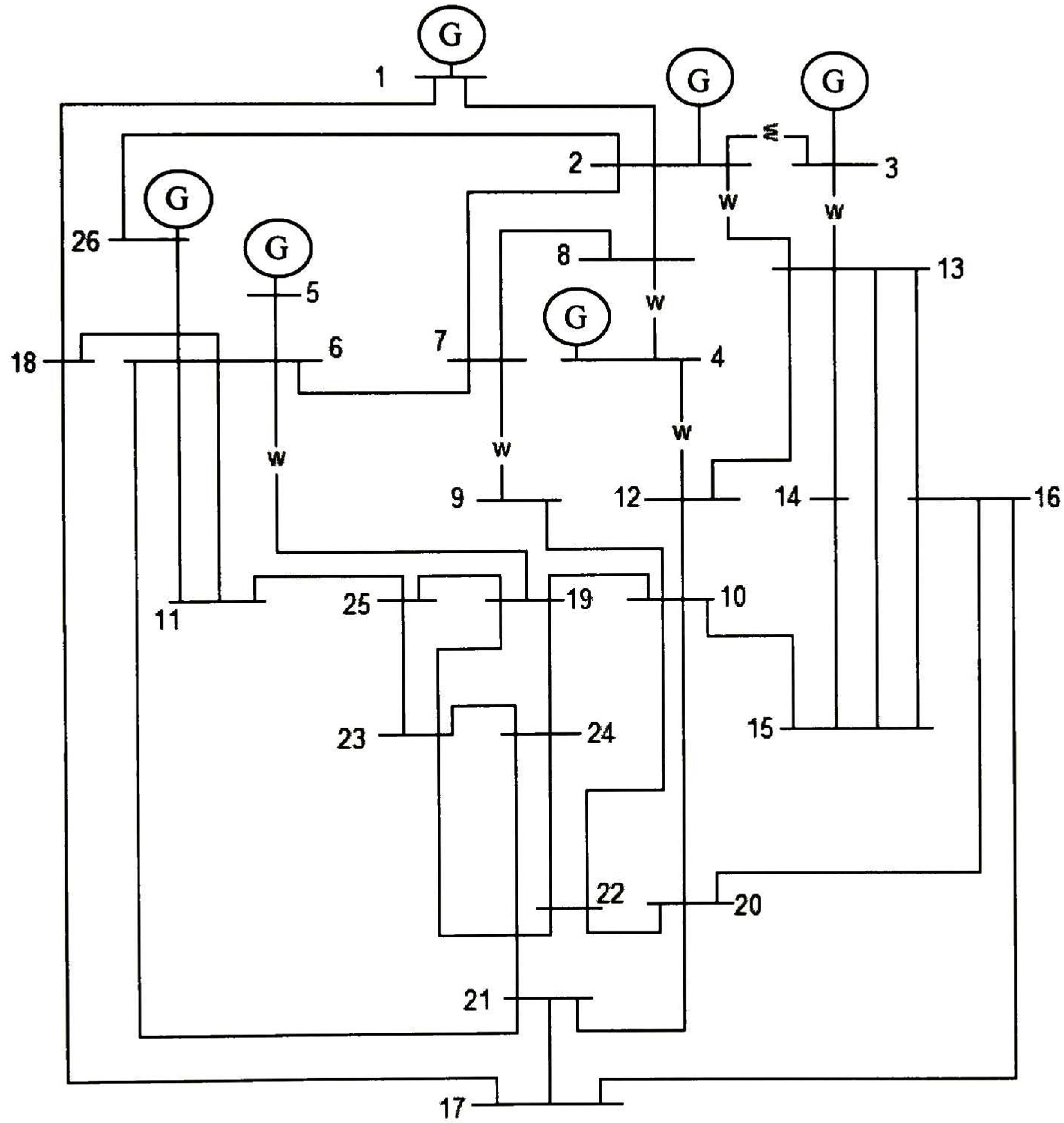

Figure 4.4: The 26-bus test system

For the purpose of comparison, the MOPSO and MGA techniques have been applied to the IEEE 26-bus system in two different tests, as follows.

4.2.1 Case 2.1: System without constraints

In this case study, network constraints are no considered. The 26-bus test system has six generators at buses 1, 2, 3, 4, 5 and 26, and therefore five active power outputs; seven transformers with off-nominal tap ratio in lines 2–3, 2–13, 3-13, 4-8, 4–12, 6-19 and 7–9. The number of control variables in this case is 18. The control parameter settings used by the MOPSO and MGA algorithms for this case are given in Table 4.5.

Table 4.5
Control parameter settings of MOPSO and MGA, case 2.1

	MOPSO	MGA
Parameter	**Setting**	
Population size	100	4
Mutation rate	0.7	0.008
Crossover rate	-	0.7
Adaptive grid subdivision		20
Second elitism		1330
Maximum number of generationts	220	13300

The Pareto optimal fronts for this case obtained by MOPSO and MGA methods for this case are illustrated in figure 4.5.

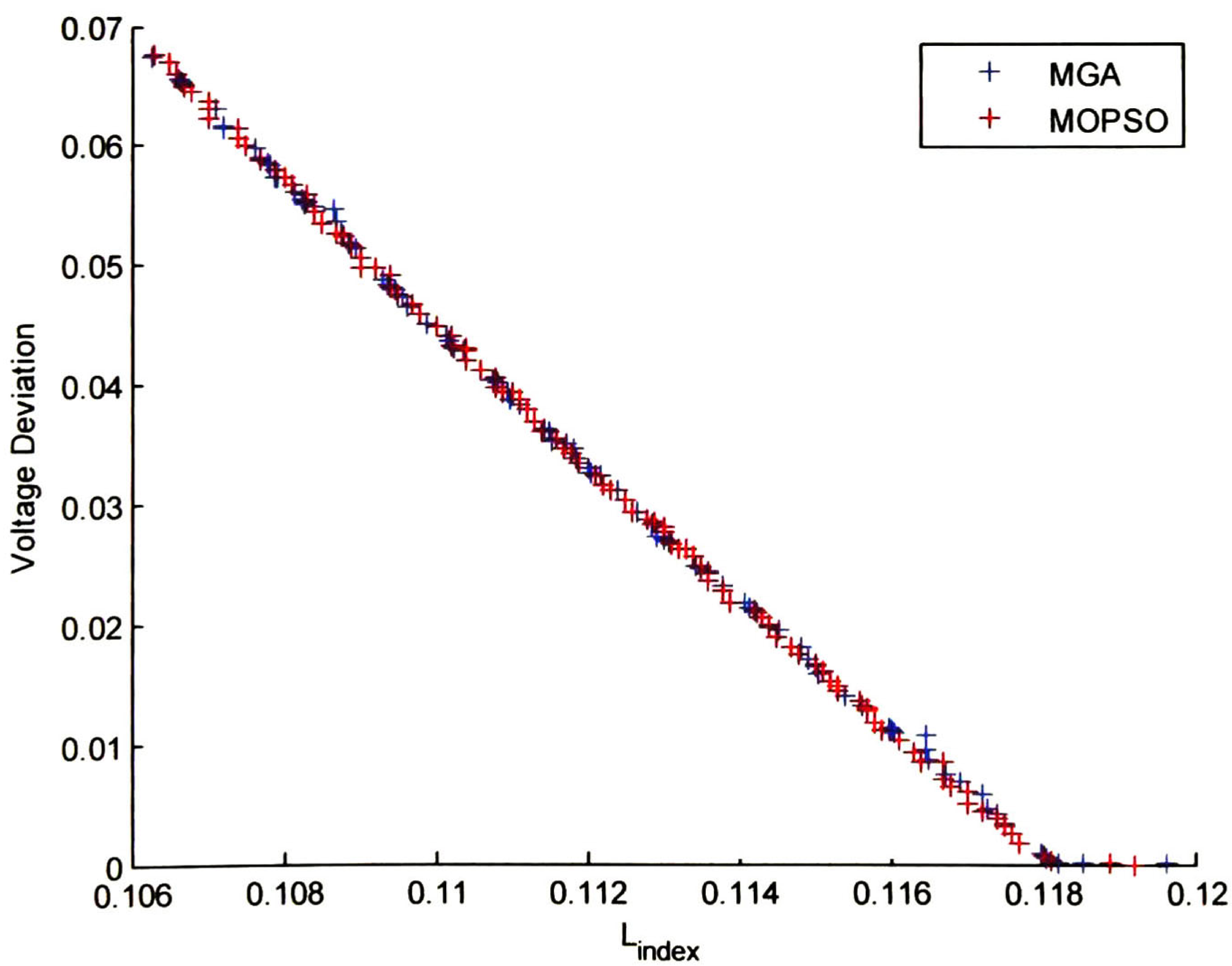

Figure 4.5 Comparison of the Pareto optimal fronts, case 2.1

It can be observed that there is good agreement between MOPSO and MGA, which prove that the problem is efficiently solved by both techniques. The best solutions obtained out of ten runs applying the two different algorithms are summarized in Table 4.6.

Table 4.6
Comparison of the best solutions of MOPSO and MGA algorithms for case 2.1

Variable	Limits		MOPSO		MGA	
	lower	upper	Lindex	VD	Lindex	VD
Vg1(p.u)	1	1.05	1.0498	1.0445	1.05	1.0429
Vg2(p.u)	1	1.05	1.0416	1.0322	1.05	1
Vg3(p.u)	1	1.05	1.05	1.029	1.05	1
Vg4(p.u)	1	1.05	1.0416	1.0196	1.05	1
Vg5(p.u)	1	1.05	1.0487	1.0386	1.0429	1.0286
Vg26(p.u)	1	1.05	1.0384	1.0217	1.0357	1.0286
Pg2 (MW)	50	100	80.8557	74.6438	51.2147	58.198
Pg3 (MW)	20	80	34.6603	50.3352	44.5465	34.9139
Pg4 (MW)	90	150	122.5639	128.6227	141.818	139.701
Pg5(MW)	280	400	333.5387	306.2301	307.284	329.295
Pg26(MW)	50	90	74.4517	70.5222	70.9426	69.3211
T3(p.u)	0.9	1.1	0.9138	1.0862	0.9129	1.0613
T6(p.u)	0.9	1.1	0.9	1.0754	0.9065	1.0613
T8(p.u)	0.9	1.1	0.9007	0.9675	0.9	0.9774
T9(p.u)	0.9	1.1	1.0318	1.0295	1.0613	0.9968
T10(p.u)	0.9	1.1	0.9041	0.9069	0.9065	0.9323
T15(p.u)	0.9	1.1	0.9	0.9076	0.9	0.9065
T18(p.u)	0.9	1.1	0.9	1.0591	0.9065	0.9323
Ploss(MW)			23.33	18.31	23.25	17.3
Lindex			**0.1063**	0.1192	**0.1063**	0.1196
VD			0.0676	**0**	0.0675	**0**
SM			**2.60E-04**		3.50E-04	

It can be observed that the MOPSO and MGA converge practically to the same solutions for the voltage stability and voltage deviation functions. It is evident that the non-dominated solutions obtained by these algorithms have good diversity characteristics. However, the results show that MOPSO exhibits better diversity characteristics.

4.2.2 Case 2.2: System with constraints

In this case, the transmission line constraint is included. The maximum allowable load through the line (10-12) is 108 MW. The control parameter settings used by MOPSO and MGA algorithms for this test are given in Table 4.7.

Table 4.7
Control parameter settings of MOPSO and MGA, case 2.2

	MOPSO	MGA
Parameter	**Setting**	
Population size	100	4
Mutation rate	0.7	0.008
Crossover rate	-	0.7
Adaptive grid subdivision	-	25
Second elitism	-	1800
Maximum number of generationts	100	18000

The distribution of the non-dominated solutions in the Pareto-optimal front using the proposed MOPSO and MGA algorithms is shown in figure 4.6. It can be observed that the non-dominated solutions of the proposed MOPSO approach have better diversity characteristics.

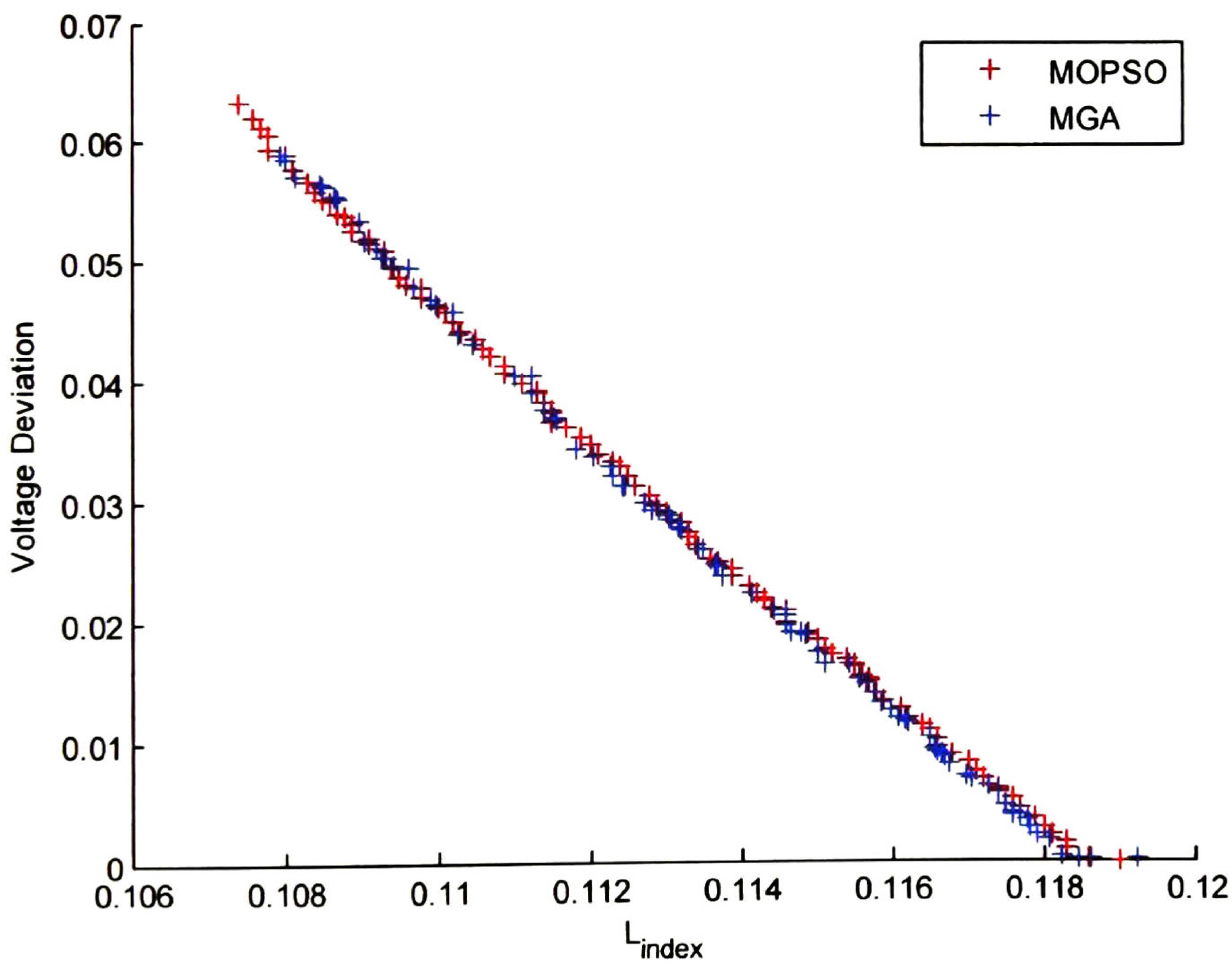

Figure 4.6: Comparison of the Pareto optimal fronts, case 2.2

The best solutions obtained out of ten runs applying the proposed algorithms are summarized in Table 4.8.

Table 4.8
Comparison of the best solutions of MOPSO and MGA algorithms for case 2.2

Variable	Limits		MOPSO		MGA	
	lower	upper	Lindex	VD	Lindex	VD
Vg1(p.u)	1	1.05	1.0493	1.0497	1.05	1.05
Vg2(p.u)	1	1.05	1.0285	1.0322	1.05	1.0429
Vg3(p.u)	1	1.05	1.0293	1.0265	1.0357	1.0071
Vg4(p.u)	1	1.05	1.0406	1.0349	1.0429	1.0143
Vg5(p.u)	1	1.05	1.0202	1.0217	1.0357	1
Vg26(p.u)	1	1.05	1.0416	1.0413	1.0214	1.0214
Pg2 (MW)	50	100	83.7284	81.8594	92.0706	60.0232
Pg3 (MW)	20	80	41.7733	43.7334	58.574	20.7984
Pg4 (MW)	90	150	102.3267	100.2135	93.1644	101.83
Pg5(MW)	280	400	390.295	385.5824	364.988	397.788
Pg26(MW)	50	90	59.8118	58.4668	88.779	51.1038
T3(p.u)	0.9	1.1	0.9	1.0628	0.9516	1.0484
T6(p.u)	0.9	1.1	0.902	1.0125	0.9	0.9323
T8(p.u)	0.9	1.1	0.9041	1.0696	0.9065	1.0807
T9(p.u)	0.9	1.1	0.9698	1.0056	0.971	1.0355
T10(p.u)	0.9	1.1	0.9025	0.9453	0.9129	0.9323
T15(p.u)	0.9	1.1	0.901	0.9016	0.9	0.9
T18(p.u)	0.9	1.1	0.9	0.9451	0.9194	0.9194
$S_{10\text{-}12}$(MW)	0	108	107.49	106.71	107.39	105.86
Ploss(MW)			23.99	20.77	20.75	23.36
Lindex			**0.1074**	0.119	**0.1079**	0.1192
VD			0.0633	**0**	0.0591	**0**
SM			**2.50E-04**		3.80E-04	

It is noteworthy that all control and state variables remain within their permissible limits in both case studies for the IEEE 26-bus test system. Also, results show that the MOPSO attains the minimum value of SM and therefore the best diversity characteristics in both case studies of the IEEE 26-bus test system.

The closeness of the non-dominated solutions of the MOPSO and MGA demonstrates good performance to solve the problem for this test system.

4.3 Case study 3: Ten generator test system

This system contains 10 generating units, 39 buses and 46 transmissions lines. The transmission lines' parameters and load data are taken from [3]. The single-line diagram of this system is shown in figure. 4.7.

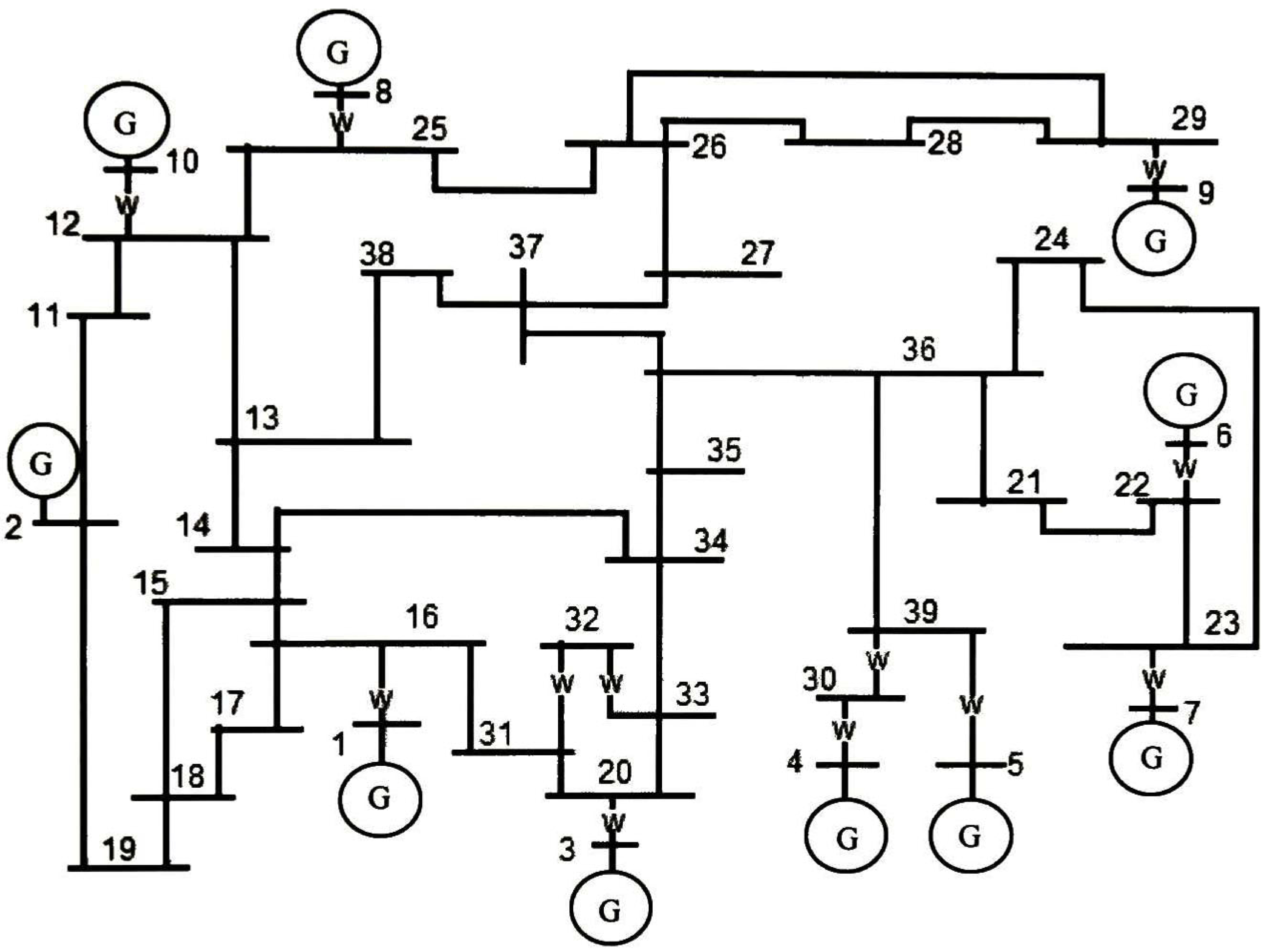

Figure 4.7: Ten generator test system

For the purpose of comparison, the proposed MOPSO and MGA algorithms have been applied to the ten generator test system in two different tests, as follows.

4.3.1 Case 3.1: System without constraints

In this case, network constraints are no considered. The system has a total of 31 control variables, being: ten generators and therefore nine active power outputs and twelve transformers with off-nominal tap. The control parameter settings used by MOPSO and MGA algorithms for this case are displayed in Table 4.9.

Table 4.9
Control parameter settings of MOPSO and MGA, case 3.1

	MOPSO	MGA
Parameter	**Setting**	
Population size	100	4
Mutation rate	0.65	0.004
Crossover rate		0.7
Adaptive grid subdivisión	-	25
Second elitism		2500
Maximum number of generationts	180	25000

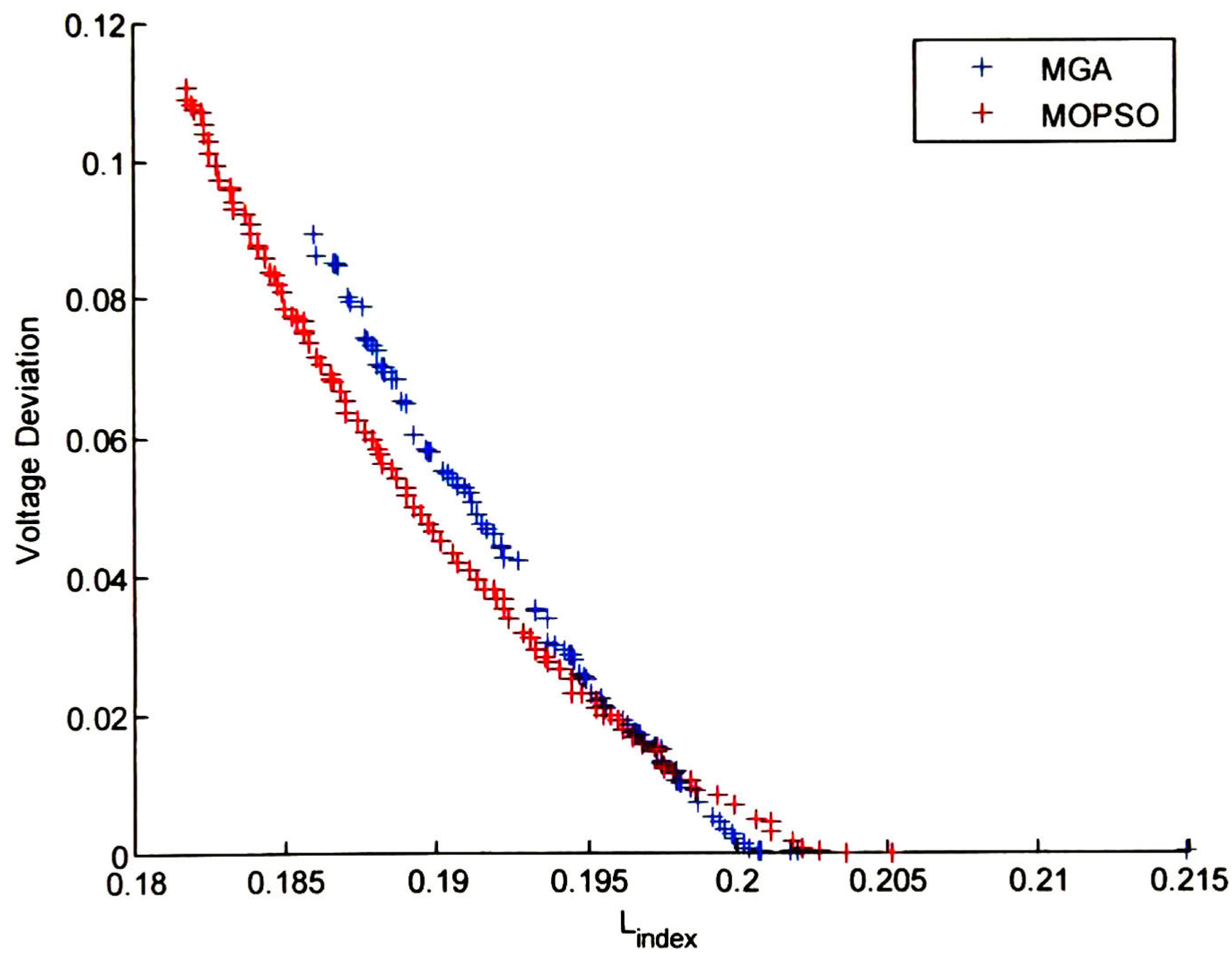

Figure 4.8: Comparison of the Pareto optimal fronts, case 3.1

The distribution of the non-dominated solutions in the Pareto-optimal front using the proposed MOPSO and MGA algorithms for this case is depicted in figure 4.8. It is evident that the non-dominated solutions of the MOPSO approach have better diversity characteristics and better non-dominated solutions.

In this case, the MGA's performance is degraded with the increased problem complexity. The best voltage stability and best voltage deviation solutions obtained out of ten runs are given in Table 4.10.

Table 4.10
Comparison of the best solutions of MOPSO and MGA algorithms for case 3.1

Variable	Limits		MOPSO		MGA	
	lower	upper	Lindex	VD	Lindex	VD
Vg1(p.u)	0.98	1.05	1.05	1.0307	1.04	1.04
Vg2(p.u)	0.98	1.05	1.0499	1.0395	1.05	1.05
Vg3(p.u)	0.98	1.05	1.0499	1.0441	1.05	1.05
Vg4(p.u)	0.98	1.05	1.0492	1.0326	1.05	1.05
Vg5(p.u)	0.98	1.05	1.05	1.0473	1.05	1.05
Vg6(p.u)	0.98	1.05	1.0484	1.0472	1.05	1.05
Vg7(p.u)	0.98	1.05	1.0489	1.0306	1.04	1.04
Vg8(p.u)	0.98	1.05	1.05	1.0169	1.03	1.03
Vg9(p.u)	0.98	1.05	1.0454	1.0247	1.05	1.05
Vg10(p.u)	0.98	1.05	1.049	1.0281	1.03	1.03
Pg2 (MW)	950	1400	1368.121	1348.8489	1381.09	1381.09
Pg3 (MW)	550	750	595.4809	596.557	567.756	568.268
Pg4 (MW)	450	600	524.3367	519.9305	491.906	487.804
Pg5(MW)	500	750	698.4651	652.4778	560.472	560.472
Pg6(MW)	450	600	450	462.288	542.593	542.593
Pg7(MW)	400	600	405.1213	462.5501	507.779	526.066
Pg8(p.u)	400	650	624.4318	577.7122	542.827	507.83
Pg9(p.u)	500	900	771.2427	779.0989	817.711	582.289
Pg10(p.u)	150	350	258.6252	274.6497	341.949	158.051
T35(p.u)	0.9	1.1	1.1	1.0738	1.1	0.9
T36(p.u)	0.9	1.1	1.0966	1.0017	1.0871	0.9194
T37(p.u)	0.9	1.1	0.9183	0.9389	0.9323	1.0677
T8(p.u)	0.9	1.1	1.0264	1.0138	0.9903	0.9065
T39(p.u)	0.9	1.1	1.1	1.0761	1.0548	0.9387
T40(p.u)	0.9	1.1	1.1	1.065	1.029	1.0226
T41(p.u)	0.9	1.1	1.1	0.9365	1.0807	0.9516
T42(p.u)	0.9	1.1	1.1	1.0768	1.1	0.9065
T43(p.u)	0.9	1.1	1.1	1.0852	1.0871	0.9258
T44(p.u)	0.9	1.1	1.0751	0.9002	1.0871	1.0613
T45(p.u)	0.9	1.1	1.0953	0.9154	1.1	1.1
Ploss(MW)			38.51	50.89	36.28	38.88
Lindex			**0.1817**	0.2036	**0.186**	0.2148
VD			0.1091	**0**	0.0896	**0**
SM			**4.82E-04**		0.0014	

The results show that the MOPSO obtained the best solutions for the voltage stability and voltage deviation functions as well as the best diversity characteristics in the non-dominated solutions.

4.3.2 Case 3.2: System with constraints

This case incorporates the constraints on the voltage magnitudes of the load buses as well as the transmission line loadings. The maximum allowable load through the line (16-17) is 415 MW and through the line (21-22) is 585 MW. The limits of the voltage magnitudes of the load buses are: $0.975 \le V_{Li} \le 1.025$ for *i =14... 18.*

The control parameter settings used by MOPSO and MGA algorithms for this case are given in Table 4.11. The diversity of the Pareto optimal fronts are compared in figure 4.9.

Table 4.11
Control parameter settings of MOPSO and MGA, case 3.2

	MOPSO	MGA
Parameter	**Setting**	
Population size	100	4
Mutation rate	0.65	0.004
Crossover rate	-	0.7
Adaptive grid subdivisión	-	25
Second elitism	-	3000
Maximum number of generationts	150	30000

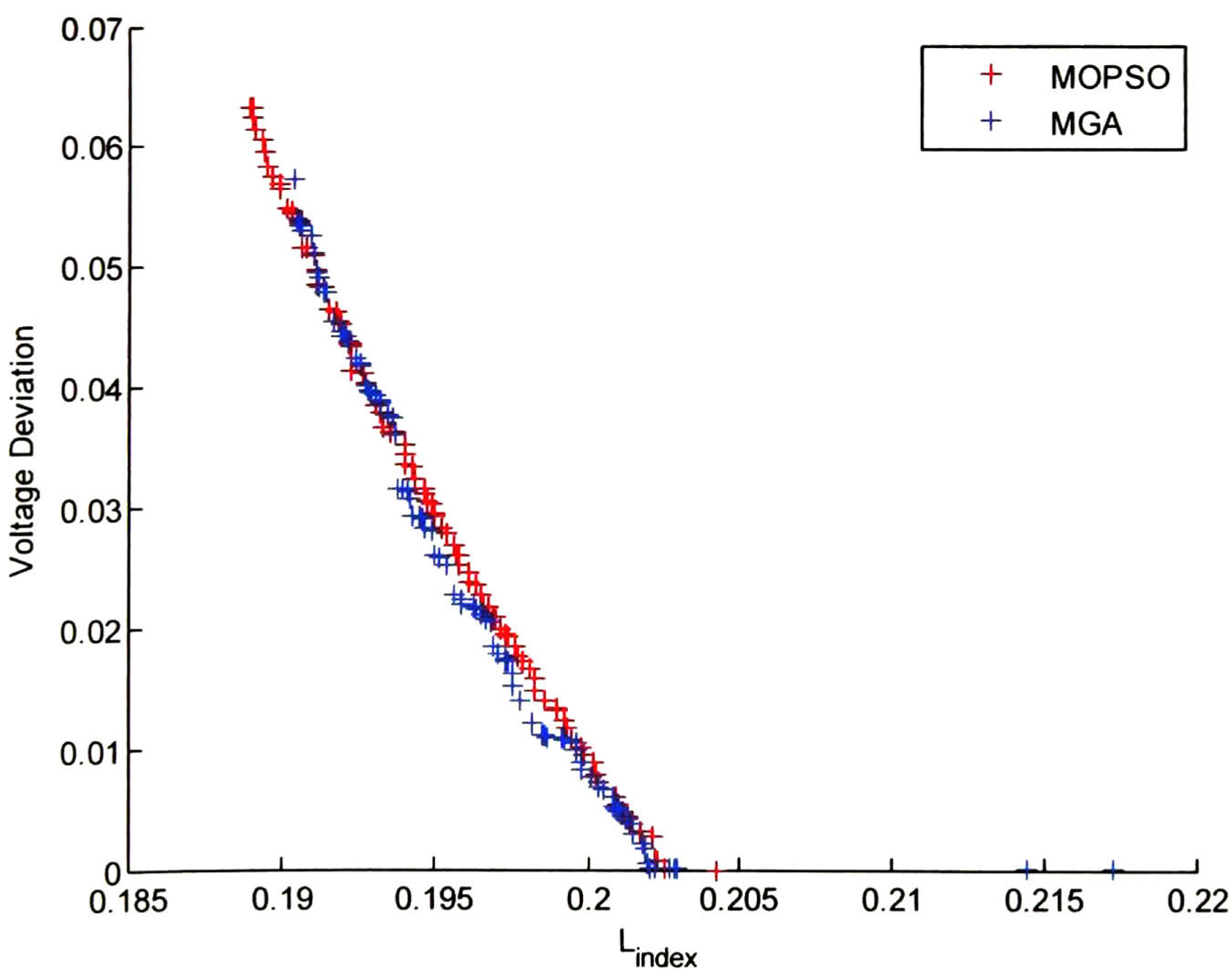

Figure 4.9: Comparison of the Pareto optimal fronts, case 3.2

Table 4.12
Comparison of the best solutions of MOPSO and MGA algorithms for case 3.2

Variable	Limits		MOPSO		MGA	
	lower	upper	Lindex	VD	Lindex	VD
Vg1(p.u)	0.98	1.05	1.0461	1.0394	1.05	1.05
Vg2(p.u)	0.98	1.05	1.0399	1.0485	1.02	1.02
Vg3(p.u)	0.98	1.05	1.0468	1.0474	1.05	1
Vg4(p.u)	0.98	1.05	1.0436	1.0457	1.04	0.99
Vg5(p.u)	0.98	1.05	1.0493	1.0495	1.05	1.04
Vg6(p.u)	0.98	1.05	1.0485	1.0486	1.04	0.99
Vg7(p.u)	0.98	1.05	1.0456	1.049	1.04	0.99
Vg8(p.u)	0.98	1.05	1.0479	1.0132	1.03	1.05
Vg9(p.u)	0.98	1.05	1.0481	1.0423	1.05	1.02
Vg10(p.u)	0.98	1.05	1.0494	1.0288	1.04	1.02
Pg2 (MW)	950	1400	1287.7313	1321.4804	1392.33	1325
Pg3 (MW)	550	750	562.0143	570.7668	617.94	648.514
Pg4 (MW)	450	600	542.9121	546.6559	474.089	535.918
Pg5(MW)	500	750	512.3916	521.3608	570.795	525.445
Pg6(MW)	450	600	584.4023	588.0919	503.69	590.597
Pg7(MW)	400	600	577.5718	577.488	470.272	446.638
Pg8(p.u)	400	650	540.9562	538.4097	547.923	595.418
Pg9(p.u)	500	900	884.7529	851.0744	842.065	761.717
Pg10(p.u)	150	350	185.4424	188.9824	272.66	217.953
T35(p.u)	0.9	1.1	1.0593	1.0619	1.0936	0.9774
T36(p.u)	0.9	1.1	1.1	1.0318	1.0807	1.0161
T37(p.u)	0.9	1.1	0.9209	0.9294	0.9258	0.9065
T8(p.u)	0.9	1.1	0.9114	0.9164	0.9194	1.0807
T39(p.u)	0.9	1.1	1.0867	1.0274	1.0613	0.9903
T40(p.u)	0.9	1.1	1.1	0.9	1.0548	1.0355
T41(p.u)	0.9	1.1	1.0492	1.0331	1.1	1.0226
T42(p.u)	0.9	1.1	1.0987	1.0842	1.1	1.0936
T43(p.u)	0.9	1.1	1.1	0.9942	1.0807	0.9
T44(p.u)	0.9	1.1	0.9043	0.9102	0.9839	1.0871
T45(p.u)	0.9	1.1	1.0684	1.0688	0.9903	1.029
Ploss(MW)			43.18	46.19	39.2	43.95
Lindex			**0.1889**	0.2042	0.1904	**0.2172**
VD			0.0634	**0**	0.0574	**0**
SM			**4.70E-04**		0.0014	

The best solutions obtained by the MOPSO and MGA are compare in Table 4.12. The results show that MOPSO obtained the minimum value of SM and therefore has better diversity characteristics, regarding the MGA method in both cases.

The value of the state variables obtained by the MOSPO and MGA for this case is summarized in Table 4.13.

Table 4.13
Comparison of the state variables of MOPSO and MGA algorithms for case 3.2

Variable	Limits		MOPSO		MGA	
	lower	upper	Lindex	VD	Lindex	VD
VL14(p.u)	0.975	1.025	1.024	0.982	1.025	0.993
VL15(p.u)	0.975	1.025	1.019	0.994	1.016	1.003
VgL16(p.u)	0.975	1.025	1.021	0.997	1.017	1.008
VL17(p.u)	0.975	1.025	1.01	0.988	1.006	0.996
VL18(p.u)	0.975	1.025	1.01	0.987	1.004	0.9954
$S_{16\text{-}17}$(MW)	0	415	352.071	341.72	328.62	350.62
$S_{21\text{-}22}$(MW)	0	585	562.37	561.87	453.48	492.14

It is worth noting that all control and state variables remained within their permissible limits. The results show that the MOPSO can solve the problem more efficiently compared to MGA for both case studies of the ten generator test power system.

4.4 Remarks

In this chapter, two multi-objective evolutionary algorithms have been compared and successfully applied to the reactive power and voltage control or volt/var control (VVC) optimization problem. The problem has been formulated as a multi-objective optimization problem with voltage deviation and voltage stability objectives.

In order to demonstrate the effectiveness of the proposed algorithms to solve the VVC problem, these algorithms have been applied in three power systems with different complexity. The Spacing Metric described in a previous chapter has been used to compare the diversity among the techniques.

4.5 References

[4.1] Peter W. Sauer, M. A. Pai. *Power System Dynamics and Stability*. Prentice Hall. 1997.

[4.2] S. Hadi. *Power System Analysis*. New York: McGraw-Hill, 2004.

[4.3] K. R. Padiyar. *Power System Dynamics: stability & control*. Anshan, first edition. 2004.

Conclusions and future work

The conventional power system optimal power flow (OPF) objective is to obtain a startup and shutdown schedule of generating units to meet the required demand at minimum production cost, satisfying units' and system's operating constraints, by adjusting the power system control variables. Such ones include the active power supplied by each available generator, the tap position of transformers, and the reactive power generation of the VAR sources.

The advanced OPF takes into account security constraints. It has been formulated as a nonlinear, nonconvex, and large-scale, mixed-integer, optimization problem. Several techniques have been used for solving such one, for instance: optimization methods involving derivative-based techniques, and heuristic optimization techniques such as genetic algorithms. Likewise, very important contributions on optimization applications in power systems have been published.

The aim of this work is to propose a multi-objective formulation to the reactive power and voltage control or volt/var control (VVC) optimization problem. The selected objectives are the bus voltage deviations and one measure for the voltage stability. Likewise, the evolutionary techniques: MOPSO and MGA are proposed as effective tools to solve the multi-objective VVC problem. The heuristic strategies allow to reduce the penalty functions' bad-conditioning problem.

In this thesis, an approach for the solution of the OPF with security constraints through the use of heuristic strategies is proposed. Conventionally, the PSO handles constraints by penalizing the objective function. In this work, a multi-objective formulation is used. Results confirm the potential of multi-objective evolutionary algorithms to solve nonlinear constrained multi-objective optimization problems like the VVC problem.

In order to demonstrate the effectiveness and robustness of the proposed algorithms to solve the VVC problem, the algorithms have been applied in three power systems with different complexity. Results are very satisfactory from the power system's operation point of view.

The results presented show that the proposed approaches achieved acceptable results in the VVC multi-objective optimization problem. The non-dominated solutions in the obtained Pareto-optimal fronts are well distributed and exhibit satisfactory diversity characteristics. It was also noted that the minimum spacing metric was attained by the MOPSO, which proves that the non-dominated solutions of this approach have better diversity characteristics in all analyzed cases.

It is evident that the non-dominated solutions obtained by the MGA are well distributed and have satisfactory diversity characteristics for the nine-bus and IEEE 26- bus test systems. However, when the system increases its complexity the performance of MGA is degraded.

Based on the comparisons and discussions, it can be concluded that MOPSO has better robustness than MGA technique for the VVC optimization problem, since the Pareto optimal fronts with satisfactory diversity characteristics have been obtained in the studies.

Contributions

A complete model that allows to include restrictions associated to the context of steady state of an electric power system is presented, which take into account inequality constraints to bound important variables on the power system operation.

The proposed methodology is able to manage the outlined optimization problem, which is highly restricted, through a multi-objective formulation.

The used methods are able to solve objective functions and no-convex restrictions, what allows the handling of realistic models in the components of the electric power systems. This is an advantage respect to many of the conventional optimization problems that are based on simplifications on modeling.

For their capacity, the proposed multi-objective formulation exhibits a bigger probability to find the global solution, compared to the conventional optimization techniques that are dependent on the starting point, and with an inappropriate election of this, they can be caught in a local minimum.

At the end of the process, the proposed methodology is able to find a group of feasible solutions close to the global, what implies an important advantage on the conventional optimization techniques, in the sense that they provide to the power system operator with choices. This is useful for those cases where it is not possible to implement the best solution for any reason.

Recommendations and future work

One of the main features of the evolutionary algorithms is to have parameters that control their performance. Choosing the value of the different parameters of an evolutionary algorithm is crucial for good performance. There is also evidence that using different values of the parameters in the various stages of the evolutionary process seem to be something appropriate.

The algorithms used in this work used a fixed mutation factor. Thus, a recommendation based on the above premise is to modify the program code to include a random mutation factor for each mutation point.

Another multi-objective evolutionary algorithms' limitation is the computation time when the problem exhibits a high degree of complexity. As future work, to consider the use of parallel computation using a network of computers to reduce the execution time in order to improve the algorithm's performance is proposed.

The inclusion of a dynamic formulation to alleviate power system stability's problems must be taken into account into future works.

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Despacho óptimo de reactivos mediante una formulación multi-objetivo

Reactive optimal management by a multi-objective formulation

del (la) C.

Miguel Alfonso de Jesús MEDINA LÓPEZ

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